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WIRELESS SECTION: CHAIRMAN'S ADDRESS

By E. B. MOULLIN, M.A., Sc.D., Member.*

"THE MOLECULAR NATURE OF A DIELECTRIC"

(*Address received 7th December*, 1939.)

For this Chairman's Address I had planned to give a lecture on the nature of dielectrics and to enliven it with some models and demonstrations. I had hoped to amuse an audience for an hour by describing molecular models in a rather free-and-easy way. I dare say much of what I should have said would have been familiar to most electrical engineers, but if it had been new to some I think it would have opened for them a way of approach as fascinating as it was pictorial, as severely practical as it was eminently rational. Now that war has come the lecture with its lighter touches and asides cannot be given and the Address can only be printed in all its academic seriousness.

(1) DIELECTRIC CONSTANT AND ITS MOLECULAR INTERPRETATION

(a) A first rough model.

It has been known for the larger part of a couple of centuries that the capacitance of a condenser is increased considerably by filling its air space with such substances as paraffin oil, glass or indiarubber: this experience leads directly to a definition of "dielectric constant." We may be content to accept the fact and call it another unexplainable mystery of nature, or we may prefer to ease our memories and try to comprehend it in terms of the inverse square law of force. If we succeed we shall have only this difficult and, in a sense, improbable hypothesis as the whole basis of electrical science. If we can tolerate only the inverse square law, then we have to find some more of our "invention, electricity" in the insulator. It is as though the condenser plates were closer together than they really are, or as though there were some invisible pieces of conducting material embedded in the insulator. Such conducting material would, *ipso facto*, contain the needed electricity. Science has a title to our respect from its results and not from the hypotheses by which it arrives at these results: the hypotheses are always more or less laughable, and it may take long to stop laughing at a new one and see what it is capable of producing.

Instead of scorning the idea of conducting particles we will render them invisible by having them rather small and sparsely scattered, and as a first shot will simplify mathematical tedium by making them spherical. As we focus our attention on one of them we see a conducting sphere in a sensibly uniform electric field which is due jointly to the electricity on the condenser plates and the separation of electricity in all the other spheres. Here is an old problem, and we know that the separated or induced charges on the sphere endow it with an electric moment which is equal to the volume of the sphere multiplied by the uniform electric field in which it is placed. It is then a very simple matter to calculate the capacitance of a condenser having n such spheres scattered sparsely in every unit volume, and we find readily that the dielectric constant resulting from their presence in the air space is given by the equation

$$\kappa\dagger = \frac{1 + (\text{twice the total volume of spheres})}{1 - (\text{the total volume of spheres})} \quad . \quad (1)$$

Equation (1) shows that the net result does not depend on the radius of the spheres, which need not, therefore, be all equal: the analysis, however, does rely on the total volume being much less than unity. We can now say that if some material has a dielectric constant of 2 it is as though one-third of it consisted of conducting spheres. Obviously this is not true, at any rate in the sense in which we have pictured it. Let us see if perhaps the trouble is not that the picture was essentially impossible but that it was too naïve.

(b) A second model, this time molecular.

We have been driven to realize that our invented stuff, electricity, cannot be subdivided indefinitely, and have had to cumber the first simple hypothesis by making it consist of identical fragments called electrons. This concession has paved the way to the picture that molecules differ in kind largely because they contain a different number of electrons disposed like planets round a positive charge, called a proton, as sun. In Fig. 1 we draw a naïve picture of such an atom (hydrogen). Here P is the proton and e the electron in a circular orbit of radius a, the plane of the ecliptic being perpendicular to an electric field ϵ. The electron attracts the proton with a force e^2/a^2, and vice versa. What maintains the radius a or the perpendicularity of the ecliptic to the electric field we refuse to inquire here. The electric field exerts an upward force ϵe on the proton and a like downward force on the electron, and this must cause the proton to move away from the centre of the circular orbit. If it moves a distance x so small that the distance between the two charges is increased insensibly, the force between them will be unchanged but will now have a component perpendicular to the orbit equal to

$$\frac{e^2}{a^2} \times \frac{x}{a}$$

* University of Oxford.

† See List of Symbols on page 127.

which is equilibrated by the external stretching force $e\epsilon$ provided by the superposed field. The atom now has an electric moment in the direction of the applied field, and its value is

$$M = ex = a^3\epsilon \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Had the atom been a conducting sphere of radius a, it would also have had a moment of the same magnitude. Hence equation (1) would still be valid, though now we should write total volume of molecules vice total volume of spheres.

Without, however, relying on the direct observations of X-ray crystallography, of which we shall speak later, we are able to deduce the volume of a molecule by interpreting certain simple measurements in terms of the kinetic theory of gases. Hence we are able to test the

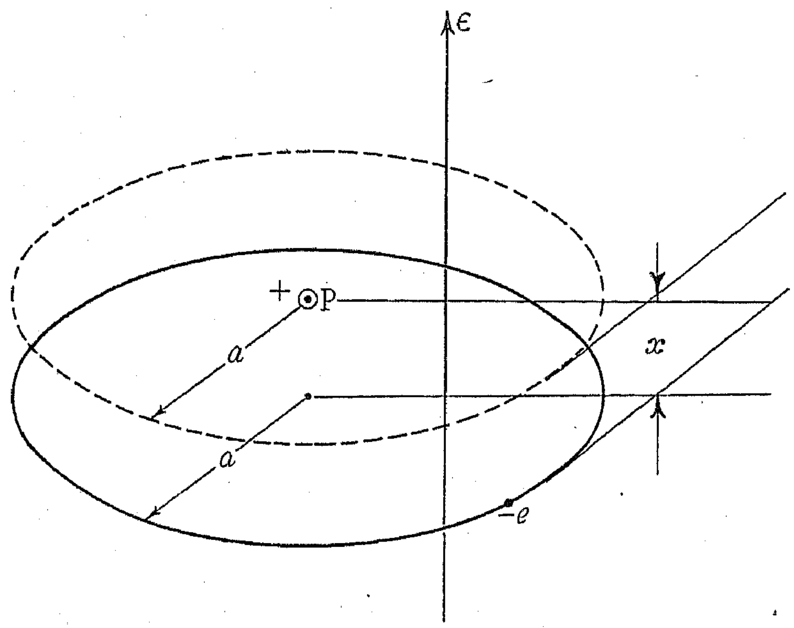


Fig. 1.—Distortion of hydrogen atom in an electric field.

enlarged interpretation of equation (1), and can compare the molecular volume as deduced from measurements of dielectric constant with the molecular volumes as deduced from measurements which are in no way electrical: let this ratio be called z, where z would be unity if the model were perfect. The values of z for four very different molecules are shown in Table 1. The mean value is 0·599: so close an agreement is remarkable and encouraging.

Table 1

Molecule	z
H_2	0·488
A	0·52
C_2H_4	0·67
C_5H_{12}	0·72

A similar table for 28 ions of inert-gas structure has been compiled by F. C. Frank,[1] and in this the mean value of x is 0·701: the range is from 0·166 to 1·5, with seven cases where the value is greater than unity. Thus we see that the dominant factor in determining the dielectric constant of a substance is what we engineers would call the " space factor " of the material packed between the condenser plates: another factor is the z value of that material. Recent years have seen the introduction of ceramic-type materials having large dielectric constants. One well known material is composed largely of titanium dioxide. Now the value of z for the titanium ion is 1·04 and for the O_2 ion is 1·2, and hence the molecule is inherently capable of giving a dielectric constant larger than most. The material is polymorphic and exists in three forms: the importance of space factor with a given molecule is well illustrated by Table 2.*

The model pictured in Fig. 1 operates by virtue of the proton being strained by the field out of the plane of the electron orbit: the action depends essentially on the valency electrons, and the mechanism is often called electronic polarization and will occur in each atom of a polyatomic molecule. We picture chemical union as due to exchange or sharing of these valency electrons between the two or more atoms of a molecule, and such exchange

Table 2

	Rutile	Brookite	Anatase
Density	4·21	4·11	3·87
Dielectric constant ..	114	87	48

causes each component atom to be charged or ionized: it is, however, exchange and not robbery, so that the molecule is not charged as a whole any more than its component atoms were charged before the union occurred, though they may be said to be charged after the union. The electrical centres of such a molecule may be strained apart by an electric field, and this would give rise to a polarization which is additional to the electronic polarization of the dismembered atoms. Such mechanism is called atomic polarization: it involves bulk movement of heavy atoms in contrast with movement of electrons within the atom. The first may be likened to two large masses connected by a spring, and the second to a very small mass connected (by a comparable spring) to another which is some 2 000 times greater. The natural frequency of the first will be vastly less than that of the second, and hence the practical possibility of an approach to resonance with the first is much greater than with the second. If the dielectric constant of a given material has an appreciable contribution from atomic polarization, we must be prepared to find the dielectric constant dependent on frequency in the infra-red region of the frequency spectrum.

(c) Effect of temperature.

All molecules are agitated by a ceaseless chaotic motion, the energy of which is familiar to us as temperature. In the gaseous state the molecules, spinning violently, travel many diameters between collisions. In the liquid state the packing is so close that the free path is only of the order of a diameter, and the spin can seldom turn the molecule through a complete revolution between

* In this connection, pages 517 and 518 of Reference (1) should be consulted.

successive collisions: on the average it will take hours for a given molecule to move its average position by, say, 1 mm. or to turn its average direction through a complete revolution. In the solid state the molecules have an assigned place in the crystal lattice and vibrate and twist about this mean position. But whatever the state, each molecule will on the average have kinetic energy $\frac{1}{2}kT$ in every degree of freedom possible to it: here k is the same for all molecules (it is Boltzmann's constant and equal to $1 \cdot 37 \times 10^{-16}$ erg) and T is the absolute temperature. Thermal agitation and the principle of equipartition of energy has forced itself loudly on the attention of radio engineers; for it is responsible for the background noise in circuits and sets a limit to the degree of amplification which can be used with benefit. I and my associates have filled so many pages of the *Journal* with

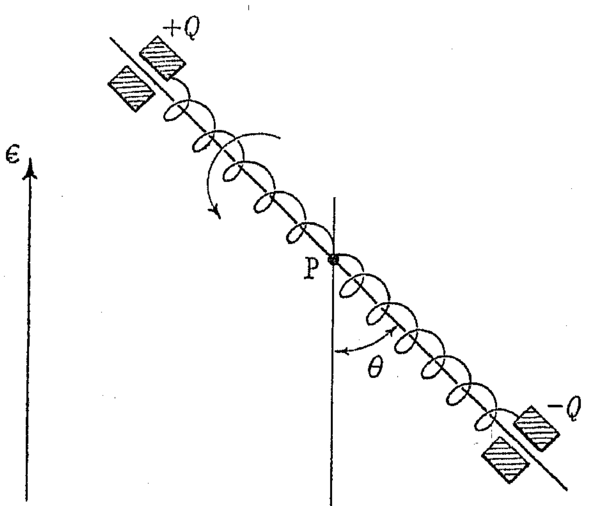


Fig. 2

discussion of thermal agitation that I need not trespass further on its space: let this introduction suffice to remind the reader of the mechanism which we must think of once more if we are to understand dielectrics.

Thus we now admit that the model atom depicted in Fig. 1 is moving as a whole and also twisting round in the field. Will this affect the result? It will not affect it, because the controlling force is proportional to the distance through which the proton moves away from the ecliptic plane: the time average of the moment will not be altered by the heat motion. For the gaseous state we may examine the problem in terms of classical mechanics very simply: it is the same as examining the motion of the mechanism shown diagrammatically in Fig. 2. Two massless charges Q can slide freely on a rod fixed perpendicular to a horizontal axle P which turns in frictionless bearings: Q is attached to P by a helical spring. The system is set rotating and then left free in an electric field ϵ perpendicular to the axle P. During each revolution the electric field will cause the charges to move to and fro along their guide rods. The electric moment will be $\alpha\epsilon \cos\theta$, where α is the spring constant and the component moment in the direction of the field will be $\alpha\epsilon\cos^2\theta$. This pulsating moment will cause the angular speed to hunt about its mean value: it is necessary to find the time average of $\cos^2\theta$ during a whole revolution. It can be shown that:—

$$\frac{\alpha\epsilon\int_0^t \cos^2\theta dt}{t} = \frac{\alpha\epsilon}{2}\left(1 + \frac{3}{8}y^2\right)$$

where t is the time of a revolution and $y \equiv \alpha\epsilon/(2kT)$. We can soon show that y is a very small quantity. For, according to Fig. 1, $\alpha = a^3$, and for all atoms a is of the order of 10^{-8} cm. (1 Ångström unit): thus if ϵ is 3 kV per cm. (10 e.s.u.) then $\alpha\epsilon$ is of the order of 10^{-23}. If T is 300° abs., $kT = 4 \cdot 1 \times 10^{-14}$, and thus y^2 is of the order 10^{-20}.

It is of fundamental importance to realize that electron polarization is not affected by temperature, and hence it follows that the temperature coefficient of dielectric constant, a parameter vitally important to radio engineering,

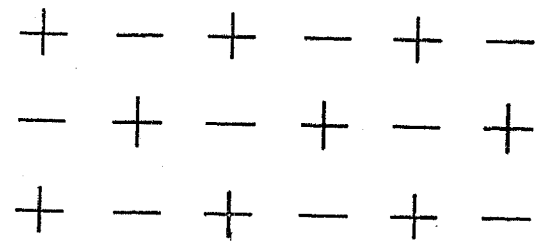


Fig. 3

is due only to change of density and is therefore essentially negative. If it is desired to have a positive or zero temperature-coefficient, it is useless to employ a material which responds only to electron polarization. It follows readily that, for a material of density ρ,

$$\text{Temperature coefficient of } \kappa \equiv \frac{1}{\kappa}\left(\frac{\partial\kappa}{\partial T}\right)_P$$
$$= \frac{(\kappa - 1)(\kappa + 2)}{3\rho\kappa}\frac{\partial\rho}{\partial T} \quad . \quad . \quad (3)$$

and is negative since $\partial\rho/\partial T$ is negative: also $\frac{1}{3}\frac{\partial\rho}{\partial T}$ is equal to the temperature coefficient of linear expansion. The atomic polarization in the molecule itself is very small compared with the electronic polarization in the individual atoms, and is consequently difficult to determine: for example, for ethylene (C_2H_4) the ratio is about $4\frac{1}{4}$ % and for normal pentane (C_5H_{12}) it is about $2\frac{1}{4}$ %. It is believed [see, for example, Reference (2)] that the atomic polarization, like the electronic, is unaffected by change of temperature or change of state. However, when ionized molecules are arranged in a solid crystal lattice, as indicated diagrammatically in Fig. 3, they contribute an atomic polarization which is subject to temperature variation. A rise of temperature expands the lattice: an increase of distance between the ions tends to decrease the attractive forces between them and thus makes more pronounced a concertina action when the whole aggregate is placed in an alternating electric field. Thus the contribution to dielectric constant from such a mechanism will have a positive temperature-coefficient. We are familiar with ceramic materials which have positive temperature-coefficients and with others in which this coefficient is negative, and we know that the two are sometimes combined in small trimming condensers whose

temperature coefficient is adjustable through zero. I suspect such ceramics act by virtue of a nicely selected mixture of materials, one of which exhibits pronounced atomic polarization. Frank[1] cites calcium fluoride as an interesting example of the opposed tendencies of electron and atom polarization in respect of temperature-changes: the coefficient due to the density change is $-0\cdot57 \times 10^{-4}$ and the estimated coefficient due to loosening of the crystal is $+1\cdot37 \times 10^{-4}$, giving a resultant of $+0\cdot87 \times 10^{-4}$ which in fact is much less than the observed positive coefficient.

Having attuned our minds to the concept of the forces within the atoms and the distortions which provide the dielectric constant, we will make an estimate of their order of magnitude. The electron charge is $4\cdot8 \times 10^{-10}$ e.s.u. and the radius of all atoms is of the order of 10^{-8} cm.: hence the field at an atomic radius from an electron is of the order of 15 000 megavolts per cm., in comparison with which the fields we can superpose sink into insignificance. Again, suppose that Fig. 1 represents a proton and a valency electron separated by one Ångström unit (10^{-8} cm.); then, from equation (1),

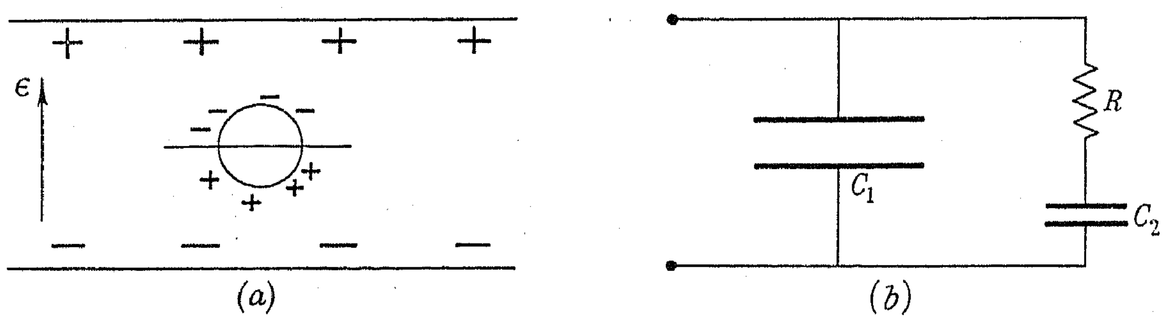


Fig. 4.—Conducting sphere between the plates of a condenser, and equivalent circuit diagram.

$$\frac{x}{a} = \frac{a^2\epsilon}{e} \simeq \frac{2}{10^6}$$

if ϵ is equal to 3 kV per cm. Thus the distortions which we produce, and which do in fact provide the dielectric constant, are incredibly small in fractional amount. Though the charge on a single electron is so small as to seem to some of us of little account as such, it must not be forgotten that we are concerned with distances whose inverse square is very large. A healthy respect for electronic forces will perhaps be engendered by a numerical example, due to Debye: A gramme-atom of sodium ions (23 g.) at the north pole of the earth would attract a gramme-atom of chlorine ions ($35\cdot5$ g.) at the south pole with a force equal to the weight of 52 tons. At any given distance the electrical attraction between a sodium ion and a chlorine ion is 10^{33} times the gravitational, and thus it follows that it is the electrical forces alone which matter appreciably.

(d) Energy loss in dielectrics.

Energy loss in dielectrics is a familiar and distressing experience. Some will have encountered it through the attenuation it produces in telephone cables or by the limit it sets to the selectivity of circuits, and others by the melting of high-frequency insulators. Is there any mechanism for this loss in the atomic picture given so far? The answer is "No." Movements of the valency electrons in the atoms cannot cause an absorption of energy in the atom as such, nor can they increase its kinetic energy (save at a resonance which can occur only in the far ultra-violet), which would in time be transferred by collision to neighbouring atoms and be disclosed as a rise of temperature of the whole. In a sense there ought to be no such thing as dielectric loss and in one view of experience there is indeed precious little. Thus good lenses absorb very little light, and the damping of a quartz piezoelectric crystal is exceedingly small (hence its valuable properties) even when accompanied by mechanical concertina action. Indeed, it seems certain there is no dielectric loss, barring perhaps sundry narrow absorption bands in the infra-red, in pure and regularly built crystals.[3] The loss we experience is often due to conducting impurities or to stray ions which move as a whole in the field and transfer the kinetic energy they acquire to the surrounding material. It is instructive to examine the mechanism of loss due to the presence of conducting particles.

Thus imagine a solid sphere of conducting material between the plates of a condenser [Fig. 4(*a*)]. Negative charge will be induced over the upper hemisphere and positive over the lower, having a magnitude and distribution such that it produces inside the sphere a uniform electric field equal and opposite to the field ϵ due to the charged condenser plates; the resultant internal field being precisely zero. If the charge on the plates is alternating there will be a flow of electric current inside the sphere, the lines of flow being parallel to the applied field [i.e. vertical in Fig. 4(*a*)]. Since current is the rate of change of charge it follows that the current must increase in proportion to the frequency if the induced charge on the hemispheres is to equal the value it would have in the instantaneous value of field, if static: thus the energy loss tends to increase as the square of the frequency. The resultant field inside the sphere cannot, however, remain zero with increasing frequency, but must equal the field necessary to drive the current through the resistance of the sphere. Hence the charges induced on the hemispheres will decrease with increasing frequency and thus the current will not increase so rapidly as we supposed at first, since the net internal field is equal to the current density multiplied by the specific resistance: thus the current density cannot exceed the value ϵ/ρ and the energy loss must approach a constant value when the frequency becomes very high. Clearly, then, there is some frequency at which the loss is a maximum. At very low frequencies the sphere increases the capacitance of

the condenser because it fills up some of the space between the plates: at very high frequencies the net field inside the sphere tends to the value ϵ and the sphere might as well be absent in respect of its ability to increase the capacitance of the condenser. It is almost obvious that Fig. 4(*a*) can be redrawn in terms of familiar circuit elements as Fig. 4(*b*), and it can be shown that the lead angle of this combination is less than $\pi/2$ by the angle δ, given by

$$\tan\delta = \frac{RK}{1 + R^2p^2K^2\dfrac{C_1 + C_2}{C_1}}$$

where $$K \equiv \frac{C_1C_2}{C_1 + C_2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and $$p = 2\pi \times \text{frequency}.$$

It follows that $\tan\delta$ is a maximum when

$$RpK = \sqrt{\left(\frac{C_1}{C_1 + C_2}\right)} \quad . \quad . \quad . \quad . \quad (5)$$

and hence

$$(\tan\delta)_{max} = \frac{RK}{2}$$

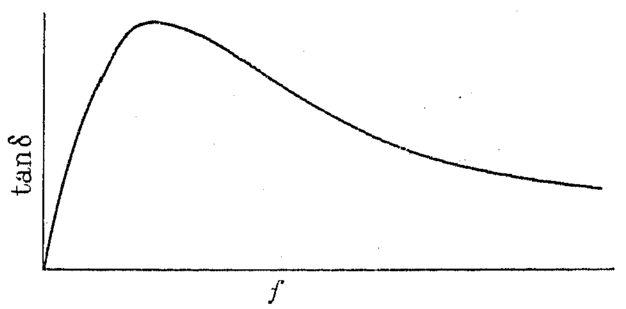


Fig. 5

It is obvious from Fig. 4(*b*) that for very low frequencies the effective capacitance is sensibly equal to $(C_1 + C_2)$ and for very high frequencies sensibly equal to C_1. The sphere in Fig. 4(*a*) is intended to represent one of many small pieces of conducting impurity or droplets of moisture: the only case of practical interest is when their total volume is small, and then C_2/C_1 is much less than unity, and (5) may be written

$$RpC_2 \simeq 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Consideration, however, will show the reader that for a sphere R increases in proportion to the radius, while C_2 varies inversely as the radius, and hence RC_2 is a constant depending only on the specific resistance. Less commonplace analysis shows that (6) should then be replaced by

$$\frac{3}{4\pi}p\rho = 1$$

that is $$f = \frac{2}{3\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

Thus the condition for maximum $\tan\delta$ depends neither on the radii of the spheres nor on their total number: the maximum value of $\tan\delta$ is proportional to C_2 and thus to the total number of spheres, or, in other words, to the fractional amount of moisture or impurity present in the dielectric. The resistivity of ordinary tap water at room temperature is about 10^5 ohm-cm. units, and for this $\tan\delta$ would be a maximum at a frequency of 6 Mc./sec.

I have now given in brief outline the classic analysis of K. W. Wagner. According to it, the curve relating $\tan\delta$ and frequency (expressed in terms of that which makes $\tan\delta$ a maximum) is shown in Fig. 5, and this needs but a glance to tell us that it is quite contrary to all ordinary experience.

The cause of the discrepancy is the assumption that the droplets of moisture are spherical, and it is easy to see that the behaviour of the dielectric must be very sensitive to the shape of the particles. For if the particles become thin needles they may ultimately form a conducting

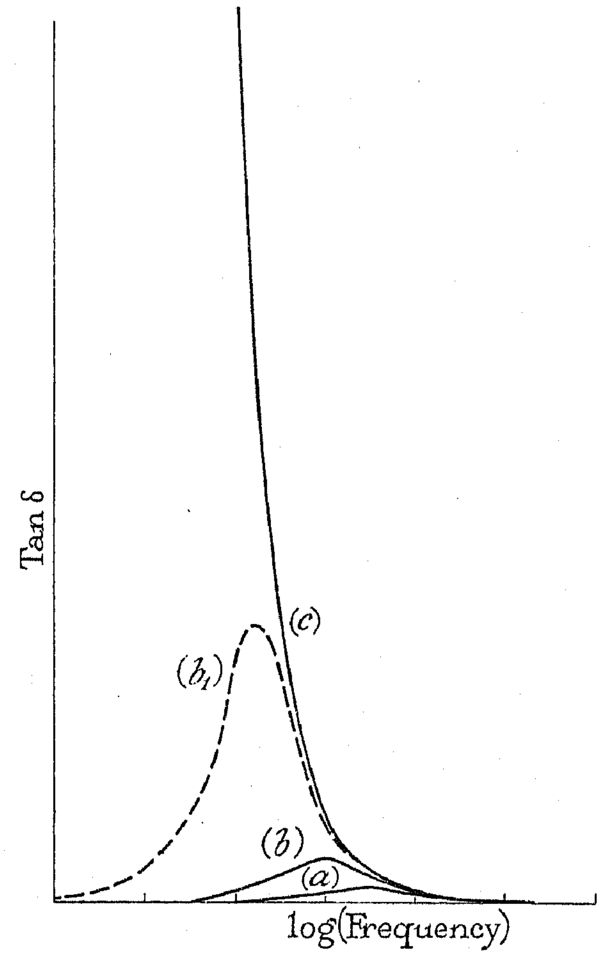


Fig. 6.—" Loss angle " caused by a given quantity of conducting material in the form of (*a*) sheet, (*b*) spheres, (*c*) cylinders.

bridge between the condenser plates; the resistance of this shunt being equal to $\rho d/v$, where v is the fractional volume of moisture in the dielectric and d is the distance between the condenser plates. Then it follows readily that

$$\tan\delta = \frac{2v}{\rho f}$$

Thus with such a shape of particle the maximum corresponding to that shown in Fig. 5 would be at zero frequency, and then its value would tend to infinity. The other extreme is to suppose the moisture is spread in a uniform sheet parallel to the plates: then it can be shown that the maximum value of $\tan\delta$ occurs when $f = 2/\rho$ vice $f = 2/(3\rho)$ for spheres: also $\tan\delta_{max} = v/2$ vice $3v/2$ for spheres. The relation between $\tan\delta$ and frequency is shown for these three shapes of particle by the three full-line curves in Fig. 6. It is now clear that a given amount of moisture

in a dielectric is capable of producing almost any power factor depending on the shape of the particles: the power factor may rise to a maximum at any frequency between zero and $2/\rho$. Since the droplets of moisture are likely to assume a variety of shapes, the power factor/frequency curve of a practical dielectric is likely to be the resultant of

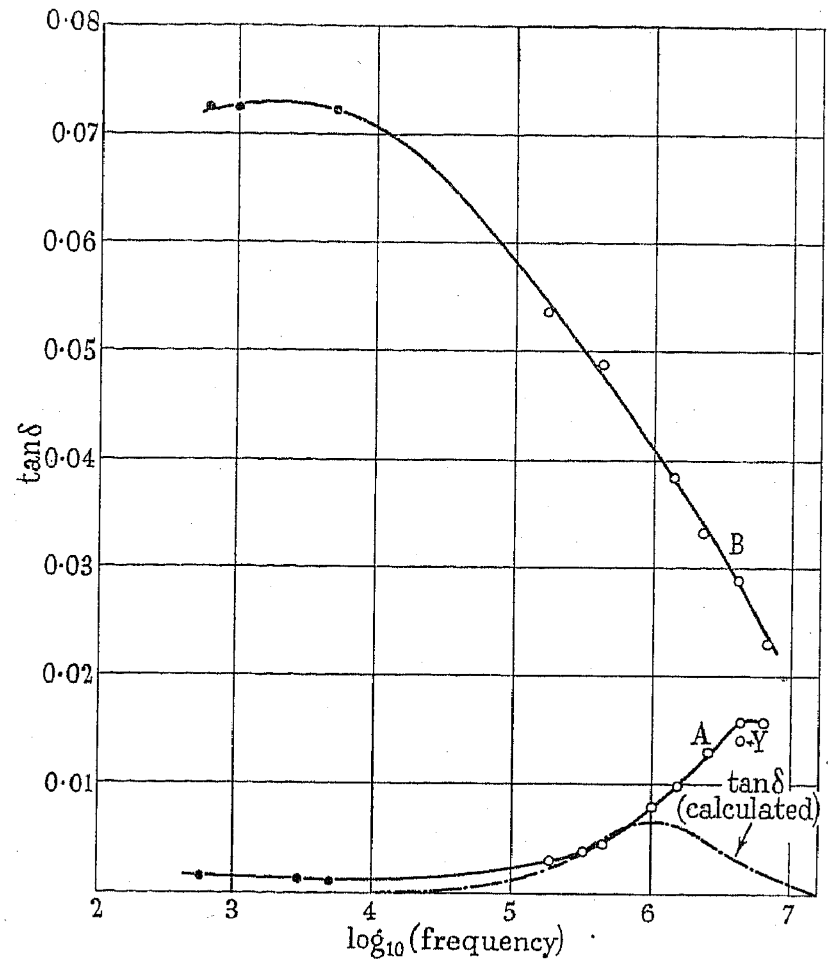


Fig. 7

Curve A. 9 deg. C. below solidifying point.
Curve B. Room temperature.

many curves typified by Curves (*a*), (*b*) and (b_1) in Fig. 6, and hence any maximum is likely to be very blunt: this is in accordance with experience. The effect has been shown experimentally in a very beautiful manner by R. W. Sillars, who produced a moisture-laden dielectric by shaking up 25 g. of water in 300 g. of molten paraffin wax. His idea was that the droplets would be spherical in the molten wax and remain approximately spherical so long as the wax was hot enough to be soft; but that when the wax hardened and contracted, minute internal cracks would form into which the moisture would run and thereby alter enormously the particle shape. The variation of tan δ with frequency was observed both when the wax was maintained at 9 deg. C. below its setting point (i.e. at about 47° C.) and when it had been allowed to cool to room temperature. The results obtained are shown respectively by Curve A and Curve B in Fig. 7, and the contrast is as instructive as it is striking. Thus at 100 kc./sec. the power factor due to 6·7 % of water was about 0·2 % and increasing with frequency when the droplets were roughly spherical: when the wax had hardened, the power factor at this frequency had risen to 5·5 % and was decreasing with frequency. In the first case the effect of the moisture was almost insignificant, whereas in the second case the dielectric would have been useless in practice. Could there be more convincing proof than Fig. 7 of the fact that a mere statement of moisture content in a dielectric can be no guide to the electrical behaviour of a dielectric?

(2) POLAR MOLECULES AND THE EFFECT OF THEIR PRESENCE IN A DIELECTRIC

All molecules are electrically neutral, but it does not follow from this that the positive and negative charges are distributed with sufficient symmetry to give no external electric field. Thus, consider a simple molecule such as water, H_2O: are the two positive hydrogen ions arranged as in (*a*) or (*b*) or (*c*) of Fig. 8?

Arrangement (*a*) would have no external moment and the force would vary as the inverse fourth power of the distance. Arrangements (*b*) and (*c*) would clearly have a net moment and the force would vary as the inverse cube of the distance. It can be shown that Arrangement (*b*) is essentially unstable and hence must be ruled out: also that Arrangement (*c*) is stable only if θ is 32° or 55°.

The problem of whether or not any particular molecule has an inherent electric moment can be tested experimentally by firing a beam of molecules, reminiscent of the electron beam in a cathode-ray tube, through a non-uniform electric field. It is not difficult to see that the deposit pattern on the target will be modified by the electric field only if the neutral molecules are inherently polar: further, it must be possible to deduce the magnitude of the moment from the shape of the deposit pattern. Such experiments have shown[4] that some molecules are polar and some are not. Water (H_2O), hydrochloric acid (HCl), ammonia (NH_3) and ethyl alcohol (C_2H_5OH) are examples of polar molecules; while nitrogen (N_2), methane (CH_4), carbon tetrachloride (CCl_4) and pentane (C_5H_{12}) are examples of non-polar molecules. All polar molecules have an inherent electric moment (μ) whose value is of the order of that due to a pair of electronic charges separated by the atomic radius, namely $\frac{4\cdot77}{10^{10}} \times \frac{1}{10^8}$; thus the

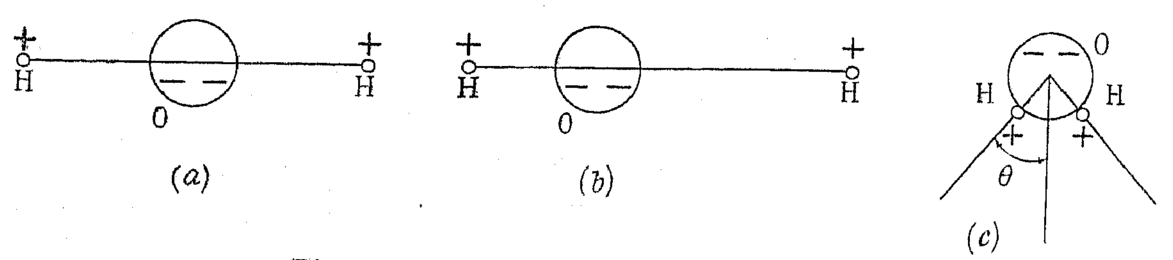


Fig. 8.—Linear and triangular H_2O models.

moment of nitrobenzene is $3 \cdot 9 \times 10^{-18}$ and of water is $1 \cdot 51 \times 10^{-18}$. Referring now to Fig. (8c), it can be shown that if $\theta = 32°$ then $\mu = 1 \cdot 34 \times 10^{-18}$, whereas if $\theta = 55°$ then $\mu = 4 \cdot 32 \times 10^{-18}$. Thus it is concluded that the smaller of the two possible values of θ is the true one. This is a fascinating example of the way in which the structure of the molecule itself can be deduced, but the reader may well ask what this has to do with dielectrics and with radio communications.

If the molecules of water are inherently polar, surely they will try to set themselves parallel to an electric field (like the corresponding behaviour of the supposed magnetic particles of iron) and endow water with a very large dielectric constant. It is well known that the dielectric constant of water is about 80, in striking contrast with a value of 2 or 3 for most other materials. Here then we have at last a rational explanation of this outstanding exception among common materials. Let

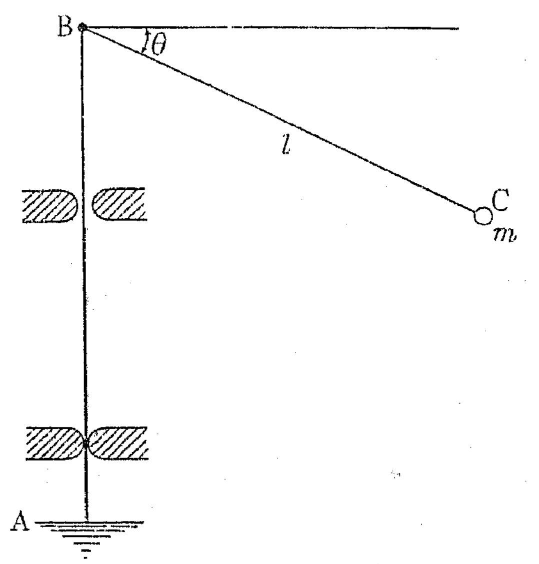


Fig. 9

it not be forgotten, however, that water has a dielectric constant of 80 at all radio frequencies, but that this falls to about 2 for light waves. Familiar experience suggests at once that the polar orientation becomes ineffective at very high frequencies and then there remains only the effect due to straining the shape of the molecule (valency electrons) discussed fully in the opening sections of this Address.

Let us now try to picture how a dipole can orientate itself in the direction of an applied field. First of all it is certain that the orientation cannot be complete, for, if it were, the dielectric constant would tend to infinity and there would be marked saturation effects, reminiscent of magnetized iron: both these things are contrary to experience. First consider polar molecules in a gaseous state and remember that now the medium consists of widely separated particles, all moving hither and thither with velocities of the order of that of sound, colliding with one another and with the walls of the container, producing on it the effect we call pressure. Also the collisions cause them to spin and twist while performing their straight-line paths between collisions. Though every conceivable velocity of translation and rotation is to be found, yet there is one which is vastly more probable than all others. The average translational energy (or mean square velocity) depends only on the temperature and not on the kind of molecule or on the molecular density. The average translational energy is $\frac{3}{2}kT$, where k is Boltzmann's constant ($1 \cdot 37 \times 10^{-16}$ erg) and T is the absolute temperature. On the average the energy of rotation bears a definite relation to the energy of translation and is equal to $\frac{1}{2}kT$ for rotation about each of the three principal axes of inertia: commonly this is expressed by the statement that on the average there is energy $\frac{1}{2}kT$ in every mode of motion.

Now any orientation effect must occur while the molecules are free between collisions, and, further, the time they are in contact is negligible. Evidently the twisting effect of the field on the dipole will be resisted by the gyrostatic effect of its spin, and the net result is likely to be small. To understand the mechanics of the effect, consider the following simple problem, which we will state first in terms of a mechanism such as an engine governor.

Let the light rod AB (Fig. 9), mounted vertically in frictionless bearings, have attached to it by a smooth hinge at B a light rod BC of length l to which is attached at C a small mass m. Let there be some device (such as a peg in a hole) which is capable of holding BC perpendicular to AB. Let the shaft AB be spun up to an angular velocity ω and then left free: it will continue to rotate at ω indefinitely, since the bearings are frictionless. At any time ($t = 0$) let the peg be withdrawn so that C starts to fall. CB will then oscillate for ever, and θ will change periodically from zero to a maximum value. It follows from simple dynamics that

$$\dot{\theta}^2 = \omega^2(s \sin \theta - \tan^2 \theta)$$

where $$s \equiv \frac{mgl}{\frac{1}{2}ml^2\omega^2}$$

Hence it follows that $\dot{\theta} = 0$ when

$$\sin \theta = \frac{\sqrt{(1 + 4s^2)} - 1}{2s} \simeq s, \text{ if } s << 1.$$

Hence if s is small, θ is always small, and we may write

$$\dot{\theta}^2 \simeq \omega^2(s\theta - \theta^2)$$

$$\therefore \quad \ddot{\theta} = \frac{\omega^2}{2}(s - 2\theta)$$

Hence it follows that BC will oscillate about its mean position ($\theta = s/2$) with frequency $\omega/(2\pi)$. Now suppose that BC is extended a distance l in the direction CB and that the single mass m in a gravitational field g is replaced by equal and opposite charges e (at C and C′) in an electric field ϵ. The motion will be unchanged, but now we must write

$$s \equiv \frac{e\epsilon l}{\frac{1}{2}ml^2\omega^2} \equiv \frac{2e\epsilon l}{kT} \equiv \frac{2\epsilon\mu}{kT} \quad . \quad . \quad . \quad (7)$$

Thus the applied field causes the plane of rotation to oscillate and in consequence the rotating dipole has a very small component moment in the direction of the

field: the component fluctuates between zero and a maximum each revolution, but its average value is

$$\overline{M} = \frac{\mu s}{2} = \frac{\mu^2 \epsilon}{kT}$$

in the direction of the field.

Now consider a dipole which is revolving about an axis P perpendicular to the field, as indicated in Fig. 10. While θ increases from 0 to π the sense of the couple due to the field is such as to assist rotation: as θ increases from π to 2π it impedes rotation. Hence the angular velocity, when in the applied field, is not constant but hunts above and below the mean value ω: it is a minimum when $\theta = 0$ in Fig. 10. Hence the positive charge spends more time below the pivot P than it spends above it, and consequently the time average of the moment in the direction

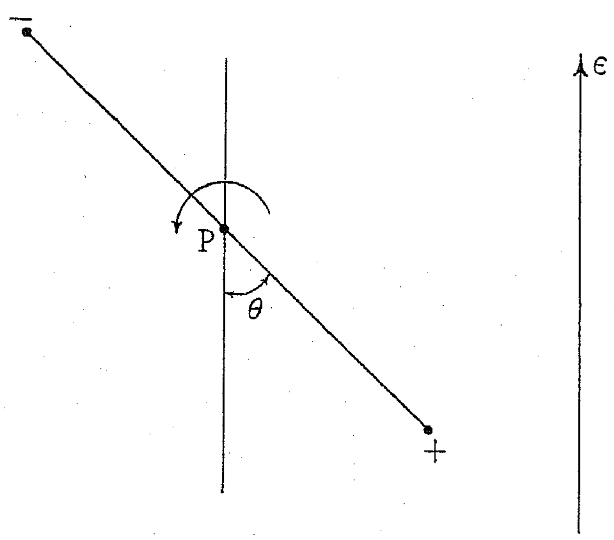


Fig. 10

of the field is not zero, but has a small value *downwards*. Treatment analogous to that used previously shows that now

$$\overline{M} = -\frac{\mu^2 \epsilon}{2kT}$$

the negative sign meaning that the direction is against the field.

In a representative sample of molecules it may be supposed that one-third are spinning about an axis parallel to the field, and one-third about each of two mutually perpendicular axes. Hence the net moment for N molecules would be

$$\overline{M}_N = \frac{\mu^2 \epsilon}{kT}\left[+\frac{N}{3} - \left(\frac{N}{3} \times \frac{1}{2}\right) - \left(\frac{N}{3} \times \frac{1}{2}\right)\right] = 0$$

Thus arises the paradox, which I have never resolved to my satisfaction, that while every molecule is affected by the field the net result is zero.

This, however, is certainly contrary to experience, for a gas of polar molecules has a dielectric constant very different from that of a non-polar gas: moreover, it is contrary to the result of a calculation made according to the most general principles of statistical mechanics and having no reference to a particular mechanism. There is a fundamental proposition known as the Maxwell–Boltzmann distribution theorem which is applicable to any aggregation of molecules which are in thermal equilibrium and which are in a field of force. Thus this general theorem tells us at once that the distribution of particles in an isothermal atmosphere must be exponential from the ground upwards: this involves no use of the gas laws. When this theorem is applied to polar molecules in an electric field, it shows that the result will be as though each dipole were at rest and had a moment $\overline{M} = \mu^2\epsilon/(3kT)$ in the direction of the field. Hence by some means or other they do not neutralize completely. Where is the mistake? Is it in the simplification introduced in the analysis by presuming that

$$\tfrac{1}{2}kT \equiv \tfrac{1}{2}\mathrm{I}\omega^2 >> \mu\epsilon$$

Taking $\mu = 1{\cdot}5 \times 10^{-18}$ and $\epsilon = 3$ kV per cm., then $\mu\epsilon = 1{\cdot}5 \times 10^{-17}$; if $T = 300°$ abs., $kT = 4{\cdot}11 \times 10^{-14}$: hence $\mu\epsilon/(3kT) \simeq 1/10^4$, and thus is very small compared with unity.

Apparently the mistake has come about by assuming that all molecules are spinning with the same energy, whereas in fact all spin energies, from zero upwards, are represented. Thus it seems that the predominant contribution is provided by those molecules which have very little spin. All are affected by the field: all those whose spin is near the average value mutually neutralize one another and might as well not be present; those which so to speak "do the trick" are those nearly at rest. I suspect it would help our knowledge of how to make good dielectrics if our minds could visualize step by step a vast series of collisions and see that it is only the unenergetic ones which matter. However, what I will call the paradox of the Boltzmann theorem is in fact familiar to all of us if we stop to think. Let us compare a gas molecule in the atmosphere to a perfectly elastic tennis ball, and let the atmosphere be so rarified that collisions do not occur. The ball will descend from the skies, hit the ground with the velocity appropriate to the temperature, rebound and ascend once more to a certain height, coming very slowly to rest, for a moment, before descending again. It will spend most of its time high up in the atmosphere, and in such circumstances the atmospheric density will increase from the ground upwards. In such a world an airman and his engine would experience breathing difficulties only when low down. Somehow the collisions and the resulting distribution of velocities make things happen as we know them and as Boltzmann statistics predict, but do any of us really see that it just could not be otherwise? Again, suppose that on the ground there is a container full of hydrogen gas at atmospheric pressure and temperature: if the cover is removed the hydrogen will ascend and distribute itself with a density decreasing exponentially from the ground upwards. Somehow the centre of gravity of the hydrogen is raised enormously. If we liken the air and the hydrogen to agitated tennis and ping-pong balls respectively, is it obvious that the ping-pong balls must on the whole be driven upwards by collisions with the tennis balls? It is puzzles like these which make it hard to think clearly about the behaviour of dielectrics.

Be this as it may, however, it is quite certain that if a

molecule has an inherent moment μ then equation (2) will have to be written as

$$M = \left(a^3 + \frac{\mu^2}{3kT}\right)\epsilon$$

and hence it follows that the dielectric constant will decrease as the temperature rises. This effect is well known in gases and liquids which are polar, such as ammonia or nitrobenzene. Indeed, the effect is used as the ordinary method of measuring the dipole moment μ, since at present the precision of the dielectric-constant method much exceeds that of the more direct molecular-beam method.

A polarizability depending on temperature is in very marked contrast to all that was said in Section 1(c), and the reader should be careful to note that what was said there in respect of temperature coefficient is valid only if the dielectric is not inherently polar.

We have seen that $\mu\epsilon/(3kT)$ is of the order of 10^{-4}, and thus we can if we like say that on the average every molecule contributes about 1 part in 10^4 of the moment it is endowed with inherently. It is tempting to use a simplified and, I suspect, misleading picture in which the molecules have no spin energy and are free to turn themselves along the field but are continually being knocked out of alignment by the thermal collisions of translatory movement. Thus suppose a person in an agitated and closely-packed crowd were determined to keep his eye directed at a mark on the wall: bumps from his neighbour would largely frustrate his aim, but not completely so. Such a model is easy to visualize, and I suspect is tacit in much that has been written on this subject: it must be wrong and I suspect it is misleading. But at least so far as the net effect is concerned it is permissible to say it is as though the molecules had no spin and no inertia and were distributed with directions at random, and each were acted on by a restraining couple equal in magnitude to $2kT$.* You may put it which way you like—inertia and no restraining force, or no inertia but restraining force present. These are two alternative methods of stating the same result.

What will happen when the applied electric field is alternating? There must be a time effect in the Boltzmann distribution law, which can surely be true only after an infinite time for adjustment. Accordingly it will surely take time for the result $\mu^2\epsilon/(3kT)$ to come to pass, and hence it will never be fulfilled accurately in an alternating field. As a guess, if the alternation becomes very rapid there will be no time to reach even approximately the Boltzmann distribution, and thus the dielectric constant will decrease with increasing frequency. This we know is the case with water, where the dielectric constant falls from, say, 80 to 2 as the frequency increases from the infra-red to blue light.

The treatment of polar molecules in an alternating field is due almost entirely to P. Debye. Before giving his treatment, we will look at the problem by analogy. Gas in a container is in the gravitational field and, as we know, the density distribution is not uniform; though in a small container we do not usually bother to take count of the difference of barometer reading between the top and the bottom. Suppose now that in some unspecified manner the gravitational field is screened off: a finite time will be required for the density to become uniform by diffusion. At any given point the density will change exponentially with time and the initial state will not be *relaxed* instantly. Just as we talk of the time-constant of an RC circuit we might talk of the relaxation time of the gas in the container: it would depend on the gas and not on the size of the container.

It is a problem of diffusion and would involve the agitation velocity and mean free path in the same manner as these are involved in the coefficient of viscosity. Thus it would be permissible to express the relaxation time of the gas in terms of its viscosity: it may, however, be misleading to picture the mechanism as unagitated molecules creeping along against a viscous drag on their surface. By analogy we shall expect a relaxation time for dipole polarization when the applied field is removed. But here the mechanism is much more difficult to picture, since it does not consist of bulk transfer of molecules. Thus, consider Fig. 10, in which the uniform angular velocity of the molecule about the axis through P has a pulsation imposed on it by the action of the applied field. If the field is removed the pulsation disappears instantly, without a transient term, and uniform rotation continues at the angular velocity obtaining at the instant when the field was removed: this would be equally true if the rotational energy were small and the molecule oscillated without complete rotation. Thus in a gas it would seem as if the polarization effect disappeared instantly but that each molecule was left with an energy which depended both on its previous collision and on the time between that collision and the instantaneous removal of the field. Thus the velocity distribution would be not quite Maxwellian and a finite time would elapse before this state was reached: the relaxation time looks to be one of velocities rather than of polarization. However, it is possible that this is apparent and not real, and that a polar gas which is not in an electric field exhibits no net polarization effect only if the velocity distribution is Maxwellian. It seems likely that the redistribution of velocities would result in a slight increase of temperature of the whole, and thus we should encounter dielectric loss.

Once grant a time factor in the attainment of the Boltzmann distribution function and I think we are committed to a dipole polarization which dies away exponentially with time when the field is removed; and thus a condenser with a polar dielectric must be representable by some equivalent circuit such as that of Fig. 4(b), and in which the equivalent leakage resistance must be related in some manner with the macroscopic viscosity of the fluid dielectric. If this be so, then electrical tests alone cannot suffice to distinguish between energy loss due to Wagner conduction and due to polar molecules. In the first the heating is caused by ions drifting through liquid droplets and thereby adding to their thermal agitation, and in the second it is caused by a slight redistribution of velocities. It is important to realize that curves typified by that of Fig. 6 are likely to arise from almost any mechanism and hence do not alone suffice to disclose the mechanism: it is all the more strange that they are unfamiliar in practice.

So far we have in the main considered a gas rather than

* The number 3 and not 2 has appeared previously through the process of integration over a sphere.

a liquid. In a liquid the molecules are closely packed and always exert large forces on one another and thus are never free in the sense that they are free between collisions in a gas. Their thermal motion is vibratory with a frequency some 10^6 times as great as the collision frequency in a gas. An analysis such as that used in relation to Fig. 9 will now be very inadequate, for the twisting dipole is in this case acted on both by the strong electric field of its neighbours and by the applied field, and also only a tiny portion of a revolution will occur between consecutive collisions. The inter-molecular field will, on the average, be expressible in terms of kT, since there is equipartition of energy between the potential and the kinetic energy. The Boltzmann distribution theorem is still applicable for a dilute solution of dipoles in a non-polar liquid, and hence it may well be that now the equation $\overline{M} = \dfrac{\mu\epsilon}{3kT}$ is fulfilled because the dipole is restrained from turning by a true resisting couple as well as by its inertia, whose effect now may be relatively small. Debye[5] introduces a resisting couple due to viscosity, but dismisses this very difficult concept in very few words. The term "viscosity" is a way of expressing what happens during the brief intervals of collision, and it refers to an effect which is absent between collisions. It looks rather as if Debye were reckoning the same thing twice over, calling it first the disordering effect of thermal agitation and then the resisting effect of viscosity. No doubt the apparent confusion can be resolved and the treatment justified.

Debye's treatment leads me to the picture of a dipole devoid of rotational energy and whose rotation is resisted both by a restoring couple $2kT$ and by a viscous couple ζ; thus giving a time-constant or relaxation time $\tau = \zeta/(2kT)$. It is important to know whether this picture is intended to be real or merely an equivalent. Be this as it may, however, it follows from what has been said just previously that the curves of $\tan\delta$ and frequency will be the same as for the Wagner mechanism (see Fig. 5), and that $\tan\delta$ will be a maximum when $p\tau = 1$.

Debye wishes to make a rough estimate of τ and in order to do so applies Stokes's formula for the viscous drag on a sphere; as a first approximation he writes

$$\zeta = 8\pi\eta a^3 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where a is the radius of the polar molecule and η is the viscosity of the liquid. He is careful to explain that the treatment is rough and drastic, but reminds his readers that a similar treatment has already been found successful in calculations of the mobility of ions in an electrolyte.

Many measurements have been made of the energy loss in liquid dielectrics containing a dilute solution of polar molecules. All have fulfilled precisely the behaviour typified by Fig. 5: and in fact the curves of $\tan\delta$ and frequency of these dielectrics have had that form which theory would suggest but which is encountered so seldom in practice. Moreover, and somewhat surprisingly, equation (8) has been shown by Bridgeman[6] to be valid for large molecules such as zein, gliadin and haemoglobin.

But perhaps no measurements have illustrated the effect more strikingly than those made by W. Jackson on a sample of the commercial dielectric known as permitol (a mixture of isomers of tetrachlorodiphenyl) and reproduced in Fig. 11. Here $\tan\delta$ is plotted as a function of temperature at eight frequencies in the range 50 to $1\cdot09 \times 10^7$ c./sec. Heretofore we have regarded the temperature as constant and the frequency as variable, but brief consideration will show that these conditions differ little from those of constant frequency and variable temperature. We have seen that the governing factor is the ratio of the periodic time of the field to the time-constant (relaxation time) of the dielectric. The relaxation time depends on the viscosity, and this is markedly a function of temperature. Hence if the temperature is varied the relaxation time is variable over a wide range, and somewhere in this range it is likely to agree with the periodic time of the field: if the temperature is constant we must vary the frequency till it agrees with an assigned relaxation time. Here, in Fig. 11, we see at last a commercial dielectric behaving in the manner predicted by

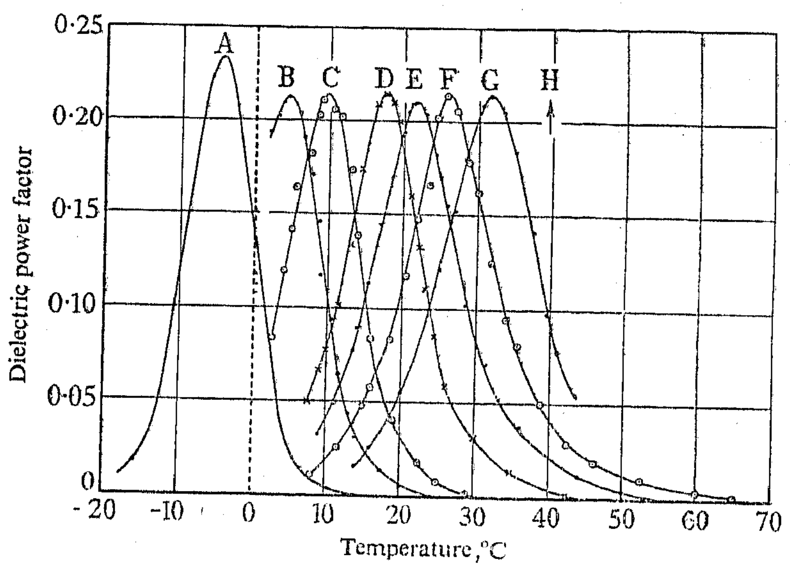


Fig. 11*

Frequency values, in cycles per sec.: A, 50; B, 10^3; C, 6×10^3; D, 10^5; E, $2\cdot95 \times 10^5$; F, $9\cdot5 \times 10^5$; G, $2\cdot75 \times 10^6$; H, $1\cdot09 \times 10^7$.

Wagner but almost unrealized heretofore. That the mechanism is not due to the conduction process of Wagner, but to the dipole rotation of Debye, is in a sense hardly relevant. Here is a dielectric which misbehaves only in a very small range of temperature, instead of behaving with the uniform mediocrity so familiar to engineers. Thus, suppose permitol is to be used at a frequency of 1 kc./sec.: it is only necessary to keep its temperature above about 20° C., and it is an admirable dielectric. If it is to be used at 1 Mc./sec. it must be kept above 50° C. or below 10° C. Though the material is capable of being very bad, it is only necessary to use it with knowledge and understanding to obtain a first-rate dielectric.

At each maximum in Fig. 11 the relaxation time of the dielectric is equal to $1/(2\pi f)$, and the relaxation time should be proportional to the viscosity. Hence if the reciprocal of the frequency is plotted against the temperature for the maximum value of $\tan\delta$, the resulting curve should represent, to some undetermined scale, the viscosity/temperature curve of the liquid; and this curve can be found by viscometer measurements. The full-line curve in Fig. 12 shows the measured viscosity of the

* This Figure is due to W. JACKSON: *Proceedings of the Royal Society*, A, 1935, **153**, p. 161.

permitol for temperatures greater than 10° C. (At temperatures less than about 10° C. permitol is a glassy substance and viscometry becomes impracticable.) The dotted curve in the same Figure shows log $1/f$ plotted against T, the scale being chosen so as to make the two curves agree at the point marked "a." This Figure shows conclusively that the relaxation time of the dielectric is proportional to the viscosity of the liquid, and that this is true even when the permitol has become a glassy solid. It is now clear that the behaviour of a polar liquid at every temperature and frequency can be predicted accurately if the curve of tan δ plotted against T is measured at any convenient frequency, and the viscosity curve is available. Recognition of this fact should be of great practical value to engineers. I feel it would obscure the issue to say that all this necessarily confirms

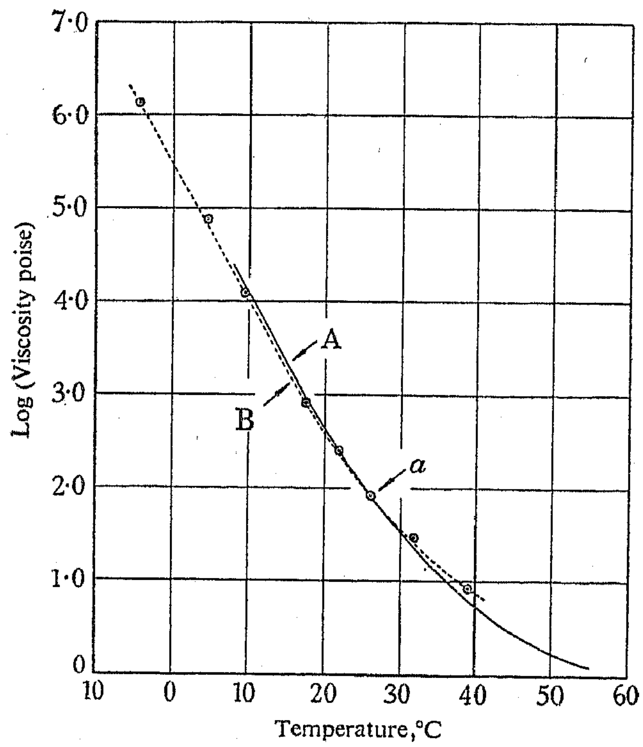


Fig. 12

A ———— measured viscosity/temperature curve.
B - - - ⊙ - - - calculated curve as determined from the position of the peaks of the curves in Fig. 2.
For the purpose of comparison the curves have been made to coincide at the point *a*.

Debye's treatment or his picture of dipole rotation, for I think the result is much more general than that. Granted only that the all-essential Maxwell–Boltzmann distribution needs time to establish itself, a polar dielectric must have a relaxation time: experiment shows that this time is proportional to the macroscopic viscosity. It would be somewhat surprising if it were otherwise, since the mechanism of viscosity, whatever it be, is one by which local agitations are distributed through the bulk medium, and we are familiar with the close connection between the viscosity of an electrolyte and its resistivity, and also between the viscosity of a gas and its thermal conductivity.

Of course, we shall wish to go further and visualize, as Debye and (more recently) Andrade[7] have done, the mechanism of dipole rotation and the mechanism of viscosity respectively; but do not let us confuse the essential experimental facts, which are conspicuous by their simplicity, with these further reachings-out into a theory of the mechanism. We can use Fig. 12 to reach out like Debye and, by means of equation (8), to estimate the volume of the dipole molecule. Numerical computation from equation (8) yields a value of about 26 cubic Ångströms for this volume, whereas chemical information suggests the volume of the molecule must have been greater than 80 cu. Å. It is interesting to find that the two values are comparable: it is fruitless to speculate about the discrepancy until more is known about the inherent rigidity of the polar group in the molecule and until the mechanism of rotation has been established more firmly.

Granted the picture of a static and restrained dipole frozen into a sticky liquid, curves such as those of Fig. 11 are susceptible of vivid interpretation, as follows. When the liquid is very cold the dipoles are frozen tight and therefore cannot respond appreciably, and consequently there will be little energy loss. When the liquid is hot the dipoles can turn freely with very little friction and so once more there is very little loss: in intermediate stages the response will be finite and the friction considerable, resulting in considerable loss of energy. It is an instructive picture, but I suggest one which is better used as an aid to memory than as a picture of reality. It is a picture which would be very suitable if the dipole were a large colloid particle floating in a liquid: then the discrepancy between the mass of the particle and the surrounding molecules would make the Brownian rotations very small and leave the particle turning very slowly and acted on by a viscous drag calculable by Stokes's law. In such circumstances Debye's treatment would be fully justified and it is near these circumstances that Bridgeman, using haemoglobin in benzene, finds it fulfilled. But, when the dipoles are of the same size as the solvent molecules, surely we must be cautious lest we build too much on this picture.

(3) DIELECTRIC LOSS IN POLAR SOLIDS

The idea of viscous restraint on a polar molecule might lead one to suppose that rotation would be impossible in a crystalline solid, but such is not the case. Thus in 1924 J. Errera published a curve of tan δ against f for ice at $-40°$ C., and this showed a maximum and was generally reminiscent of the behaviour of a polar liquid. The behaviour of a polar solid is of great scientific interest and obviously has very great technical importance. We have been studying this problem in the Engineering Laboratory of the University of Oxford since 1933 and have been able to make some contributions to knowledge which may some day be helpful in the manufacture of dielectrics. Since there was scarcely any information available, it seemed essential to seek for it without any regard as to whether the materials examined had any great technical importance: in fact, to study the basic theory of dielectrics rather than to search blindly for materials which were good in the technical sense.

Thus the line of attack was to study the effect of adding known and chemically pure polar substances to a nonpolar solid: the first problem was to find possible materials. Those chosen were esters added to paraffin wax. Since we hoped to deduce from electrical measurements the antics of certain molecules in the presence of others, it is essential to know what the molecules look like: hence a digression on molecular structure is necessary. The chemical formula of a paraffin wax is

C_nH_{2n+2}, and the chemist often draws it like a picture of a centipede (Fig. 13). The formula and the picture imply that a range of substances can be made by the process of hooking together the appropriate number of identical links, as one might form a chain. Thus we might visualize pentane as a chain 5 links long and ordinary paraffin wax as a chain of about 26 links: that pentane is a liquid at room temperature and paraffin wax a very hard solid is irrelevant so far.

For further information we must go to X-ray crystallography, which in a very different scale of wavelength is a process very similar to our radio technique. Thus it would clearly be possible, though laborious, to deduce the spacing of an antenna array by directing at it a parallel beam of radio waves and observing the polar diagram of the reflected signals: it would be necessary to presume only that the spacing was regular. In the X-ray process each antenna of the vast array is a single molecule or single atom or ion, and in a crystal the spacing turns out to be regular. In some substances all similar ions have the same spacing in every direction, and then the molecule of the liquid state has lost its

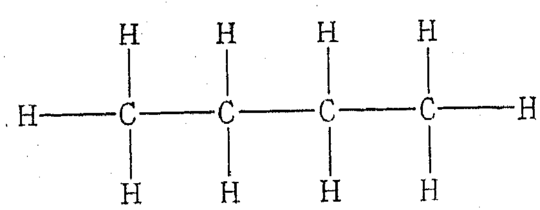


Fig. 13

identity and the crystal consists of a perfectly regular lattice structure. Carbon can take up three different structures: (i) diamond, which consists of carbon atoms regularly spaced in every direction; (ii) graphite, which consists of layers of regularly-spaced carbon atoms and may be said to consist of layers of sheet molecules, or if you prefer, of flat sheets one atom thick. (iii) carbon can also exist in bundles of rod molecules, and then we call it a paraffin (comparison with a Franklin array is here irresistible).* However, the length of the rods in the paraffin crystal is finite and definite, and depends on the number of carbon atoms in the molecule of the vapour phase, and thus it may be said that the molecule preserves its identity from vapour through liquid to solid. Indeed, the centipede diagram of Fig. 13 turns out to be a true picture.

The vapour of paraffin wax consists of widely separated rod molecules, one atom thick: when the wax is solid these are packed regularly side by side and may appropriately be likened to packets of candles. The molecule can be represented by Fig. 14 (which is much more than a mere diagram), i.e. by a series of spheres fixed together so that their centres are all in one plane. Each sphere represents a carbon atom, and the measured value of the diameter is $1 \cdot 54 \times 10^{-8}$ cm.: the measured value of α is 109° 30′, which is the angle of a regular tetrahedron. The two hydrogen atoms belonging to each carbon are too small to show up in the X-ray picture, but undoubtedly they are situated at the other two apices of the tetrahedron having two apices at the two contact points on each sphere. The centre distance between the axes of successive rods is either 4·08 Å or 3·67 Å, and the clearance between the ends is 2·1 Å. These distances are the same in every paraffin; it is only the chain length which changes from member to member of the series. When the crystal is heated the length does not expand, but the side spacings increase, one at three times the rate of the other. All the evidence shows that the rods are incredibly rigid—that they are properly called rods and not strings. It is proper to say they are as rigid as a diamond, for the work required to break the bond between two carbons is known to be the same as in a diamond and is about 80 kilogramme-calories.

The mechanical weakness of paraffin wax is due to the relatively small cohesion (Van der Waals forces) between

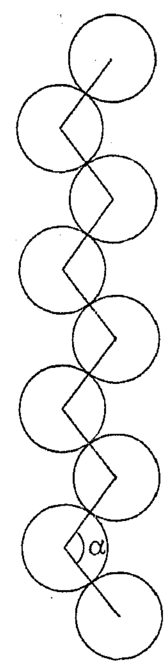
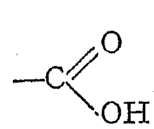
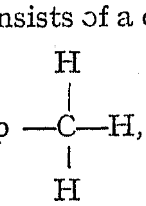


Fig. 14

the sides and ends of rods and not to the rods themselves, which are mechanically unbreakable, being thousands of times stronger than steel.

The symmetry of each atom precludes the possibility of a permanent dipole; and no paraffin, whatever its length, is in the least degree polar. The dielectric constant can be due only to strain of the valency electrons in each atom, and, barring density effects, it is obvious that all paraffins must have the same dielectric constant. Thus paraffin wax is essentially a non-polar crystalline solid having a precisely known and simple structure: it is admirably suited to our experiments if we can find polar materials to add to it.

If the last link in the chain consists of a carboxyl group , vice a methyl group , the rod molecule is called a fatty acid: the rod to which the carboxyl is attached may have any length from zero (in formic acid) to 17 carbons (in stearic acid, etc.). The carboxyl group is not symmetrical and, according to expectation, has a permanent dipole.

* It is amusing to think that if we could sever every horizontal tie rod in a diamond without severing vertical ties, the net result would be, roughly speaking, to convert the diamond into paraffin wax.

Now it seems probable that an *acid rod* would pack in without much fuss among paraffin rods, provided perhaps it was shorter than, but not too short compared with, the paraffin rods. The Van der Waals force from the polar end will be considerable and may cause some misfit: in the acid crystals it tilts the rod through about 40°, and this may perhaps be the angle between the direction of the dipole and the axis of the rod. The crystal is divided into unit cells each of which very often contains

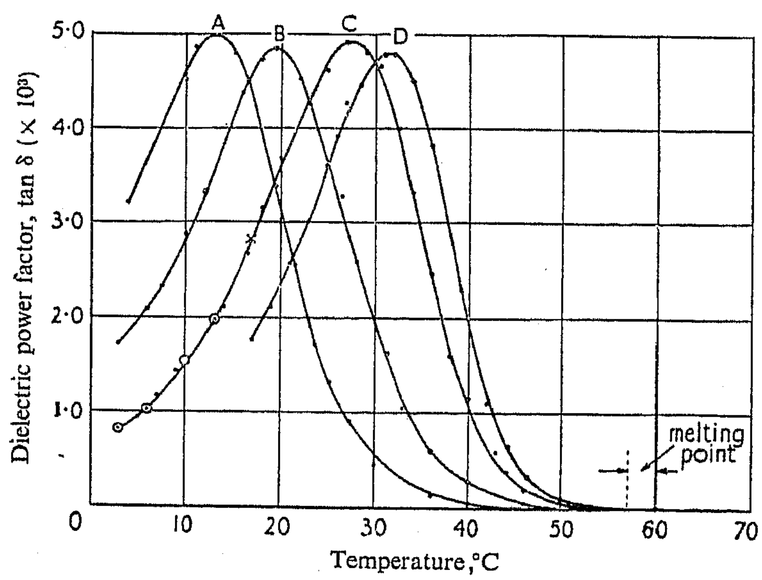


Fig. 15

Frequency of measurement, in cycles per sec.: A, $6 \cdot 6 \times 10^5$; B, $2 \cdot 8 \times 10^6$; C, $7 \cdot 78 \times 10^6$; D, $1 \cdot 42 \times 10^7$.

four molecules. Now it may be that, say, three acid and one paraffin fit into a unit cell of the wax better than two and two, or one and three, but whatever the arrangement of a unit cell which does contain an acid molecule it would seem probable that any degree of solid solution should be possible up to that amount in which every cell has its complement.*

If the last link on the chain is hydroxyl, —OH, the rod molecule is called an alcohol: thus structurally the difference between acids, alcohols and paraffins is extremely small, and indeed, the higher members look indistinguishable in bulk. The unsymmetrical hydroxyl group is also polar, and so here we have another range of substances which should mix happily into the wax. But alcohol rods and acid rods will readily weld together and form a straight, rigid rod having a dipole at the junction of the two parts: such a rod is called an ester. Because the dipole in the ester, with its accompanying Van der Waals field of force, is not at the end of the rod, it seemed reasonable to suppose it might fit better than an alcohol or an acid into the paraffin lattice. For this reason and others it was decided to use esters as the polar material to mix into the wax.

A high-grade paraffin wax of setting point 57°–60° C. was available: this setting point indicates that the dominant chain-length was about 26 carbon atoms. The material spermaceti consists dominantly of the ester cetyl palmitate, which has the polar COO group with 15 carbons on one side and 16 on the other: thus it is almost symmetrical, and the total length of the rod is equivalent to about 33 carbon atoms, and this is about 25 % longer than the wax molecules. Spermaceti and wax appeared to mix without difficulty, and it was hoped that they went into solution.* It was found that the value of tan δ for the wax was not greater than 5×10^{-5} in the temperature range 0°–80° C. but that the addition of $4 \cdot 5$ % of spermaceti increased this one-hundredfold at a sharp maximum at a temperature depending on the frequency. A set of tan δ/frequency curves is shown in Fig. 15, and they are characteristic of a polar liquid. A curve connecting tan δ_{max} and the concentration is exhibited in Fig. 16, and this shows that the dielectric loss is certainly due to the addition of the permanently polarized ester molecules.[10] The magnitude of the dipole moment can be deduced from Fig. 15, and the figure obtained was $12 \cdot 5$ % less than that found from measurements in benzene solution. There can thus be no doubt that the behaviour of polar molecules in a crystal lattice is not substantially different from their behaviour in liquid solution. The relaxation time of a given molecule in a given wax at a given temperature is still a measurable quantity, but we can no longer follow Debye and estimate what this relaxation time will be because, firstly, there is so far no certain knowledge that the dipole cannot swivel round the long axis of the molecule without twisting with it the long carbon chains on either

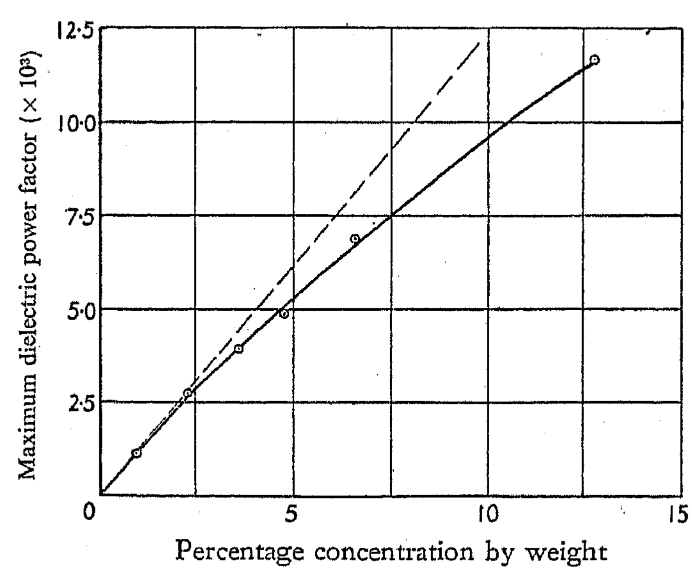


Fig. 16

side. Secondly, with or without this uncertainty, is it fair to treat the molecule either as a sphere or as a smooth rod? Lastly, and perhaps most pertinently, what magnitude is to be assigned to the viscosity in the solid state? Viscosity, according to definition, no longer has any meaning, yet presumably there will exist a dissipative mechanism in the solid which plays a role corresponding to that of viscosity in a liquid. If in Fig. 15 the temperature at the maxima is plotted against the logarithm of the reciprocal of the corresponding frequency, the points lie on a straight line. This implies that the parameter which for convenience we will still call viscosity varies exponentially as the reciprocal of the temperature: in this respect there is continuity with the

* In point of fact, however, the evidence of X-ray and melting-point phenomena shows that the active ends of the acid rods unite and form a double molecule: possibly acids will form solid solutions with paraffins only if the acids are less than half as long as the paraffin.

* The chemical side of these first experiments was guided by Dr. F. C. Frank, and the electrical work was performed by Prof. W. Jackson.

liquid state. If the volume of the whole molecule is substituted in Debye's equation, equation (8), then the viscosity in the solid wax at 40° C. is less than in the molten wax at 80° C.: if the volume of the rotating part is taken to be that of the polar group alone the apparent viscosity is about equivalent to that of castor oil. With liquid permitol the measured viscosity was a useful guide for predicting the behaviour of the dielectric, but for the solid there is no such guide available at present. At least it is possible to investigate the effect of the size of the molecule by using esters of different length: if the polar group is free to swivel in the rod, the length of the rod should be of no consequence. The result of such an investigation[11] is summarized in Fig. 17, and relates to esters ranging from 21 to 33 carbon atoms long. At any given temperature, say − 20° C., the viscosity of the wax will have the same value for the rotation of all the esters. Yet for butyl palmitate (21 carbons), $2\pi\tau$ is equal to 0·2 microsec., and for cetyl palmitate (33 carbons) $2\pi\tau$ is equal to 98 microsec. Thus the length of the rod does affect the relaxation time, and hence the whole rod must turn rigidly with the dipole. These measurements have thus disclosed something about the molecular structure of ester molecules, namely that the whole rod is rigid round its long axis; at every bond both C—C and C—O.

We have described the formation of an ester as the welding-together of an alcohol and an acid chain. Thus an ester of given total length can be made by welding an alcohol to an acid of equal length, or a short alcohol to a longer acid: in other words, the weld (that is, the dipole) can be placed at any desired point along the length of the finished rod. Thus it is possible to see whether the relaxation time depends on the position of the dipole: the answer is that it does not depend on the position but only on the total length. Thus ethyl stearate and butyl palmitate are both 21 carbons long; in the first the dipole is placed at 10 % of the length, and in the second at 20 %. Yet the electrical behaviour of these two esters has been found to be indistinguishable (see Fig. 17), and this is not the only test of this property which has been made. But since increasing the chain length from 21 to 33 carbons increases $2\pi\tau$ by 500 times, it is quite certain that the relaxation time cannot be proportional to the volume of the rod. Hence we now know that the very form of Debye's equation for τ is invalid in the solid state.

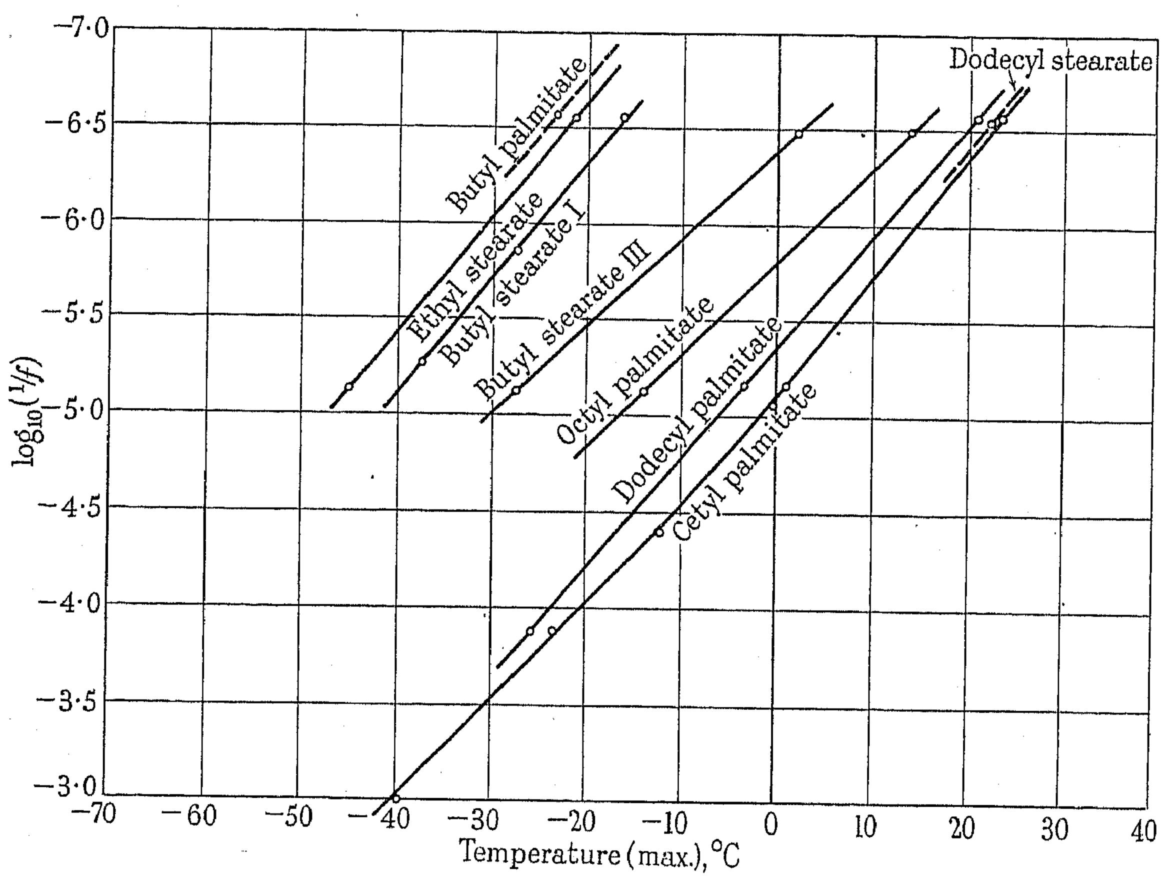


Fig. 17

There is one more very fascinating experiment to recount, and one which has obvious possibilities of technical use as well as being of deep scientific interest. Seeing we have proved that the dipole is rigid with the rod, it should be possible to obtain either twice the effect or zero effect by making a rod having two dipoles facing either in the same or in opposite senses. The sense can be arranged by using an odd or an even number of carbon atoms between the two dipoles: this is illustrated by Figs. 18(*a*) and 18(*b*) for dioctyl sebacate and dioctyl azelate respectively. Experiment showed[8] that the azelate gave the familiar sharp maximum of tan δ (0·33 % for 3·73 % concentration), whereas the sebacate gave no maximum and tan δ was less than 0·025 % between − 25° and + 60° C. Hence the oppositely directed dipoles were able to neutralize one another and render the material effectively non-polar so far as dielectric loss is concerned. Such a disposition of dipoles has obvious technical possibilities for the production of low-loss materials: these have been explored tentatively for certain alkyd resins.[9]

(4) SOME PROBLEMS YET TO BE SOLVED

This Address has reached already an inordinate length, yet there is still much fundamental work that has not been described. I must hope later to contribute to the *Journal* a paper on electrical and mechanical properties of paraffin waxes both with and without polar molecules. With this Address as an introduction I could then discuss the mechanism of the phenomena which the electrical engineer must try to visualize if he hopes to make low-loss materials having assigned qualities. It is fairly certain he will have to use polar materials, because so very many organic substances are naturally polar. Also it seems to me that he will want to avail himself of the powerful Van der Waals forces of the dipoles in order to raise the melting point and increase the mechanical strength.

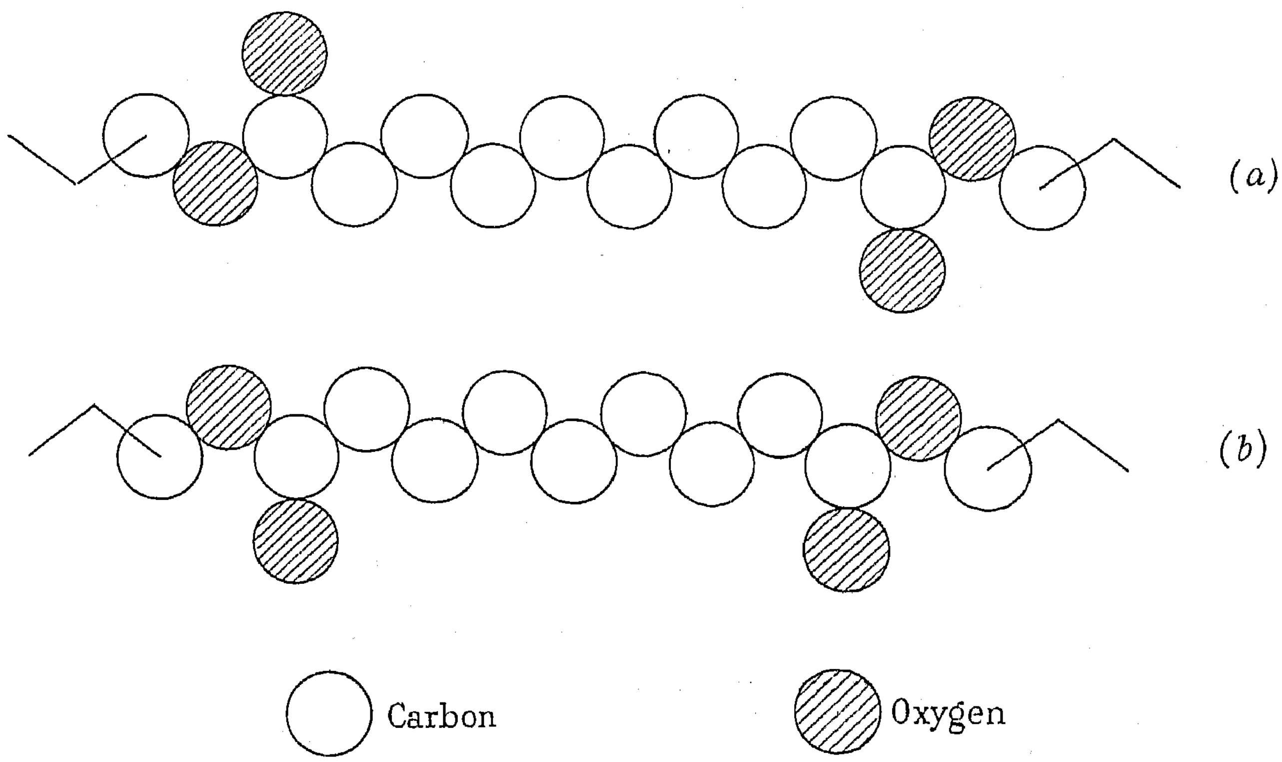


Fig. 18.—Part of the molecules of (*a*) dioctyl sebacate and (*b*) dioctyl azelate, showing how the dipole moments are in the opposite sense in (*a*) and in the same sense in (*b*).

He dislikes their electrical properties, and therefore he will render them innocuous as regards dielectric loss by using them in opposed pairs; or he will take steps to adjust their relaxation time so that maxima will be situated at a temperature far below, or far above, the working range. May one put it that he will gladly use rivets in his structure but either he will close them tightly or lubricate them freely to avoid friction loss. But before he can do that he must think out the mechanism of viscosity in a solid and learn the effect on the relaxation time of the shape of the molecule.

I will close on an imaginative note. May the day come when the mechanical, electrical and thermal properties of a desired dielectric are first specified; then the drawing office is instructed to design a molecule of the size and shape which will do the job; then this working drawing is handed over to the *chemical smith* to hammer up sufficient of the desired parts; and finally the erectors put it all together. The picture is no doubt fantastic, but not impossible. If it is fulfilled, we shall work with more mental satisfaction and certainty than in the days of the witch's cauldron of metaphorical herbs and frogs' legs which reigned supreme till a few years ago.

LIST OF SYMBOLS

a = radius of orbit.
C_1, C_2 = capacitances.
d = distance between condenser plates.
e = electronic charge.
f = frequency
g = acceleration due to gravity.
$\mathbf{I}$ = moment of inertia.
K = $C_1C_2/(C_1 + C_2)$.
k = Boltzmann's constant.
l = length.
M = electric moment of atom due to field.
m = mass.
N = number of molecules.
n = number of spheres per unit volume.
P = pressure.
p = $2\pi f$.
R = resistance.
s = $mgl/(\frac{1}{2}ml^2\omega^2)$.
T = absolute temperature.
t = time.
v = fractional volume of moisture in dielectric.
x = distance moved by proton.
y = $\alpha\epsilon/(2kT)$.
z = $\dfrac{\text{molecular volume from measurements of } \kappa}{\text{molecular volume from non-electrical measurements}}$
α = spring constant.
δ = $\pi/2$ — (lead angle of combination shown in Fig. 5*b*).
ϵ = electric field.
θ = angular displacement.
κ = dielectric constant.

LIST OF SYMBOLS—*continued*.

μ = inherent electric moment of molecule.
ρ = density of material and electrical resistivity.
ω = angular velocity.

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VIBRATION OF OVERHEAD LINE CONDUCTORS*

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SUMMARY

This paper is a résumé of available data concerning the causes of and remedies for wire failure in overhead line conductors subject to the " eddy " vibration due to steady winds ranging from 5 to 25 m.p.h.

The theory of eddy vibration is first outlined and tests are detailed dealing with the effect on wire failure of such factors as tension and the design of clamp and conductor. Studies of clamp and conductor motions during vibration are also described. It is shown that wire failure is caused mainly by the " nicking " action between wires occasioned by the clamping stresses, and to a lesser degree by the " hammering " between conductor and clamp. The " nicking " weakens the wires locally and thereby prevents full use being made of their fatigue strength. Though it is difficult to reduce the " nicking " action, other data showed that a reduction of wire failure could be obtained by a lowering of the working tension, and that the effects of " hammering " could be reduced by a suitable design of clamp.

Geographical factors favourable to vibration are then discussed, and data are set out which indicate that in general light and hollow conductors are more susceptible to vibration than other types. The " non-vibrating " conductor due to Preiswerk is also described. A discussion then follows of the various methods which have been employed in practice to reduce wire failure (i.e. methods of reinforcing the conductor at the clamp against the bending caused by vibration, the use of dampers, festoons, etc.). All these methods have been more or less successfully employed, and each has its advocate. It appears, however, that wire failure can be reduced by a suitable application of either, though it has not been found possible to make a definite statement concerning their relative costs, which in any case must depend on local conditions. In recent years, however, the tendency has been to employ dampers.

The " dancing " of conductors is then discussed. This is not a very frequent occurrence, and there is consequent difficulty in formulating mitigative measures.

CONTENTS

CONTENTS—*continued.*

(1) INTRODUCTION

It is now well known that steady winds of velocity below about 25 m.p.h. sometimes induce mechanical vibrations in overhead transmission line conductors, these vibrations taking the form of vibrating loops varying from about 6 ft. to 30 ft. in length. The vibrations subject the conductor wires to alternating bending stresses, which cause fatigue and, finally, wire failure. For reasons given later these failures occur usually just in or adjacent to the clamps.

Investigations with the object of reducing this wire failure have been carried out in various parts of the world, and there is now a vast literature available. The experimental work of the E.R.A., described herein, has been confined, however, to specific points of special interest to its members. It was, nevertheless, recognized that the reduction of wire failures due to vibration was one of those many-sided problems best solved by the pooling of information gathered from as many sources as possible. A résumé of published literature was therefore made, emphasis being laid on the more important points of immediate economic interest. This résumé, together with the work of the E.R.A., is presented in this paper, and conclusions are drawn therefrom.

At this stage it will be convenient to mention the generally accepted theory concerning the formation of vibrations in overhead conductors.†

When a smooth cylindrical body is placed in a fluid moving with steady velocity at right angles to the body,

* Official communication (Report Ref. F/T121) from the British Electrical and Allied Industries Research Association.

† P. D. MORGAN and S. WHITEHEAD: "A Critical Study of Mechanical Vibrations on Overhead Transmission Lines." (E.R.A. Report Ref. F/T 39.)

the formation of eddies in the fluid behind the body is dependent upon the expression

$$\frac{vd}{\nu}$$

where

v = velocity of moving fluid (in feet per second).
d = length of obstacle (diameter of conductor in feet).
ν = dynamic viscosity of fluid ($= 1 \cdot 59 \times 10^{-4}$ for air).

This expression, known as the Reynolds number, is a critical variable, so that when it attains a certain value there is a change from steady stream-line flow to eddy or turbulent conditions. Now the frequency of the eddies formed when this critical value is reached is given by

$$N \text{ (in periods per sec.)} = \frac{v}{d} \times f\left(\frac{vd}{\nu}\right)$$

In the case of air, though the expression vd/ν may vary from about 600 to 30 000, the function vd/ν over this range has been shown to remain approximately constant and equal to $0 \cdot 185$, so that for air

$$N = 0 \cdot 185 \frac{v}{d}$$

The theory assumes that the vibration of overhead line conductors is associated with the coincidence of this eddy frequency of the air with one of the elastic periods of the line, thereby causing resonance which initiates and maintains vibrations. Although in a particular case other factors such as elasticity of tower arms, etc., may be involved, the theory has not been seriously challenged, even though there have been cases where the observed frequency of vibration has differed from that deduced from theoretical considerations.

(2) INVESTIGATIONS CARRIED OUT BY THE E.R.A.

(a) Preliminary Laboratory Tests*

Concurrently with field observations of vibrating conductors detailed under (*b*), preliminary laboratory tests were made with 100-ft. lengths of 37/·110 in. steel-cored aluminium conductor (equivalent to 0·175 sq. in. copper section) to determine under given controlled conditions of vibration:—

(i) The behaviour of conductor and clamps.
(ii) The maximum bending stresses induced in the conductor.

With knowledge of (i) and (ii) it was considered that it would be possible to deduce the maximum safe stringing tension so that the fatigue range of the aluminium wires under vibration would not be exceeded. This idea assumed that wire failure was due solely to the repeated bending stresses, and it was only in later investigations that the harmful effects of such factors as "nicking" and abrasion (which cannot be allowed for mathematically) were appreciated. The tests yielded useful data, however, and are therefore detailed herein.

* These were carried out by Mr. H. W. B. Gardiner, of the G.E.C. Research Laboratory.

The general method of testing adopted for this and subsequent investigations was as follows:—

A conductor 100 ft. long was strung with cone-type clamps between two "A" frames simulating tower arms. A trunnion-type suspension clamp was fixed midway in an inverted position and attached via porcelain insulators to a concrete base such that the angle between the conductor and the horizontal (at the clamps) could be varied between 2° and 5°. Vibrations were imparted to the conductor by a cord tied to a rocker arm driven by the flywheel of a small motor. Each conductor was vibrated into 6 loops with a maximum displacement, in the centre of a loop, varying from ½ in. to 1½ in. according to the condition of test.

(i) Behaviour of Conductor and Clamps.

Vibration tests were first made with a maximum displacement in the centre of a loop of 1½ in. and with a tension of 8 000 lb.* Stroboscopic observations showed that the conductor and clamps did not vibrate in phase with each other but that there was a certain time-lag or displacement between them. This displacement was due to the restraining action of the clamps and it caused a relatively sharp local bending of the conductor about the clamp mouths. Accurate measurements were not made, but the displacement and bending appeared greater at the suspension than at the tension clamp (the latter, of course, being pivoted at one end, was better able to follow the motion of the conductor than the suspension clamp). After about 1 million vibrations, wire failures were found at the suspension but not at the tension clamp. It was also observed that those portions of the suspension-clamp shims and conductor which came into intermittent contact during vibration were marked by black patches and lines. This phenomenon of intermittent contact is subsequently called "hammering." It was found that some of the outer-wire failures had occurred at points marked by "hammering," and it was therefore concluded that "hammering" was a factor accelerating wire failure. Further tests with maximum displacement amplitudes of ½ in. instead of 1½ in. showed that the severity of "hammering" decreased with amplitude, and was greater at the suspension than at the tension clamp—owing no doubt to the greater displacement between conductor and suspension clamp.

(ii) Maximum Bending Stresses Induced in the Conductors.

In order to compute these stresses the bending formula

$$\frac{f}{y} = \frac{M}{\mathbf{I}} = \frac{\mathbf{E}}{r}$$

was applied.

M (conductor bending moment) was found from knowledge of the conductor tension and its displacement amplitude from the mean position.

The displacements were measured by attaching pointers to the clamps and to points along the conductor. These pointers traced the conductor and clamp motions on sensitized paper wrapped round revolving drums. The maximum curvature of the conductor $1/r$

* The tension was purposely made higher in these preliminary tests in order to obtain results fairly quickly.

was also found from these traces, and the product $M \times r (= \mathbf{EI})$ then calculated. **I** (moment of inertia of conductor section) and **E** (modulus of elasticity of the composite conductor) could not be determined independently since the conductor was stranded and made of two different materials. It was found that the product **EI** increased with conductor tension but decreased with an increase in conductor curvature. Now in an earlier report (Ref. F/T39)* formulae were developed for calculating the conductor bending stresses for two extreme conditions. The first assumed that during vibration the conductor bent as a whole about its own neutral axis, in which case y is equal to the conductor radius. The second assumed that each wire in the conductor bent separately, when y becomes equal to the radius of the individual wires. For 37/·110-in. S.C.A. conductor the corresponding values of **EI** calculated in the above report were found to be 130 000 and 3 560 lb./sq. in. Now in practice it is clear that for this size of conductor the values of y and **EI** must lie between their respective extreme values quoted above, since under different conditions of vibration a varying number of the conductor wires will bend individually (or partially so) and the remainder all together. The values of **EI** and y for a number of these different conditions were calculated, and a curve was drawn showing the relationship between them.

The value of y corresponding to the value of **EI** deduced from the test data (i.e. from $M/\mathbf{I} = \mathbf{E}/r$, M and r having been found) was read off from this curve, and the maximum conductor stress (f) then calculated from the formula

$$\frac{f}{y} = \frac{\mathbf{E}}{r}$$

The value of **E** for aluminium was taken as $9 \cdot 6 \times 10^6$ lb./sq. in.

This method enabled the bending stresses in the conductor at the tension clamps to be determined. Unfortunately, because of the more complicated motion of the suspension clamp and conductor it was not found possible in this case to measure M and $1/r$ with sufficient accuracy for mathematical purposes, and it was decided not to develop this line of the research any further.

The tests revealed, however, that there were irregularities in the motion of both tension and suspension clamps. They also showed that the suspension clamp lagged behind the conductor on the side to which vibrations were imparted, and "led" the conductor on the other side, thus amplifying the stroboscopic observations mentioned under (i). The irregularities were found to be due to the presence of a second harmonic, which in turn was found to be mainly caused by the presence of the suspension clamp itself. A certain amount of the complexity in the conductor motion also appeared to be due to the slight rolling of the clamp about its clevis pin, since the substitution of knife-edge pivots for the clevis pin resulted in a more sinusoidal motion (and a decrease in conductor curvature) being obtained.

(b) Observation on Existing Transmission Line

Simultaneously with the laboratory tests described in (*a*), measurements were made of the vibrations on a 800-ft. span of double-circuit 66-kV latticed-tower transmission line in N.E. England. The measurements were made with a stroboscope and a tuned-reed frequency meter, and vibrations were recorded with a cinematograph camera.

It was found that the motion of the conductors (37/·102 in. steel-cored aluminium), which then had a tension of about 4 300 lb., was often very erratic. The vibration amplitudes and loop lengths were continually changing, while at intervals beats of large amplitudes were imposed on vibrations of smaller amplitudes. These beats may have been due to waves reflected from the towers. It was noticed that conductors in the same span did not always vibrate at the same time, and that vibrations were not always transmitted through the suspension clamps; thus the conductor on one side of a clamp sometimes vibrated strongly, while on the other side it remained almost motionless. The maximum amplitude recorded in the centre of a vibrating loop was approximately 1·25 in., corresponding to a frequency of 900 cycles per minute and a loop length of about 35 ft. In general it appeared that the most common vibration frequency for this size of conductor and tension was between 900 and 1 100 cycles per minute, and it was concluded that the most severe condition likely to persist for any length of time corresponded to a maximum amplitude of $1\frac{1}{2}$ in. and loop length of 30–35 ft. These values were adopted as a guide in the investigations described below.

(c) Effects of Different Factors upon Wire Failure

This series of tests was carried out to determine to what extent wire failure under given conditions of vibration was influenced by such factors as tension and clamp weight, and also to examine in further detail the motions of suspension clamp and conductor. The method of testing was the same as employed under (*a*), except that 37/·102 in. instead of 37/·110 in. S.C.A. conductor was used, while the maximum displacement in the centre of a loop was kept at $\frac{3}{4}$ in. and the angle between horizontal and conductor made equal to about 2°, a value very common in practice. Except where otherwise stated a tension of 4 180 lb. was employed, thus simulating practical conditions for this size of conductor for temperatures below about 40° F. The tests were usually stopped after two outer-layer wires had broken (as revealed by the interruption of a paint line round the conductor at the clamps) or at the end of 5 million vibrations.

(i) Working Tension and Lay Ratio.

Tests were made with 10 conductor lengths, 5 being vibrated at a tension of 4 180 lb., and 5 at 3 300 lb. In each group of 5 there were three lengths of new conductor with lay ratios of 11·4, 13·5, and 15·5, and two lengths of old conductor (removed after two years of service) with lay ratios of 13·5 and 15·5.

At the end of the 10 tests it was found that at the higher tension there was a total of 23 broken wires as compared with 13 at the lower tension. All these breakages occurred in or near the suspension clamp, and there was a total of 26 failures in the inner layer of aluminium wires as compared with 10 in the outer layer.

* *Loc. cit.*

This showed that in practice inner-layer failures are liable to occur before those in the outer layer, despite the theoretically higher bending stresses in the outer layer. The reason for this is suggested below. No simple correlation between wire failure and lay ratio was found, neither was there evidence to show that from the vibration standpoint the "old" conductor had weakened during its period of service. Details of the test results are given in Tables 1A, 1B, and 2.

It was therefore concluded that in practice a reduction of wire failures due to vibration could be obtained by a reduction of the working tension, but that the effects of lay ratio (within the above limits) upon wire failure were very small.

A close examination of the wires at the end of this investigation showed that nearly all the failures occurred at points where the inner- and outer-layer wires crossed each other. At these points the clamping stresses had bedded the aluminium wires into one another, causing a kind of "nick" in points along each wire (see Fig. 1). The presence of small quantities of aluminium paste indicated that a certain amount of abrasion had occurred.

Table 1A

EFFECT OF WORKING TENSION ON WIRE FAILURE

<table>
<tr><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th></tr>
<tr><th rowspan="2">Test No.</th><th rowspan="2">Type of S.C.A. conductor used</th><th rowspan="2">Working tension</th><th rowspan="2">Number of vibrations to complete test</th><th colspan="2">Total number of aluminium wire failures in—</th><th rowspan="2">Total number of aluminium wire failures in conductor</th><th rowspan="2">Total number of failures in each group of 5 tests</th></tr>
<tr><th>Outer layer</th><th>Inner layer</th></tr>
<tr><td></td><td></td><td>lb.</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>Old, 15·5 lay ratio</td><td>4 180</td><td>5 296 000</td><td>1</td><td>5</td><td>6</td><td rowspan="5">23</td></tr>
<tr><td>2</td><td>New, 15·5 lay ratio</td><td>4 180</td><td>3 268 000</td><td>2</td><td>2</td><td>4</td></tr>
<tr><td>3</td><td>Old, 13·5 lay ratio</td><td>4 180</td><td>5 015 000</td><td>None</td><td>1</td><td>1</td></tr>
<tr><td>4</td><td>New, 13·5 lay ratio</td><td>4 180</td><td>4 060 000</td><td>2</td><td>3</td><td>5</td></tr>
<tr><td>5</td><td>New, 11·4 lay ratio</td><td>4 180</td><td>4 634 000</td><td>3</td><td>4</td><td>7</td></tr>
<tr><td>6</td><td>Old, 15·5 lay ratio</td><td>3 300</td><td>5 043 000</td><td>None</td><td>3</td><td>3</td><td rowspan="5">13</td></tr>
<tr><td>7</td><td>New, 15·5 lay ratio</td><td>3 300</td><td>3 700 970</td><td>2</td><td>2</td><td>4</td></tr>
<tr><td>8</td><td>Old, 13·5 lay ratio</td><td>3 300</td><td>5 041 140</td><td>None</td><td>1</td><td>1</td></tr>
<tr><td>9</td><td>New, 13·5 lay ratio</td><td>3 300</td><td>5 025 420</td><td>None</td><td>3</td><td>3</td></tr>
<tr><td>10</td><td>New, 11·4 lay ratio</td><td>3 300</td><td>5 123 600</td><td>None</td><td>2</td><td>2</td></tr>
</table>

N.B.—Note that the number of wire failures was greater at the higher tension.

Table 1B

EFFECT OF LAY RATIO ON WIRE FAILURE

<table>
<tr><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th></tr>
<tr><th colspan="5">Lay ratio of conductor and its working tension</th></tr>
<tr><th rowspan="2">Conductor lay ratio</th><th colspan="2">4 180 lb.</th><th colspan="2">3 300 lb.</th></tr>
<tr><th>Old conductor</th><th>New conductor</th><th>Old conductor</th><th>New conductor</th></tr>
<tr><td>11·4</td><td>—</td><td>7</td><td>—</td><td>2</td></tr>
<tr><td>13·5</td><td>1</td><td>5</td><td>1</td><td>3</td></tr>
<tr><td>15·5</td><td>6</td><td>4</td><td>3</td><td>4</td></tr>
<tr><td>Total number of wire failures . .</td><td colspan="2">23</td><td colspan="2">13</td></tr>
</table>

N.B.—There is no obvious correlation between lay ratio and wire failure, e.g. though (under Col. 3) failures in new conductor decreased with increase of lay ratio, the reverse was the case when the tension was lowered (Col. 5).

Table 2 (Compiled from Table 1A)

WIRE FAILURE IN OUTER AND INNER ALUMINIUM LAYERS (37/·102 S.C.A. CONDUCTOR)

<table>
<tr><th>1</th><th colspan="2">2</th><th colspan="2">3</th><th colspan="2">4</th><th colspan="2">5</th></tr>
<tr><th rowspan="2">S.C.A. conductor, and its lay ratio</th><th colspan="4">Number of wire failures in outer aluminium layer</th><th colspan="4">Number of wire failures in inner aluminium layer</th></tr>
<tr><th colspan="2">Tension 4 180 lb.</th><th colspan="2">Tension 3 300 lb.</th><th colspan="2">Tension 4 180 lb.</th><th colspan="2">Tension 3 300 lb.</th></tr>
<tr><td>Old, 11·4</td><td>—</td><td rowspan="2">3</td><td>—</td><td rowspan="2">0</td><td>—</td><td rowspan="2">4</td><td>—</td><td rowspan="2">2</td></tr>
<tr><td>New, 11·4</td><td>3</td><td>0</td><td>4</td><td>2</td></tr>
<tr><td>Old, 13·5</td><td>0</td><td rowspan="2">2</td><td>0</td><td rowspan="2">0</td><td>1</td><td rowspan="2">4</td><td>1</td><td rowspan="2">4</td></tr>
<tr><td>New, 13·5</td><td>2</td><td>0</td><td>3</td><td>3</td></tr>
<tr><td>Old, 15·5</td><td>1</td><td rowspan="2">3</td><td>0</td><td rowspan="2">2</td><td>5</td><td rowspan="2">7</td><td>3</td><td rowspan="2">5</td></tr>
<tr><td>New, 15·5</td><td>2</td><td>2</td><td>2</td><td>2</td></tr>
<tr><td></td><td colspan="4">10</td><td colspan="4">26</td></tr>
</table>

The " nicking " and " abrasion " had naturally weakened the wires locally, so that full use was not made of the fatigue strength of the aluminium; it was concluded, therefore, that these phenomena were the main factors accelerating wire failure.

The outer-layer wires, of course, only showed signs of " nicking " on one side, i.e. that adjacent to the inner layer, while the inner layer was " nicked " both on the side adjacent to the outer layer and on that adjacent to the wires in the steel core. This double weakening of the inner-layer wires probably accounted for the large number of inner-layer failures, despite the fact that these inner-layer wires were closer to the neutral axis of the conductor, and were therefore probably subjected to lower bending stresses.

It was also observed that a few of the outer-layer wires had failed at points not nicked on the under side, but at points marked by the " hammering " with the clamp shim. It was finally concluded, therefore, that though " hammering " between conductor and clamp also accelerates wire failure, it is a much smaller factor than the " nicking " and " abrasion " caused by the clamping stresses.

(ii) Types of Suspension Clamp, and Suspension-Clamp Inertia.

Vibration tests were then made with 6 new lengths of conductor employing a different type of suspension clamp in each case. The object of these tests was to examine the behaviour of each clamp and also to determine whether wire failure was affected by clamp weight.

It was realized, of course, that several tests on each clamp would have been desirable, but owing to time limitations these were not made. Such tests as were carried out showed that wire failure in the light clamps was as great as in the heavier clamps, but it was not found possible to correlate wire failure with either clamp weight, moment of inertia, or the length of conductor gripped by the clamp (details are given in Table 3A). Variations in the design of each clamp may have masked any correlation that may have existed.

In these, as in previous tests, wire failure was accelerated mainly by the effects of " nicking " and abrasion, and, to a lesser degree, by " hammering." In one of the light clamps (No. 5 in Table 3A), however, the effects of " hammering " were very severe. It was observed that in this case the " hammering " occurred just at the end of the clamp, which in this instance terminated in a sharp radius of curvature. All the outer- and inner-layer failures occurred opposite this radius, which also appeared to act as a fulcrum (about which the conductor bent) inducing high bending stresses in the conductor.

Vibration tests were then made with two new lengths of S.C.A. conductor in order to isolate the effect of suspension-clamp weight upon wire failure. These two tests were made with the same design of trunnion clamp as was used in the tests described under (*a*), but the

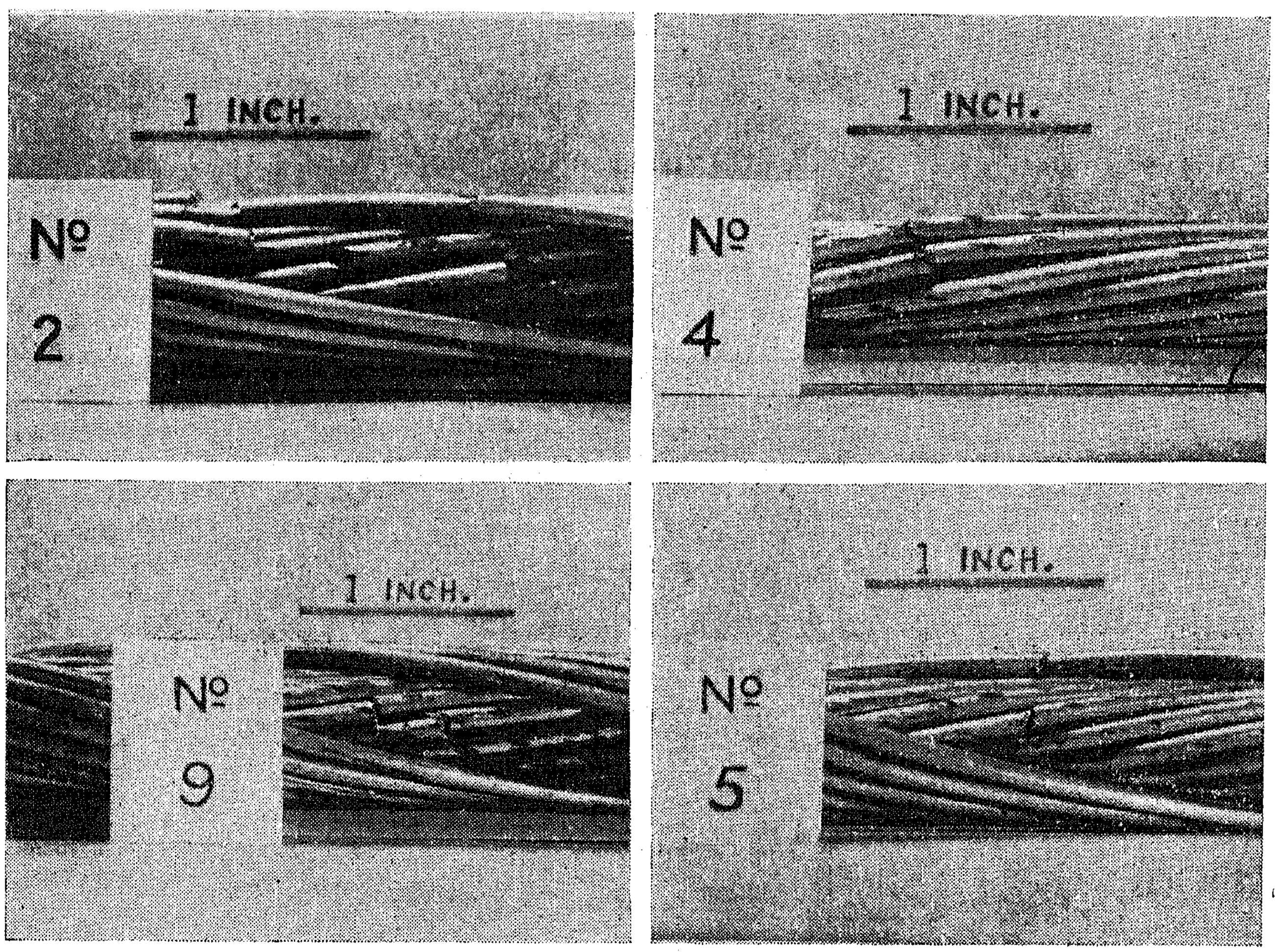

Fig. 1.—Showing wires which had broken at the end of the suspension clamp during vibration.
The scoring of the aluminium wires due to the clamping pressures is clearly seen. This scoring or "nicking," aided by abrasion, weakened the wires locally so that full use was not made of the fatigue strength of the aluminium. It will be noted how the actual failures occurred at points where the wires were "nicked."

first was carried out with a clamp cast in aluminium and weighing 6·5 lb., and the second with the ordinary malleable cast-iron type [as used in (i)], but evenly weighted with lead side-weights to bring its total weight to 12·5 lb. The results were compared with those of the earlier investigation detailed under (i), where the simple malleable cast-iron clamp weighing 10·5 lb. was used. Another series of tests was then made with the aluminium and unweighted cast-iron clamp to determine to what extent clamp weight influenced the maximum bending angle between clamp and conductor; for obviously the greater the bending angle the greater the tendency to wire failure. The angles were measured by means of small projectors (attached to clamp and conductor) which threw a beam of light on to a revolving-drum camera so that permanent records of the variations in the bending angle were recorded. Also, in order to determine whether the presence of the insulators

Table 3A

EFFECT OF DIFFERENT TYPES OF SUSPENSION CLAMP UPON WIRE FAILURE

1	2	3	4	5	6	7	8	9
Suspension clamp No.	Suspension clamp weight*	Clamp moment of inertia about horizontal axis†	Approximate length of conductor gripped by clamp	Number of vibrations to complete test	Total number of aluminium wire failures in— Outer layer	Inner layer	Total number of aluminium wire failures in conductor	Average number of vibrations to cause one aluminium wire failure
	lb.	lb.-ft.^{2}	in.					
1	10·0	0·185	6·75	1 948 850	2	1	3	649 000
2	9·25	0·641	3·75	1 276 800	2	3	5	256 000
3	8·5	0·415	8·00	2 300 550	2	1	3	767 000
4	6·75	0·409	4·25	2 363 850	3	2	5	472 000
5	5·0	0·128	7·5	1 207 470	2	3	5	241 000
6	2·6	0·115	8·5	1 217 000	2	1	3	402 000

N.B.—With these 6 different suspension clamps there is no obvious correlation between clamp weight (Col. 2), moment of inertia (Col. 3), length of clamp gripping the conductor (Col. 4), and wire failure (Col. 9).

* Excluding weight of side links.

† Taken about axis at right angles to conductor and passing through point of suspension.

Table 3B

EFFECT OF SUSPENSION CLAMP WEIGHT UPON WIRE FAILURE

1	2	3	4	5	6	7	8	9
Test No.	Trunnion suspension clamp employed	Suspension clamp weight and moment of inertia	Number of vibrations to complete test	Total number of aluminium wire failures in— Outer layer	Inner layer	Total number of aluminium wire failures in conductor	Average number of vibrations per aluminium wire failure	Remarks
1	"Light" aluminium clamp, specially made	6·5 lb.; 0·115 lb.-ft.^{2}	5 109 200	1	3	4	1 277 000	
2	"Ordinary" clamp, as used in some of the above investigations	10·0 lb.; 0·185 lb.-ft.^{2}	3 268 000	2	2	4	817 000	Taken from results given in Table 1A
3			4 060 000	2	3	5	812 000	
4			4 634 000	3	4	7	662 000	
5	"Weighted" clamp	12·5 lb.; 0·221 lb.-ft.^{2}	3 945 300	2	7	9	428 000	

N.B.—With the same design of suspension clamp wire, failures (Cols. 7 and 8) increased with clamp weight.

affected the bending angle, tests were made both with the insulator string present and with a steel strap substituted for the insulators.

In all the tests described in the above paragraph it was again found that wire failure was accelerated mainly by the " nicking " and abrasion between wires, and to a lesser degree by " hammering." Nevertheless, wire failure and " hammering " were greater with the heavier clamps, and although only one test was made on the light and weighted clamps it was found, as shown in Table 3B, that the average number of vibrations to cause one aluminium-wire failure was reduced to one-third when the clamp weight was doubled. It is important to remember, however, that lightness is not the sole criterion of the excellence or otherwise of any clamp design, and that because of the effects of " nicking " and " hammering " it is possible to nullify the beneficial effects of lightness by unsuitable design in other directions. The test results suggested, however, that the effects of " hammering " could be minimized by avoiding sharp ends or small radius of curvature in the clamp, and that it would be advantageous to employ a " transition radius of curvature " from that portion gripping the conductor to the beginning of a bell mouth at the clamp end. Later tests mentioned under (iii) also support this suggestion. The photographic records obtained showed that the angular motions of clamp and conductor (upon which the value of the bending angle depends) were complex, and it was clear that several " repeat " tests would be necessary before general conclusions could be drawn. Although it was not possible to carry out many " repeat " tests, useful data were obtained. Thus in no case was it found that the maximum conductor bending angle exceeded 30–35 minutes,* and that while the presence of the insulator string sometimes increased the complexity of the clamp and conductor motion (as compared with those cases where a steel strap was substituted for the insulators) the effect was very slight.

(iii) Types of Conductor.

Since the magnitude of the " nicking " action between wires clearly depended upon the clamping stress per unit area it was considered that its effects could be reduced by increasing the contact area between the wires Vibration tests were therefore made with eight lengths of S.C.A. " segmental " conductor, the wires of which are shaped (see Fig. 2) to give greater contact area. The same copper equivalent size conductor was used as in earlier tests, i.e. 0·15 sq. in., to correspond with the 37/·102 in. S.C.A. size previously used. The segmental conductor was vibrated with two types of suspension clamp, four tests being made with a short and light clamp which gripped the conductor by means of pressure exerted on aluminium cones, and four tests with the bell-mouthed trunnion suspension clamp used in earlier investigations.

The test results again showed the importance of properly shaping the suspension clamp mouth, viz. with the short clamp wire failure was more common than in any previous series of tests, there being a total of 40 wire failures, all of which occurred just in or at the edge of the suspension clamp, and in both the inner and outer aluminium layers. This short clamp, though exerting uniform pressure on the conductor, terminated suddenly in a sharp radius of curvature. " Hammering," which was in this instance very severe, was localized on this radius and on the conductor facing this radius.

It was concluded that this sharp radius of curvature had [as in a previous case mentioned under (ii)] accentuated the effects of " hammering " to a high degree and was therefore mainly responsible for the larger number of failures with this clamp.

In the four tests with the bell-mouth trunnion clamp it was necessary to wrap that portion gripped by the clamp with aluminium tape (as well as to employ clamp

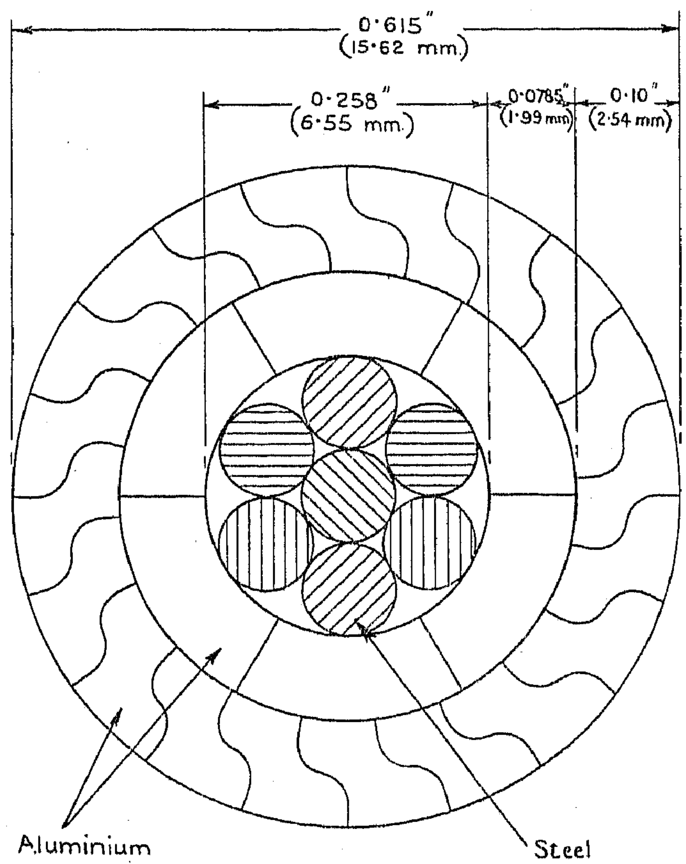

Fig. 2.—Cross-section of " lock-coil " segmental conductor.

shims), owing to the smaller diameter of the " segmental " as compared with the ordinary conductor of the same copper equivalent cross-section. As shown in Table 4, this method of clamping considerably reduced wire failure, there being only a total of 11 failures (all at the suspension clamp) as compared with the 40 mentioned above. This result, when compared with that obtained in an earlier investigation with ordinary S.C.A. conductor (see Table 4), indicated that the " segmental " was superior, though it is difficult to say how much the improvement was due to the additional protection from " hammering " and the cushioning effect of the aluminium tape.

Tests were then made with four lengths of " balanced lay " conductor made up with the ordinary circular-section wires. The same bell-mouthed trunnion suspension clamp (with aluminium shims) as had been used in earlier tests was again employed. In this type of conductor the lay ratios of each layer are " balanced out " against each other, so as to minimize the twisting ten-

* The significance of this is that bending stresses in practice are not likely to be greater than those calculated on the assumption that the conductor bending angle at the clamp will not exceed about half a degree—since the severity of these test conditions is not likely to be exceeded often in practice.

Table 4

EFFECTS OF DIFFERENT TYPES OF CONDUCTOR UPON WIRE FAILURE

1	2	3	4	5	6
			Total number of wire failures in—		
Test No.	Type of conductor and clamps	Number of vibrations to complete test	Outer aluminium layer	Inner aluminium layer	Total number of aluminium wire failures in conductor
1	"Segmental" conductor (with a short and light suspension clamp)	5 410 570	2	None	2
2		2 956 000	4	None	4
3		2 117 000	All (20)	All (6)	All (26)
4		441 000	7	1	8
5	"Segmental" conductor with the trunnion type suspension clamp (with shims) and aluminium tape	5 154 750	None	None	None
6		5 491 850	None	None	None
7		4 072 650	8	None	8
8		5 329 050	2	1	3
9	"Ordinary" 30/·102 in. aluminium + 7/·102 in. steel, with trunnion type suspension clamp and shims (no tape)	3 268 000	2	2	4
10		4 060 000	2	3	5
11		4 634 000	3	4	7
12	"Balanced lay" 30/·102 in. aluminium + 7/·102 in. steel, with trunnion-type suspension clamp and shims, but no tape	1 112 640	2	None	2
13		1 385 000	3	None	3
14		5 040 706	3	1	4
15		3 257 850	3	None	3

N.B.—Note how the performance of the "segmental" with trunnion-type suspension clamp (with aluminium tape protection) as given in Tests 5, 6, 7, and 8, was superior to its performance with the short clamp which terminated in a sharp radius of curvature (Tests 1, 2, 3, and 4). Its performance was also superior to that of the ordinary conductor (Tests 9, 10, and 11).

At first sight it may appear that the performance of the "balanced lay" was superior to the "ordinary" conductor, but, bearing in mind the number of vibrations per test, there appears little to choose between them.

dency of the conductor as a whole. It was thought possible that this reduction of torsional stress might (by reducing the net conductor stress) be sufficient to affect the wire failure accelerated by vibration. However, wire failures were again caused mainly by the effects of "nicking" and abrasion, and there was little in the results (see Table 4) to suggest that in practice the "balanced lay" will be either superior or inferior to the ordinary conductor, provided care is taken to avoid twisting while uncoiling from the drum.*

(3) RÉSUMÉ OF PUBLISHED INFORMATION ON "EDDY" VIBRATIONS

In this Section of the paper a résumé of more important data of immediate economic consequence is given, and from this and the work of the E.R.A. conclusions are drawn regarding the reduction of wire failures due to vibration.

(a) Occurrence and Measurement of "Eddy" Vibration

The available information tends to show that the "eddy" vibration of either copper or S.C.A. conductors is most common in steady transverse winds ranging from about 5 to 25 m.p.h. Vibration does not seem to occur with winds parallel to the conductors, and the few such cases reported may have been due to the winds having vertical components. Some observers* have noticed that long lines across plains are very susceptible to vibration, and that long spans are, in general, more liable to vibration than short spans.† It has sometimes been stated‡ that vibration susceptibility may be greater in winter than in summer, and test data obtained by Maass,§ with copper conductors, showed that vibration amplitudes were greater in the winter than in the preceding summer of 1930. He suggested that the heating of the conductor and the consequent disturbance of the surrounding air by direct radiation from the sun in the summer may retard the formation of "eddies," but without further data it is not possible to comment upon this, except to suggest that ordinary current heating may have comparable effects. With the prevalence of similar winds in summer as in winter, vibration troubles may, of course, be more common in the latter season, because of the higher working tension due to lower temperatures. Vibration records obtained by Gaylord,‖ however, showed no correlation between vibration and

* It must be remembered that in all the tests detailed above, care had been taken to avoid twisting of the conductor while it was being unwrapped from the drum. Had this not been the case then it is possible that the lower torsional stresses inherent in the "balanced lay" conductor might have had some effect.

* See Bibliography, (28). † *Ibid.*, (13). ‡ *Ibid.*, (28). § *Ibid.*, (23). ‖ *Ibid.*, (13).

temperature, although Koontz* and Knopp† concluded from various observations that vibration was more common at night than during the day, and Varney‡ quotes data to the same effect. Shields§ and others have observed vibration to be particularly common during the early morning or afternoon, and Jamieson‖ mentions sunrise and sunset as bad periods, though Gaylord¶ and Maass found that vibration seemed to occur at any time of the day. In general, therefore, it is concluded that vibration susceptibility cannot, as yet, be correlated with any time of the day or season, but that local topography and meteorological conditions may cause eddy-forming winds (and hence vibration) in a particular district to be more common at certain times than at others.

Vibration amplitudes and frequencies, etc., have been measured in various ways, and the methods employed by the E.R.A. have already been described. Stroboscopic observations were also taken by Maass** to measure loop lengths and frequencies by means of a telescope and a graduated scale behind the conductor. Schmitt and Behrens†† recorded vibration amplitudes and frequencies with a pointer (attached to the line) which traced on a drum, while a special carbon-pack recorder was used by Buchanan.‡‡ Wright and Mini§§ fixed special instruments to the conductors, each instrument consisting of a stylus (tracing on a clockwork-driven chart) attached to a light pendulum operated by the line vibrations. Similar vibration recorders were used by Monroe and Templin.‖‖ In laboratory tests more refined methods of recording vibration phenomena can be used. Stolte¶¶ obtained oscillograms of the motions of test wires in a wind tunnel by recording on photographic paper the light reflected from small mirrors attached to the wires, and, as previously described, a similar method was used by the E.R.A. An electro-dynamical method of inducing vibrations has been employed by Van Staveren,*** and a development of a similar method by Carroll and Koontz††† enables vibration amplitudes, etc., to be measured with considerable accuracy.

In general, however, the "field" results obtained by these various observers were similar to those obtained by the E.R.A. and already discussed, except that the vibration amplitudes were not always so large. Field tests were made by Buchanan‡‡ to determine the reflection factors at the supports and the attenuation of travelling waves in the conductor, etc. He found that the magnitude of the transmitted component of a travelling conductor wave increased with the flexibility of the clamp support.‡‡‡ Observations made during the various tests of the E.R.A. suggested that the rigidity or damping effects of the suspension clamp are not the only factors tending to reduce the transmission of vibrations through the clamp. Thus it was found that unless the frequency of the vibrations reaching the suspension clamp very nearly coincided with one of the line harmonics of the adjacent span only small amplitudes would be built up therein. In practice it is unlikely that the natural harmonics of adjacent spans will always be of the same frequency, because of the variations in wind loads, span lengths and tension, etc. This, together with the damping effects of the clamp, may account for the conductor sometimes vibrating quite vigorously on one side of the clamp and remaining nearly motionless on the other.

* See Bibliography, (19). † *Ibid.*, (18). ‡ *Ibid.*, (43).
§ *Ibid.*, (38). ‖ *Ibid.*, (17). ¶ *Ibid.*, (13). ** *Ibid.*, (23).
†† *Ibid.*, (37). ‡‡ *Ibid.*, (3). §§ *Ibid.*, (44). ‖‖ *Ibid.*, (26).
¶¶ *Ibid.*, (41). *** *Ibid.*, (39). ††† *Ibid.*, (4).
‡‡‡ Varied by means of a spring.

(b) Conductor Tension and Properties Affecting Vibration Susceptibility

As explained in the Introduction, the formation of eddies behind a smooth cylindrical body depends upon vd/ν, i.e. eddy formation in air behind an overhead conductor will depend primarily upon wind velocity and conductor diameter. When the natural period of the line (or a harmonic thereof) coincides with this eddy frequency the theory assumes that vibration occurs. The natural period of the line in turn depends upon conductor tension, weight per unit length, and modulus of elasticity. It will be realized that it is a specific combination of all these factors which will determine whether a conductor will vibrate or not, while the stiffness of the conductor (in turn affected by the number of wires and internal friction) will determine the size of the vibration amplitudes. Some of the above factors are interdependent, and an attempt is made below to state the relative importance of each.

Data regarding the effect of tension upon vibration susceptibility are rather inconclusive. It has been argued* that an increase in tension will cause greater vibration amplitudes, and some test results obtained by Koontz† and Scattergood‡ support this view, but since (with the same reasoning) the loop lengths will also increase it does not follow that an increase in tension will necessarily induce higher bending stresses in the conductor. In the extreme case where tension is reduced to zero there will of course be no vibration, but tests made by Monroe and Templin§ indicate that the incidence of vibration cannot be economically reduced by reduction of tension within practical limits, and that line conductors may be expected to vibrate whatever the tension. It appears, however, that more wire failures have occurred at high than at low tensions, though what proportion of these were caused by the higher mean stress, or the greater number of vibrations in a given time,‖ or a greater vibration susceptibility (resulting in an increased incidence of vibration), cannot be definitely stated. The work of the E.R.A., however, has shown that under similar test conditions more failures will occur at the higher tension. Whether, therefore, the incidence of vibration is or is not materially affected by a change of tension within practical limits, there is no doubt that a reduction of tension will help to minimize wire failure.

Many field tests¶ have been carried out to compare the performance of the heavy copper conductors (or steel-cored copper) with those of aluminium and steel-cored aluminium. In many cases the data obtained

* See Bibliography, (1) and (34).
† *Ibid.*, (20). ‡ *Ibid.*, (36). § *Ibid.*, (26).
‖ The frequency of vibration increases with working tension; thus, in the case of a single wire,

$$f \text{ (in cycles per sec.)} = \frac{1}{2l}\sqrt{\frac{Tg}{w}}$$

so that in a given time more vibrations will occur at the higher frequency.
¶ See Bibliography, (16), (19), (23), (29), (38).

throw light upon the effect of such factors as conductor diameter and weight upon vibration susceptibility. The test results are not always strictly comparable owing to the different test conditions employed, etc., but in general they suggest that, under practical conditions now operating, conductors of aluminium or steel-cored aluminium are more liable to vibration than those made up of the heavier copper, and that hollow conductors are more susceptible to vibration than the ordinary types. It has also been noticed that vibration troubles (wire failure) were comparatively rare in countries when the majority of high-voltage lines were of copper but increased with the introduction of aluminium, aluminium alloys, and hollow conductors,* i.e. with an increase of conductor diameter. The increase in wire failure may have been due to the lower fatigue strength of aluminium and to the greater span lengths employed. Moreover, weight for weight the diameter of a light conductor (or of a hollow type) will be greater than that of a heavier conductor, and the former will therefore receive more energy from the wind and be more liable to vibration. These facts support the view that the incidence and amplitude of vibration will be greater in the case of light conductors, and also support Shields's† suggestion that vibration susceptibility will increase with a decrease of the ratio of conductor weight (per foot) to diameter.

Various laboratory and field tests have also been made to determine the effect of the shape of conductor cross-section upon vibration susceptibility. Thus Davison, Ingles, and Martinoff,‡ first experimented (with water as a medium) with about a dozen different cross-sections, viz. circular, square, triangular, that formed by two wires twisted together, etc., and found that the greater amplitudes were usually obtained with the circular cross-sections, and the least with the triangular sections. Stranded conductors approximating a triangular section were also tested in air, and though a suppression of 20 %–40 % in amplitude was sometimes found (as compared with the conductors of circular section), the triangular conductor sometimes vibrated when the circular remained steady. Similar tests (in air) made by Maass§ with conductors of circular, oval, triangular, and fluted cross-section, showed that vibration amplitudes were in general considerably reduced by using conductors with sharply-defined profiles and of triangular cross-section. Monroe and Templin‖ carried out field tests with conductors composed of a single circular wire, 3 wires and 7 wires, and employed different working tensions. In general, the incidence and amplitude of vibration was least with the 3-wire conductor (i.e. that approximating a triangular section) and greatest with the single-wire conductor. Other tests were made with an ordinary standard S.C.A. conductor and a conductor employing a certain number of large-diameter wires in the outer layer, thus loading the conductor helically. The amplitude of vibration was again found to be greater with the standard type than with the special type.¶

* See Bibliography, (34) (Discussion). † *Ibid.*, (38). ‡ *Ibid.*, (8). § *Ibid.*, (23). ‖ *Ibid.*, (26).

¶ One power company in the U.S.A. reduced vibration in a single span by spiralling a single wire round the conductor for about 300 ft. in the centre of the span, thus "weighting" the conductor helically and breaking up its symmetry [see Bibliography, (38)]. The method was not proceeded with, however.

Stickley* describes field tests on 1 000-ft. spans with ordinary circular-section S.C.A. conductor and conductor of oval cross-section (with major axis vertical) in which the incidence and amplitude of vibration was as great with the oval as with the ordinary conductor. A reduction of up to about 50 % (as compared with the vibration of the "ordinary"), however, was obtained by twisting the oval conductor to give one complete turn every 20 ft. A reduction in vibration was also obtained by twisting two circular S.C.A. conductors together, the maximum amplitudes being about $\frac{3}{16}$ of those with the circular conductor.

In general, therefore, it appears that (neglecting such factors as a variation in internal friction that may modify the result) conductors approaching a circular cross-section are more liable to vibration than those of other section, while conductors of triangular section are the least susceptible to vibration. The cross-sections found to be least liable to vibration are rather impracticable and should also be avoided on account of larger corona effects and greater wind and ice loadings. It does not seem, therefore, that vibration can be economically reduced by a modification of the shape of the conductor cross-section.

Little direct data are available regarding the effect of conductor stiffness upon vibration.† It is obvious, however, that any changes in the physical properties of the conductor which increase its stiffness (such as an increase in wire size) will also increase the self-damping action caused by friction, and thus reduce vibration amplitude. The decrease of vibration obtained by various experimental methods of increasing the self-damping action has been very small and would be of little commercial significance. Thus, though Davison‡ found that amplitudes of conductors filled with a special "tacky" filler were often reduced as much as 20–40 % (as compared with those of an ordinary unfilled conductor), these conductors often vibrated while the unfilled conductor remained steady. The painting of the conductor§ may also temporarily effect a reduction in vibration amplitudes, though it appears that the presence of bitumen only slightly increases the self-damping. Lay ratio within the commercial limits now employed may have some effect on conductor stiffness, but there are few reliable data on this point.

(4) MITIGATIVE MEASURES

Before discussing the various methods adopted either to reduce vibration or to minimize the wire failure caused, it is desired to emphasize again the importance of "nicking" and abrasion as factors accelerating wire failure. These phenomena are bound to occur where there are high clamping stresses associated with circular-section wires, and their importance is not sufficiently emphasized in the various studies made to determine the causes of wire failure. It must also be remembered that, although mathematically it is possible to indicate (from calculations based on the measured value of the conductor bending angles) whether fatigue failure will occur, such calculations cannot take into account the

* See Bibliography, (40). † *Ibid.*, (1), (5), (9), (38). ‡ *Ibid.*, (9). § *Ibid.*, (38).

weakening effects produced by "nicking" and "abrasion."* The latter phenomena can, of course, be reduced by a reduction of the clamping stresses per unit area, though it is impossible to eliminate them altogether with any clamp that is to grip the conductor. It is therefore suggested that, in general, short clamps should be avoided.

Apart from the necessity of minimizing "nicking" and "abrasion," the various measures adopted to reduce vibration troubles can be divided into four groups:—

(a) Modification of clamp and conductor design and working conditions.†
(b) Reinforcement of the conductor at the clamps.
(c) Suppression of vibration by the provision of dampers.
(d) Other methods—festoons, eveners, etc.

(a) Modification of Clamp and Conductor Design and Working Conditions: the Preiswerk non-vibrating Conductor

The effect of various clamps and differently stranded types of S.C.A. conductor upon wire failure under given conditions has already been described in this paper. So also has the effect of conductor section and tension upon vibration susceptibility and wire failure, and it is unnecessary to refer to them again. There is, however, a development in conductor design which requires special attention, namely the non-vibrating conductor designed by Preiswerk.‡

Because of the different elastic moduli of the two metals in the ordinary steel-cored aluminium conductor the steel core and aluminium sheath are worked at different stresses. Hence if the core and sheath are tensioned separately with a load corresponding to that which each takes in the composite conductor, they will tend to vibrate at different frequencies (i.e. at different wind velocities) and with different loop lengths and amplitudes. This fact is utilized in the design of the "vibration-free" conductor. In this the annular aluminium sheath is made to fit very loosely over the steel core so that there is "play" between them (see Fig. 3). Each therefore tends to carry its load independently and, within the limits allowed by the "play," is free to rattle or vibrate in relation to the other. It is claimed that vibrations induced in the sheath by a given wind velocity and eddy frequency are damped out by the core (which normally tends to vibrate on a different harmonic), so that vibration of the conductor as a whole is reduced to a minimum. Vibration records have been published which support this claim, and it is also stated that there are no signs of "hammering" or wear between core and sheath.§

* Similar remarks were made by Noyes [see Bibliography, (8), Discussion], though with particular reference to special conductor sections where high concentrations of stress might cause surface deformation or "nicking." Nefzger [see Bibliography, (28)], however, also noticed that wire failure occurred at points of wire crossing, i.e. where "nicking" and "abrasion" occur, and also found that clamps designed to prevent the conductor from slipping seemed to favour breakages. This of course may have been because in such clamps the clamping stresses are high and hence "nicking" will be severe.

† Under this heading are included improvements which could be made in the design of single clamps. On this basis "eveners" and double suspension clamps fall into a rather different category, as in both cases the conductor is gripped in two places by auxiliary clamps pivoted to a yoke which in turn oscillates about the main point of suspension.

‡ See Bibliography, (31), (32), (33).

§ *Ibid.*, (32).

Table 5

COMPARISON OF A NORMAL S.C.A. CONDUCTOR WITH AN S.C.A. "NON-VIBRATING" CONDUCTOR OF THE SAME COPPER EQUIVALENT

1	2	3	4	5	6	7	8
Size and type of conductor	Number and diameter of steel strands in core	Radial clearance between steel core and aluminium sheath	Number and diameter of aluminium strands in inner layer	Number and diameter of aluminium strands in outer layer	Total cross-sectional area of steel	Total cross-sectional area of aluminium	External diameter of conductor
*Case I: Where the "Clearance" or "Play" between Steel Core and Aluminium Sheath in the "Non-Vibrating Conductor" is Obtained by Increasing the Inner and Outer Diameter of the Aluminium Sheath**							
0·175 sq. in. copper equivalent normal S.C.A. conductor	7/0·110 in.	Nil	12/0·110 in.	18/0·110 in.	0·0665 sq. in.	0·285 sq. in.	0·77 in.
0·175 sq. in. copper equivalent "non-vibrating" S.C.A. conductor	7/0·110 in.	0·037 in. (0·94 mm.)	16/0·098 in.	22/0·098 in.	0·0665 sq. in.	0·286 sq. in.	(3 × 0·110) (4 × 0·098) (2 × 0·037) = 0·796 in.
Case II: Where the "Clearance" or "Play" is Obtained by Decreasing the Diameter of the Steel Core†							
Normal steel‡ Aldrey conductor	7/0·098 in. (7/2·5 mm.)	Nil	12/0·098 in.	18/0·098 in.	0·0528 sq. in.	0·226 sq. in.	0·69 in.
Same copper equivalent steel Aldrey‡ "non-vibrating conductor"	7/0·071 in. (7/1·8 mm.)	0·041 in. (1·05 mm.)	12/0·098 in.	18/0·098 in.	0·0276 sq. in.	0·226 sq. in.	0·69 in.

* Data kindly submitted by The British Aluminium Co., Ltd.

† Data kindly submitted by M. Preiswerk.

‡ Copper size not given.

There are, however, certain points to be considered before designing a "vibration-free" conductor of the same current capacity as the normal type. The play (or clearance) between the steel core and the aluminium sheath may be obtained in two ways, viz.:—

(i) By reducing the diameter of the steel core so that the overall diameter of the "vibration-free" conductor remains the same as that of the ordinary type. This is

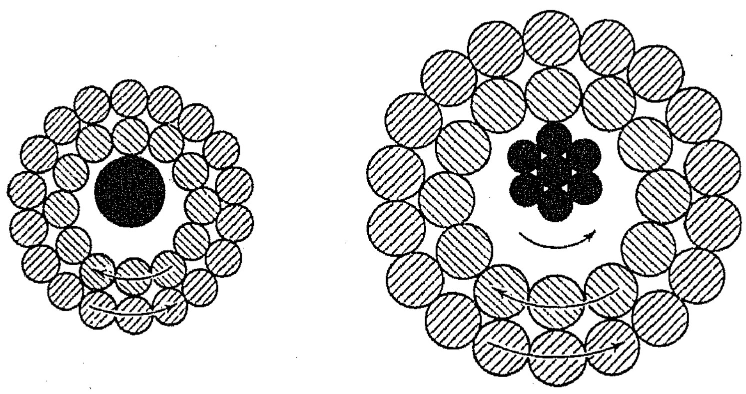

Fig. 3.—Section through "vibration-free" conductor.

the method adopted by Preiswerk, and a comparison between an ordinary and a non-vibrating conductor of equal current capacity is shown in Table 5. It will be noticed that in the non-vibrating conductor the cross-sectional area of the steel has been reduced by nearly 50 %, in order to obtain a "play" of 1·05 mm. (0·041 in.) as measured on radius.

Whether or not this is a justifiable procedure depends largely on the conditions under which the "vibration-free" conductor is to be used. If it be considered in relation to or to replace (i.e. give the same maximum sag as) a conductor of the same aluminium area, erected in accordance with the Regulations of the Electricity Commissioners, and of which the breaking load is estimated according to B.S. 215, then the procedure cannot be justified. The stress on the conductor in such conditions is considerably increased. On the other hand, if the "vibration-free" conductor is intended to replace one of which the permissible tension was estimated in accordance with the usual German practice, then the reduction in area of the steel core is generally justified. If the "vibration-free" conductor is to be used in a new line, then it is a matter of economics—balancing the cost of rather greater sags against the cost of dampers, etc.

(ii) By slightly increasing the overall diameter of the aluminium sheath. Thus the equivalent to the "grid" size of S.C.A. conductor (37/·110 in.) could be made up of the same steel-core size (7/·110 in.) with two layers of aluminium wires of 0·098 in. diameter, the first layer with 16 and the second with 22 wires. The clearance as measured on radius would then be 0·037 in. (0·94 mm.) and the overall diameter of the cable would be 0·796 in., as compared with 0·770 in. for the standard conductor. There would thus be a slight increase in the maximum wind and ice loading.

According to information supplied to the Association, about 700 miles of this conductor has been erected, or is in course of erection, in Germany.

(b) Reinforcement of the Conductor at the Clamps

The various mitigative devices grouped under this heading are shown in Figs. 4–9. The main object of all of them is to stiffen and strengthen the conductor at the clamps (particularly at the suspension clamps) so that its resistance to bending is thereby increased. In the device shown in Fig. 4 this is accomplished by the two outside members, which are fixed at the suspension clamp and attached to the conductor by means of the split sleeves S. The conductor simply rides over the suspension clamp and is here free from the normal clamping stresses. The sleeves S are slightly tapered inside so that at the span ends their diameter is a little larger than that of the conductor, thus tending to minimize the formation of a reflection point for vibrations. A device designed on similar principles is shown in Fig. 5 and has been used with partial success in Norway.* Two short cables of unequal length were attached and strung in the same horizontal plane parallel to the conductor by means of the auxiliary and the main suspension clamp. While the conductor was thus stiffened the chances of reflection points being formed were minimized by employing side cables of unequal length.

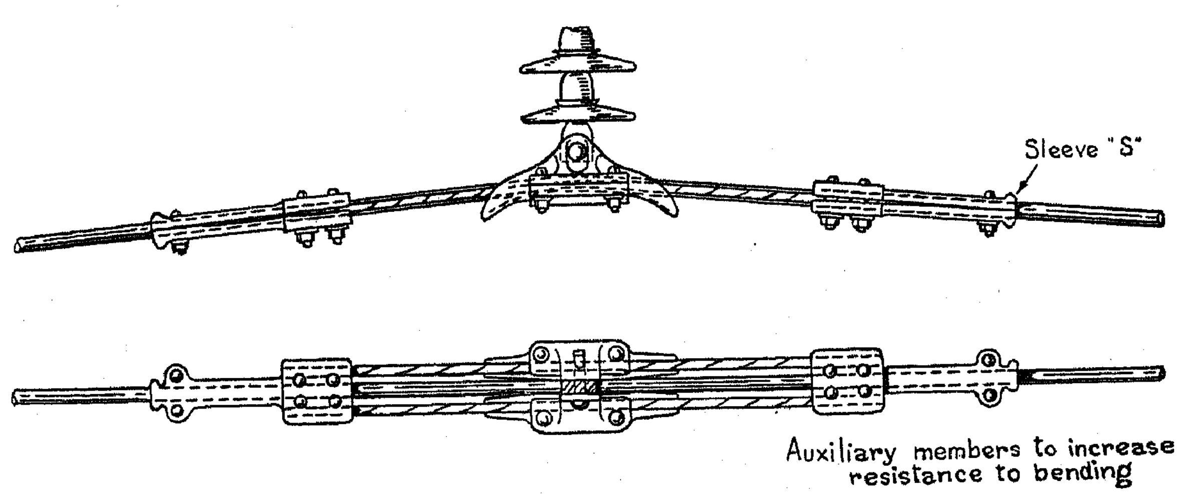

Fig. 4.—Side-member reinforcement.

The devices shown in Fig. 6 are usually known as "armour rods." The conductor is simply stiffened for a distance of about 2 to 3 ft. on either side of the clamp

* See Bibliography, (16).

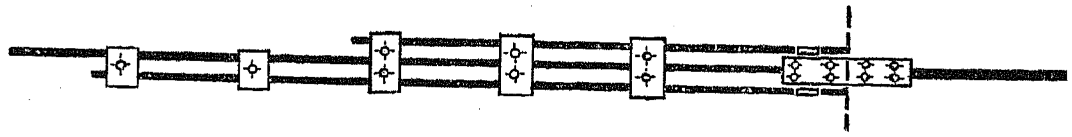

Fig. 5.—Staggered side-member reinforcement.

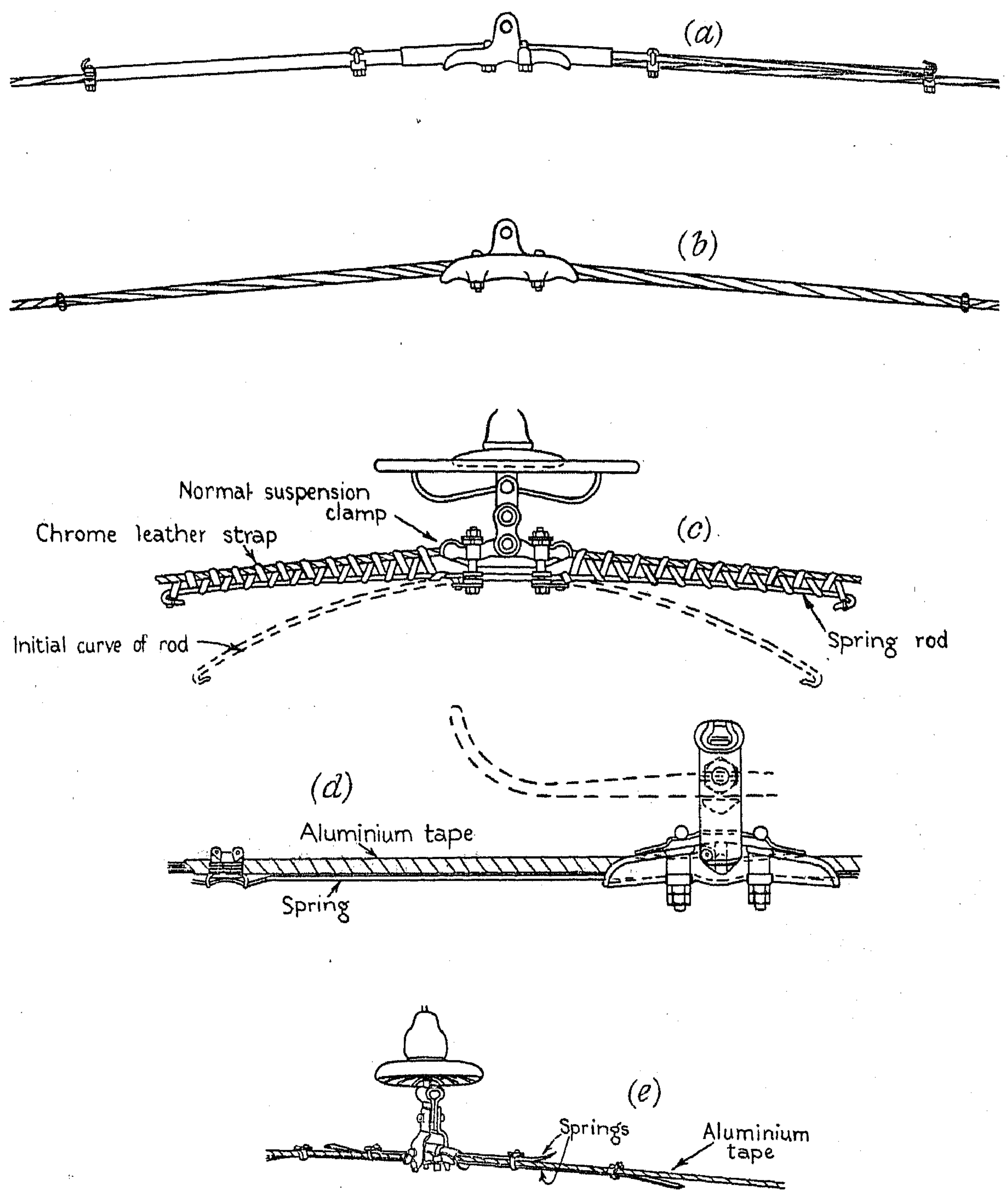

Fig. 6.—Types of conductor reinforcement.

(a) Plain armour rod.
(b) Tapered armour rod (Varney).
(c) Spring and strap reinforcement.
(d) Spring-tape reinforcement.
(e) Staggered spring-tape reinforcement.

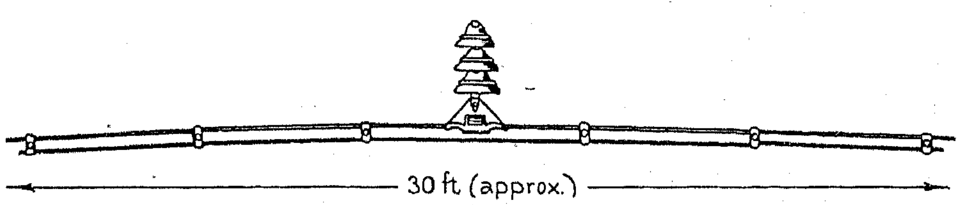

Fig. 7.—Bate damper.

Line conductor: 30 aluminium and 7 steel wires each 0·1059 in. diameter. Light aluminium clamps. Supplementary length of conductor free from suspension clamp.

by binding round and subsequently clamping a number of cylindrical rods or wires. In the type shown in Fig. 6(*b*) the rods are tapered towards the span end and spiralled round the conductor, thus minimizing sharp changes in conductor armour-rod cross-section, and reducing the formation of those reflection points which help to maintain vibrations. This probably accounts for the *reduction* in vibration amplitudes (observed in America and England) sometimes obtained when this device has been used. Larger clamps are required for armour rods, but it has been stated that wire failure in America and Canada* has been arrested by their adoption.

A form of reinforcement which has been successfully used in Germany is shown in Fig. 6(*d*). A steel spring is placed beneath the conductor and extends for about 2 ft. on either side of the suspension clamp. The ends are clamped to the conductor against the force of the spring, and the conductor is additionally stiffened by being wrapped with tape for the length of the spring. A similar arrangement is shown in Fig. 6(*e*), and it will be seen that here the provision of an additional but shorter spring on top of the conductor minimizes the formation of definite reflection points more than in the first design. A similar method of reinforcing (due to Electrical Improvements, Ltd.) is shown in Fig. 6(*c*). This has the advantage that there are no auxiliary clamping spots on the conductor at which vibration troubles due to a certain amount of wave reflection might appear. Fig. 7 shows an arrangement due to Bate† which is a little difficult to classify. An auxiliary conductor length is clamped at regular intervals beneath the main conductor, and extends for about 15 ft. on either side of the suspension clamp. Tests in Australia and England have shown that the device also damps out or greatly reduces vibrations in the span, and for this reason it has been called the " Bate damper." From Fig. 7 it is clear, however, that the auxiliary conductor stiffens and strengthens the main conductor, and because of this the device has been classified under " reinforcements " in this paper. In effect the device can be called a reinforcement which also acts as an efficient damper, though to a lesser degree, as mentioned later, this is true of other types of reinforcement.* A certain amount of wave reflection and consequent conductor bending may occur at the end of the auxiliary conductor, but there is no available evidence on this point. Applications of some of these reinforcements to tension clamps are shown in Figs. 8 and 9. The tension clamp, however, seems to give less trouble than suspension clamps. This is not because of any constructional advantage of the tension clamp, but because it is affected by vibrations from one span only (whereas there are two spans liable to induce vibration in the suspension clamp). Also, as explained in Section 2(i), the necessary pivoting arrangements enable the tension clamp to follow the motion of the conductor more closely than the suspension clamp, so that smaller conductor bending angles are induced.

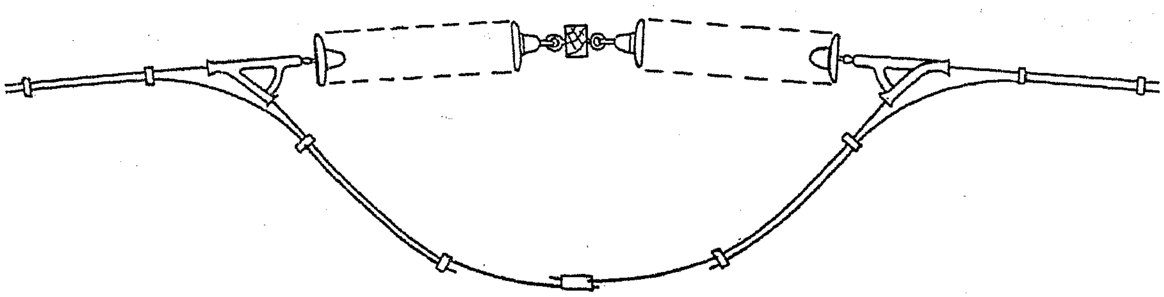

Fig. 8.—By-pass armour rod applied to tension clamp.

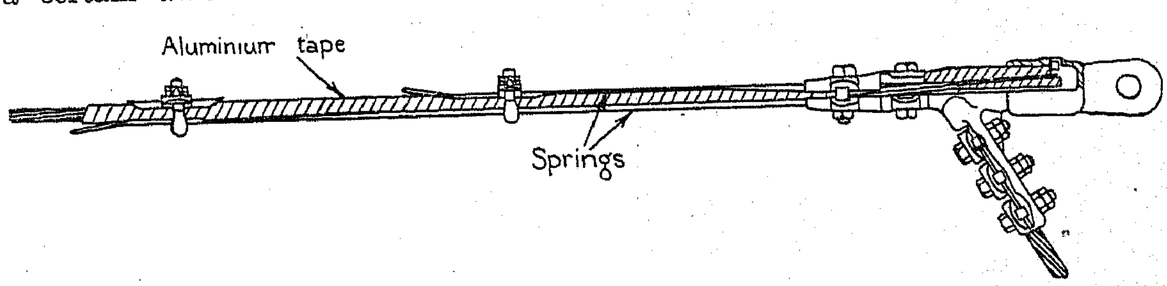

Fig. 9.—Staggered spring-tape reinforcement at a tension clamp.

(c) Suppression of Vibration by Provision of Dampers

Vibration dampers, unlike the reinforcing arrangements just described, are not designed either to strengthen or to stiffen the conductor at the clamp. They are merely devices (designed with a certain natural frequency of their own) which are attached to the line near the clamps to suppress vibrations as they arise, and thus prevent harmful conductor amplitudes from being formed. The performance of some dampers has been investigated mathematically, and laboratory tests have also been made to determine the most suitable sizes and weights for given conditions.† Briefly, however, the theory accounting for the suppression of vibration by dampers may be summed up as follows. Vibrations of the conductor induce out-of-phase oscillations in the damper. These oscillations oppose and therefore tend to damp

* See Bibliography, (27).
† Sometimes called the Bate damper [see Bibliography, (1) and (2)].

* See Bibliography, (35).
† *Ibid.*, (21), (30), and (42).

out the agency producing them, so that the line vibrations are suppressed or prevented from growing to an appreciable magnitude.

The Stockbridge damper is shown in Fig. 10. In this damper a short length of steel cable with a mass at each end is clamped at its centre to the main conductor near the clamp, and in practice the magnitude of vibrations

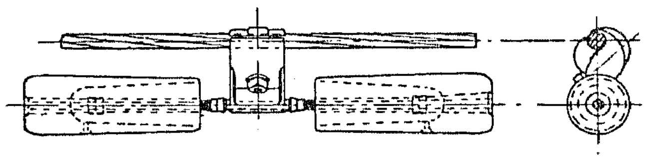

Fig. 10.—Stockbridge damper.

generated in the line is limited by the oscillation of the damper weights. Tests have shown that this damper has two natural frequencies and that it is most efficient when the period of the line vibration coincides with one of these natural frequencies.* In America, where the Stockbridge damper is very popular, it is customary to design the standard dampers so that the mean frequencies lie between the two natural frequencies, so that with variation of line frequency the damper will still remain efficient. One important point† concerning this damper is that even if it were operating at a node, the oscillation of the conductor about the node would still cause the damper weights to vibrate because of their cantilever arrangement beneath the conductor. (Similar remarks

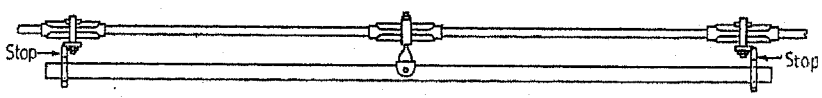

Fig. 11.—Hofmann lever damper.

In recent designs only one stop is used, though the length of the lever remains the same.

apply to the Hofmann lever damper described below.) This may partially account for the considerable reduction of vibration that the Stockbridge damper appears to effect. Much research concerning the Stockbridge damper has been carried out in America, and mathematical analyses of its action have been made.‡

The Hofmann lever damper (see Fig. 11) consists of a steel lever pivoted slightly out of centre from a clip beneath the conductor. The two ends of the lever are

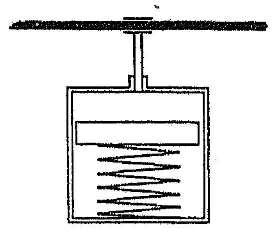

Fig. 12.—Simple type of spring-piston damper.
(As used by Holts, see Bibliography 16.)

free to move up and down between stops in guides attached to the conductor. (In recent designs, however, only one stop is used, hence less adjustment on the line is required.) In practice the lever rattles in a "see-saw" fashion, and the vibrational energy received from the line is dissipated in this way instead of being transmitted to the clamps. It has, however, been suggested that the rattling of the lever sets up trains of high-frequency vibrations which are easily damped out by the conductor itself. Tests by independent investigators* have shown that this damper is quite effective and may be efficacious over a wider frequency range than other types. As

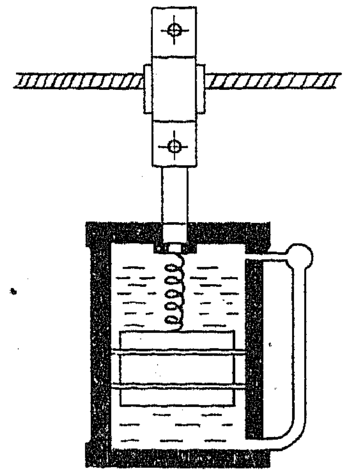

Fig. 13.—Spring-piston oil damper.

mentioned later, however, it is liable, at any rate in its earlier forms, to the drawbacks of noise and wear.

The spring piston damper shown in Fig. 12 has been successfully used in Norway.† The piston forms a loose fit inside the cylinder and constitutes an out-of-phase vibration system of its own which tends to damp out the agency (i.e. conductor vibrations) causing it to oscillate. Data published by Holts‡ showed that, as with other types, the damper was most effective when its natural period of oscillation approximated to that of the vibrating conductor loop, and it was stated that

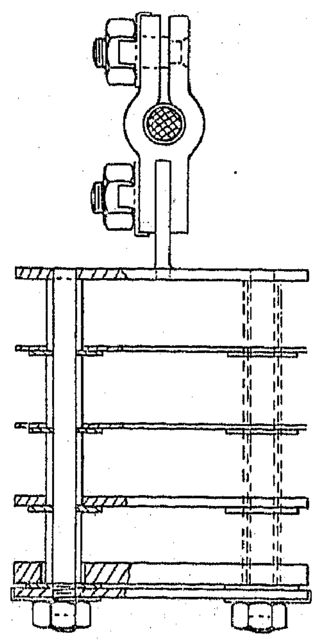

Fig. 14.—Plate damper.

under these conditions line amplitudes were reduced by about 90 %. Other types of piston damper, including the oil-pot type, are shown in Figs. 13–17. The oil-pot damper (Fig. 13) has been used in Switzerland and Germany. As a method of reducing vibrations it does not seem to be so popular as the Hofmann lever damper,

* See Bibliography, (26), also discussion.
† Advanced by P. J. Ryle.
‡ See Bibliography, (21), (42).

* See Bibliography, (24), (28), (35), (37). † *Ibid.*, (16). ‡ *Ibid.*

though it is claimed to be quite efficacious.* A plate damper in which plates of different weights (to correspond to different frequency and amplitude ranges) has also been developed in Germany and is shown in Fig. 14. It is claimed that this apparatus is quite efficient,† even if noisy in operation, but it does not appear to have been widely used.

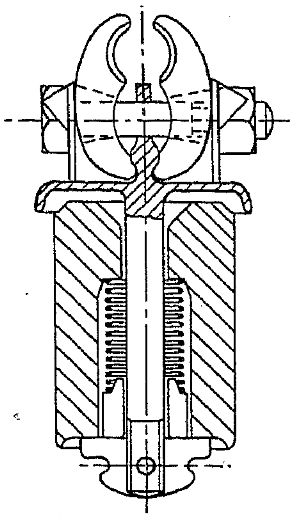

Fig. 15.—Spring-piston damper.

More modern types of spring piston dampers are shown in Figs. 15 and 16, and it is stated (test results have been quoted to the Association) that their efficiency is practically independent of line frequency. In the damper shown in Fig. 16, for example, only about 90 % of the weight is supported by the spring, the remainder being carried by the stop. When a travelling wave arrives at the clamp the weight is thrown upwards and, most of the line energy being absorbed in this way, the wave passes on to the clamp with reduced amplitude.

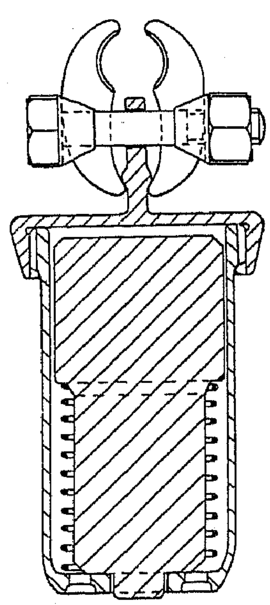

Fig. 16.—Spring-piston damper (recent design).
Claims are made that the efficiency of this damper is practically independent of line frequency.

When the weight falls it transmits a "blow" to the conductor and tends to set up a second wave therein, but since this is not in phase with the first, because of the elasticity of the spring, it is claimed that the damper has no "natural frequency" and therefore has a wide application. Some of these dampers, which combine the principles of the simple piston type and the plate type, have been installed in Switzerland, and so far seem to have given satisfactory service. Observation of their

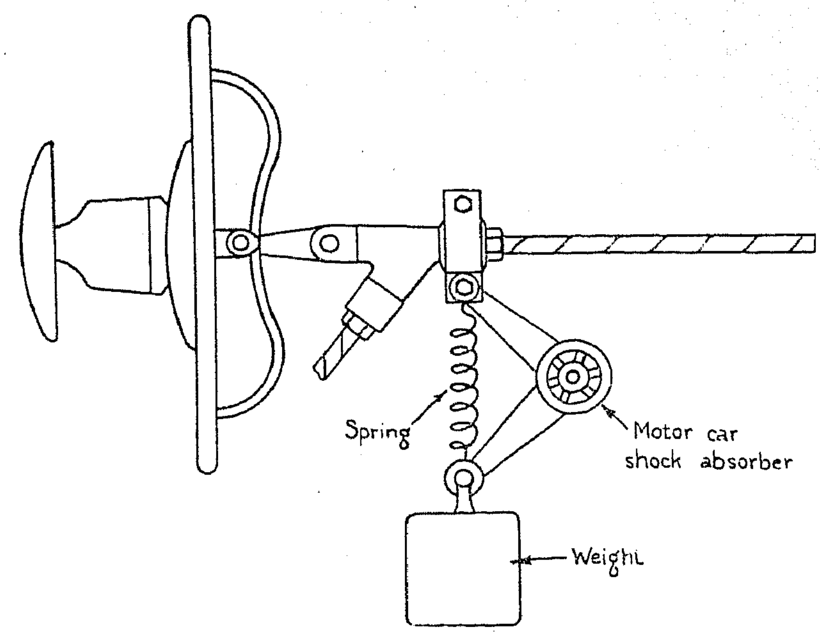

Fig. 17.—Suggested shock-absorber damper.

performance, however, is not yet completed.* The damper shown in Fig. 17 was proposed by Ryle† but it is suggested that its efficiency would be higher if attached to the line farther from the clamp, i.e. nearer the first antinodal position of the more common loop-length. A pneumatic damper developed in Germany is shown in Fig. 18, and although tests are referred to which show it can be designed to be effective, its use is not common and its cost is stated to be high.‡ Little data are available regarding its performance.

(d) Other Measures (Festoons, Eveners, and Special Clamps, etc.)

The "festoon" method of damping vibrations is shown in Fig. 19. The principle of action is that the points (A in Fig. 19) of festoon attachment to the line

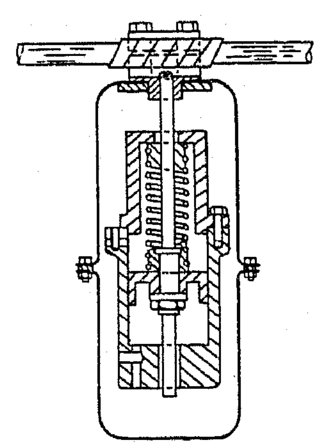

Fig. 18.—Pneumatic damper.

are designed to coincide with the antinodal positions (or points near them) of loop lengths which normally are of common occurrence. In order that the whole arrangement may be effective over a wide range of vibration loops and frequencies, the festoon loop lengths and

* See Bibliography, (24). † *Ibid.*, (34) (Discussion).

* See Bibliography, (33). † *Ibid.*, (34). ‡ *Ibid.*, (28).

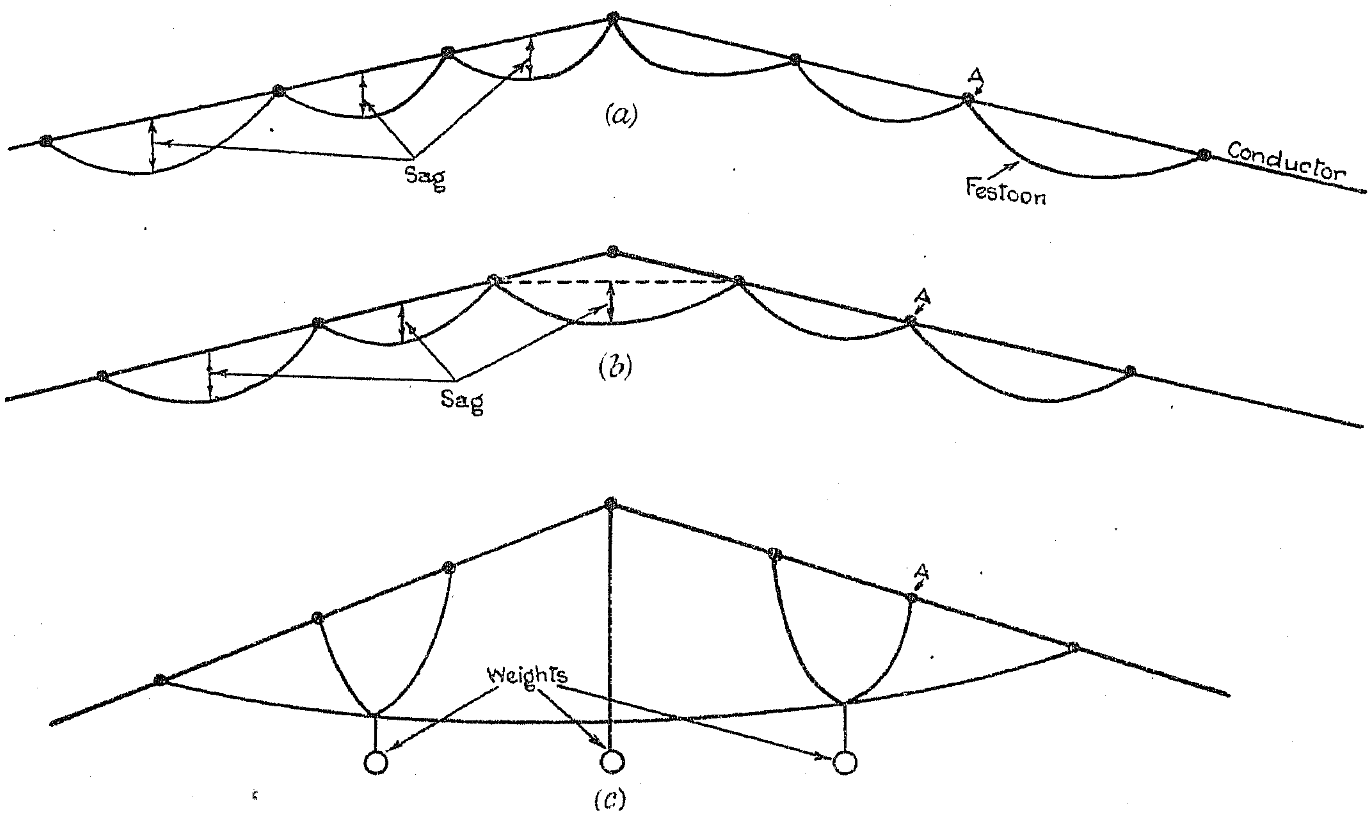

Fig. 19.—Types of festoons.

points of attachment have often been staggered. The sags of the various loops have also been made different.* In practice the festoon has usually consisted of ordinary steel rope, hemp-core galvanized-steel wire hoisting rope, or of lengths of the actual line conductor. Field tests in trial installations have shown that festoons practically eliminate vibration at the clamps, and considerably reduce it in the span. The absence of a definite reflection point may of course partially account for the reduction of vibration in the spans. Lawton† found that the maximum amplitudes observed after the installation of festoons were less than $\frac{1}{8}$ in., though the festoons consisting of lengths of line conductor were slightly more effective than those of steel rope. Festoons have been successfully used in Australia, and according to claims made are as efficient as dampers of the Stockbridge type.

" Eveners " and double-saddle clamps have been used in America and Canada. It is claimed that they allow a more gradual and uniform bending of the conductor than is the normal case, since that portion of the conductor riding immediately under or over the point of suspension is not clamped (this will be apparent from inspection of Figs. 20 and 21). The local formation of high bending stresses in the conductor, just outside the clamps, is thereby reduced. The double-saddle clamp, and also a similar type, viz. a " centre free " suspension clamp, is used in connection with the Boulder Dam enterprise, and laboratory tests‡ are referred to which show the superiority of these forms of clamp over other types.

Both these clamps (Figs. 20 and 21) to some extent reduce these effects of interference between adjacent spans, as do reinforcing devices such as are shown in Fig. 4. The difference, however, is that the " eveners " and double suspension clamps tend to separate the effects of clamping and bending without materially stiffening the conductor.

Another type of " anti-fatigue " clamp (sometimes called the Westinghouse anti-fatigue suspension clamp) has been tested in America* and is said to have given very satisfactory results, but no details are available to the E.R.A.

In field tests mentioned by Nefzger† the conductors within the suspension clamp were protected by rubber, and although " they behave well " it appears that at this stage the efficacy of this method of protection is limited by the life of the rubber. In addition, large-diameter clamps are required to accommodate the rubber, though, as mentioned previously, an improvement might be effected by protecting and stiffening the conductor within the clamp with aluminium tape.

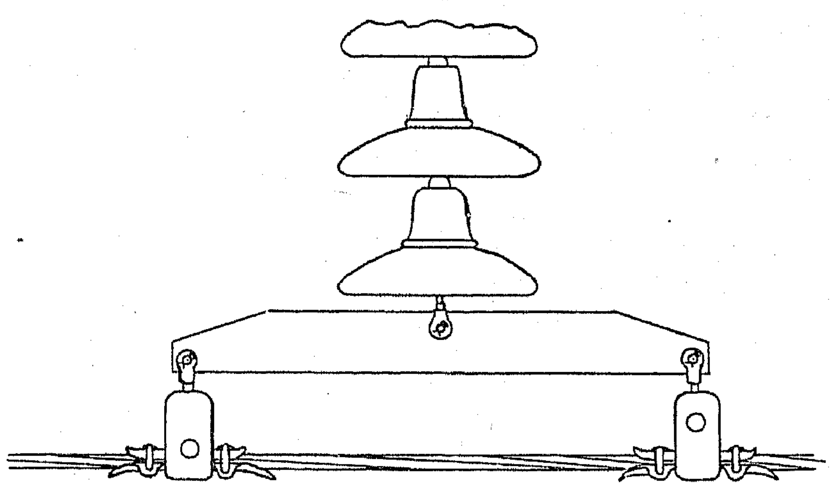

Fig. 20.—Evener.

In New Zealand‡ attempts to reduce vibration in cadmium-copper telephone wires have been made by wrapping fibrous material round the wires near the points of support (Fig. 22), and there seems to be no

* See Bibliography, (22). † *Ibid.*, (22). ‡ *Ibid.*, (36).

* See Bibliography, (12). † *Ibid.*, (28). ‡ *Ibid.*, (17).

reason why this method should not be applied to overhead conductors. For example, a layer of aluminium tape wrapped tightly round the conductor for a short distance near the clamp would act as a reinforcement and minimize wave reflection, in addition to having the advantage of simplicity.

An attempt to reduce vibration has also been made by the use of "dual conductors,"* i.e. the use of two smaller-diameter conductors [either (i) clamped a few inches apart at regular intervals or (ii) strung closely together at the same sag so that when erected they touch all along their length] instead of one of normal cross-section. The net cost per route mile of a dual conductor will of course be a little greater than that of the normal conductor of the same current capacity. The field tests mentioned by Stickley† indicated, however, that only a slight reduction in vibration was obtained when the dual conductor was strung as under (i), and though amplitudes and the incidence of vibration was considerably reduced when method (ii) was employed, it seems that the success of this method will be largely dependent upon the dual conductors always remaining in contact (i.e. having equal sags) during service. There is also the disadvantage that under (ii) considerable conductor wear may occur owing to friction between the "dual" conductors, and that the total wind and ice loading will be greater than that for the normal type.

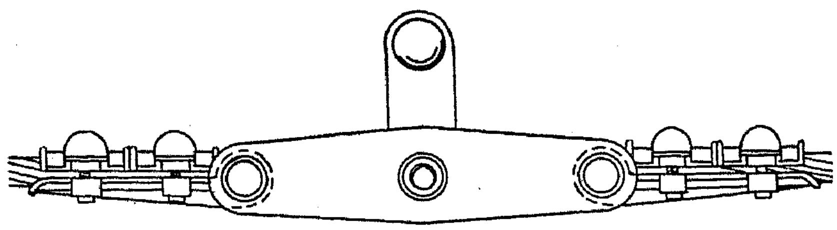

Fig. 21.—Double-saddle suspension clamp.

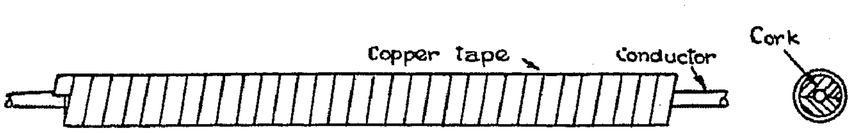

Fig. 22.—Cork damper used for telephone wires.

(5) DISCUSSION OF THE MORE IMPORTANT MITIGATIVE MEASURES

Even when working conditions associated with tension, clamp design, etc., have been modified as far as economically possible, it may still be necessary to consider the use of some of the mitigative measures described above, and the relative merits of these are discussed below.

The obvious merit of reinforcing devices is that they reduce the bending of the conductor at the clamp under all conditions of vibration. Their efficiency is therefore independent of line conditions (frequency, etc.), which is not necessarily the case with dampers. They also tend to reduce vibration in the span itself,‡ viz. in the tests detailed by Ryle,§ the Bate, Electrical Improvements, Ltd., device, and Varney's tapered armour rod, were found to reduce vibration amplitudes by about 50–80 %, 50–70 %, and 10 % respectively. In general this reduction is probably brought about by the stiffening of the conductor at the clamp, which in turn minimizes the formation of reflection points which normally help to sustain vibrations.

With the available data it is difficult to determine to what extent strand failures may still occur under the armour-rod type of reinforcement, but since the latter reduces conductor bending at the clamp it has generally been assumed that strand failure has also been reduced. An examination reported by Deck* of certain sections of a steel-cored aluminium conductor earth wire fitted with armour-rod reinforcements showed that, after some years of service, the conductor under the armouring was still unimpaired, whereas failures had occurred on sections not provided with armouring. In other cases there was no further increase in the number of strand failures after the reinforcements or armour rods had been installed. It appears, therefore, that armour rods or reinforcements reduce wire failure, though in some cases, especially where vibration is severe, it is possible that it may not be completely arrested.

Most reinforcements cannot be attached to existing lines without dismantling the clamp, and most require some modification to the clamp, as will be obvious from those shown in Figs. 6–9. They are therefore more suitable for use on new lines where it is desired to safeguard against vibration troubles than on lines already in operation.

As intimated above, however, aluminium tape could be used as a reinforcement without interfering with the clamps, though the whole theory of reinforcements suggests that an improvement could be effected by first wrapping the conductor with tape within the clamps [see remarks on possible cushioning effect of tape at the end of Section (2)].

Regarding dampers, it must be remembered that their elastic or moving parts may be subjected to high stresses and excessive wear, so that inspection and renewal may be necessary. The work of Holts,† Margoulies,‡ and Nefzger§ in Europe, and Koontz‖ and others in America, all supports the view that maximum efficiency is obtained when the line vibrations coincide with the natural period of the damper. This of course would be expected from theoretical considerations, but in some dampers, viz. the plate and spring-weight types shown in Fig. 14–16, it is claimed that under ordinary operating conditions such variation in line frequency as usually occurs has but little effect on the efficiency of the damper. The field tests of Margoulies‡ and Nefzger§ suggest that the efficiency of the Hofmann lever damper is less affected by a variation in line frequency than either the Stockbridge or the simple piston damper, and the work of Schmitt and Behrens¶ also indicates that in practice the efficiency of the lever damper remains high. As mentioned later, however, tests in England and France** have shown that the lever damper is liable to suffer from the drawbacks of noise and wear.

The information published, and also furnished to the Association, indicates, however, that if knowledge of the

* See Bibliography, (40). † *Ibid.*, (40).
‡ *Ibid.*, (26) and (27). § *Ibid.*, (35).

* See Bibliography, (10). † *Ibid.*, (16). ‡ *Ibid.*, (24).
§ *Ibid.*, (28). ‖ *Ibid.*, (26) (Discussion).
¶ *Ibid.*, (37). ** *Ibid.*, (35) and (24).

more common conditions of vibration of a given line be obtained, effective dampers of either the spring piston, lever, or Stockbridge type can be designed. Should, of course, " standard " dampers be attached to lines where the nature of the vibrations proves to be considerably different from those to which the dampers were designed, then their efficiency may be reduced, and some of the moving parts may be subjected to very high stresses.

The question arises as to how many dampers per span are necessary to reduce vibration to a negligible value and to minimize the strain imposed on each damper. Regarding the Stockbridge damper, Shields* concluded that up to 3 per span may be necessary. Koontz† recommended 4 per span, with staggered spacings from the clamp. Monroe and Templin‡ agreed with the latter in suggesting that in spans over 1 000 ft. long four dampers were sometimes necessary. They stated, however, that they had usually found two per span sufficient, and that under such conditions the damping was so complete that the advantage gained by the use of staggered spacings would be very small. Wright and Mini§ also found two dampers per span quite effective (on 770- to 875-ft. spans). Experience obtained in Switzerland with the spring piston damper shown in Fig. 16 also indicated that two dampers per span are necessary but sufficient, and the same experience has been obtained with the Hofmann lever damper though four of the latter are sometimes used on long spans, i.e. river crossings, etc. Little information is available regarding the effect of using only one damper per span, but from the above it would appear that the resultant damping may be unsatisfactory, and that there is then the likelihood that the single damper may be subjected to considerable mechanical strain and wear (leading to what is known as " drooping " of the cantilever arms in the case of the Stockbridge damper).

There is insufficient information available to show how weather conditions affect the performance of dampers—thus accumulations of ice or dirt inside the weight of the Stockbridge type or at the stops of the Hofmann lever damper might seriously impair their efficiency, i.e. by altering their weight, or weight distribution, and restraining the action of moving parts, etc. No trouble in this connection, however, has come to the notice of the Association.

The question as to which type of damper is, in general, the most effective and economical will only be settled by further experience. All the advocates of the various forms now marketed quote test and field results showing that their particular form is efficacious, and since the test conditions (working tension, span length, conductor size, wind velocity, etc.) have differed in each case, it is obviously difficult to make a fair comparison of the dampers themselves. Such a comparison might be made by a lengthy series of field tests; and Margoulies,|| Nefzger,¶ Ryle,** and others have attached various dampers to conductors in different field tests, but the results obtained are somewhat at variance. Thus Margoulies'|| and Nefzger's¶ tests indicated that the Hofmann lever damper was superior to the Stockbridge

* See Bibliography, (38). † *Ibid.*, (26) (Discussion). ‡ *Ibid.*, (26). § *Ibid.*, (44). || *Ibid.*, (24). ¶ *Ibid.*, (28). ** *Ibid.*, (35).

type, but Ryle* found that the latter was more efficient than the former, which, as mentioned before, also had the disadvantage of being noisy and subject to excessive wear. Margoulies† also found that the lever damper was noisy, although it is claimed by the makers that no complaints have been received regarding noise or wear. The variation in these test results, however, does not affect their validity, and is probably associated with the different test conditions of the investigations. Hence, in the choice of a damper (from amongst those now marketed) for a particular transmission line, simplicity and robustness of construction should be an important factor, and such factors as ease of renewals and proprietary rights must also be considered. Dampers can of course be erected with less trouble than the ordinary reinforcement, and in America have been attached to the line without interruption of supply.‡

With festoons and eveners line clearances may be slightly reduced, though an objection can also be raised on aesthetic grounds. Regarding the reduction of wire failure, festoons appear to be just as efficient as dampers§ and reinforcing devices. It has also been stated that longer conductor life has been obtained by the use of eveners, but these have not been widely used and there is insufficient information to show how they compare with other mitigative measures. There is also insufficient data to show how strand failure has been effected by the installation of the double-saddle clamps. These latter are more costly to manufacture than the normal designs and, as indicated in Fig. 21, are rather heavy. As in the case of dampers and reinforcements, it is difficult to compare the costs and economies effected by these various devices since the cost per route mile will be affected by span length, erection difficulties, and proprietary rights (and the economies determined by the effectiveness of each device under the varying conditions of line vibration). The figures given in this paper, however, give an idea of the amount of material and assembly necessary for these various mitigative measures.

(6) "DANCING" OR "GALLOPING" OF CONDUCTORS

These terms are applied to the phenomenon of waves of large amplitudes and low frequencies which cause the conductor to whip up and down and may make the whole span vibrate or swing as a single loop.

The occurrence of this phenomenon is irregular and rare, so that detailed examination is difficult. Davison,|| however, has reported amplitudes as large as 10 ft., and states that lines in several adjacent spans have been affected, while Den Hartog¶ mentions amplitudes as large as 20 ft. The dangers arising from the possibility of the conductors arcing-over, or even touching, and the greater mechanical stresses set up in the insulators and clamps during " dancing," are obvious. It appears that " dancing " is usually associated with ice- or sleet-loaded conductors (copper or steel-cored aluminium) and occurs in winds of about 25–30 m.p.h., being more common in districts subject to gusty winds, snow, and sleet storms. Ice-free conductors have also been observed to dance,**

* See Bibliography, (35). † *Ibid.*, (24). ‡ *Ibid.*, (14). § *Ibid.*, (2), (3), (22). || *Ibid.* (7). ¶ *Ibid.*, (11). ** *Ibid.*, (6), (7), (15).

and at temperatures above freezing, so that there is some diversity between the various aero-dynamical theories advanced to explain the phenomenon, and consequent difficulty in suggesting mitigative measures. The basis of most of these theories (for the case of ice-loaded conductors) is somewhat as follows:—

Sections through the conductor and adhering ice or snow may sometimes assume some kind of "streamline" or "aerofoil" shape which in certain circumstances may enable a strong or gusty wind to lift that portion of the conductor thus affected, viz. as in the case of the aerofoil section of an aeroplane wing the wind may strike the conductor and ice at certain critical angles, thereby causing "lift." A portion of the conductor or even the whole span may thus be given an upward jerk, and on springing back will generate an undulation or travelling wave, or even cause the whole conductor to vibrate as a single loop. Gusts of wind or the swinging motion of the towers, coinciding with the natural periods of these loops, will naturally help to induce or maintain "dancing."

The work of Davison,* Den Hartog,† Middleton,‡ and Nivens,§ all partially supports an aerofoil theory. Davison experimented with glazed, partially glazed, and unglazed wires, and Den Hartog† with elliptical conductors, while Nivens§ found that during wind-tunnel tests the amplitude of shellac-treated wires was greater than that of bare wires, indicating that in practice larger amplitudes would occur with ice-loaded conductors than with bare conductors. The aerofoil theory does not account for the dancing of ice-free conductors, and Middleton‡ states that more knowledge of the meteorological conditions accompanying "dancing" is necessary before a satisfactory theory can be evolved. It has been suggested, however, that a suitable gusty wind (not necessarily accompanied by snow or ice) is primarily responsible, but that such winds may be very common in snow and sleet storms. Hawley,‖ for example, observed "dancing" with ice-free conductors in two or three adjacent spans during a high but pulsating wind, when the temperature was several degrees above freezing, while Davison* frequently found that investigation of "dancing" reports showed that a wind of hurricane proportions had passed through the district.

These facts suggest that though the aerofoil shape developed by the conductor and adhering ice in a storm may partially account for "dancing," the basic factor is a suitable gusty wind. If such a wind is very common in snow or sleet storms, or arises soon after ice has formed on the conductor, it will account for the phenomenon being more frequent under these than under other meteorological conditions (while an aerofoil shape due to ice would favour its occurrence). Further information is required, however, before specific conclusions can be drawn.

(7) CONCLUSIONS

The available data lead to the conclusion that the eddies formed behind the conductor by winds below about 25 m.p.h. are primarily responsible for the vibration of overhead lines, and that vibration may occur on many of the steel-cored aluminium conductors now in service, particularly on lines where the ratio of conductor weight per unit length to diameter is relatively small, and also on hollow copper conductors. It does not follow, however, that such vibration will always cause wire failure.

In steel-cored aluminium conductors of circular-section wires the failure of the aluminium strands caused by this eddy vibration appears to be mainly accelerated by the "nicking" of the strands due to the clamping stresses (a factor which prevents full use being made of the fatigue strength of the aluminium), and, though to a smaller extent, by the "hammering" between clamp and conductor. "Nicking" and "hammering" and the resulting strand failure can be slightly reduced by the use of a "bell mouthed" and light design of clamp and by the avoidance of short clamps. There is also evidence indicating that "nicking" is less severe with a "segmental" conductor than with the ordinary type of circular strands, and that the use of aluminium tape round the conductor within the clamp reduces the effects of "hammering."

Wire failure can also be reduced by lowering the working tension, by the action of vibration dampers, or by conductor reinforcement. The latter can be applied without re-designing the line, though dampers, unlike most reinforcements, can be erected without dismantling the clamps, and special clamps are not required. All the dampers discussed in this paper seem to suppress vibration effectively when correctly designed and located for the given line conditions. Although in such circumstances the damper efficiency may vary slightly with the frequency of the line vibration, such variation appears to be unimportant. Reinforcements by strengthening the conductor at the clamp seem to reduce wire failure as well as dampers. Hence, when considering the adoption of either dampers or reinforcements, simplicity and robustness of construction should be important factors.

Wire failure can also be effectively reduced by the use of festoons, though line clearances may be thereby slightly reduced. In most cases this will be unimportant. Lack of information prevents a definite conclusion being drawn regarding the efficiency of some of the more recent devices such as eveners (where slightly greater line spacing is required) and special clamps. Such data as are available indicate that they are at least partially effective. It is also claimed that vibration can be greatly reduced by the use of the "vibration-free" conductor, of which some 500 miles have been erected in Germany.

"Dancing" or "galloping" of conductors is a comparatively rare occurrence, and its causes are not clearly understood, but it seems to be associated with certain kinds of gusty wind and to be favoured by the formation of ice or sleet on the lines.

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* See Bibliography, (7). † *Ibid.*, (11). ‡ *Ibid.*, (25). § *Ibid.*, (29). ‖ *Ibid.*, (15). ¶ *Ibid.*, (7).

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DISCUSSION BEFORE THE TRANSMISSION SECTION, 12TH APRIL, 1939

Mr. C. F. Bolton: It may be of interest to give some data in regard to the practical application of vibration damping devices and an indication of the results obtained from their use.

Following the construction of the grid, the Central Electricity Board had in mind from the outset the possibility of strand breakage due to vibration, and periodic inspections of conductors at suspension points were carried out. In due course some breakages were found on 132-kV transmission lines, the conductor of which is steel-cored aluminium, consisting of 7 steel wires and 30 aluminium wires each of 0·11 in. diameter, the copper equivalent being 0·175 sq. in. The stringing tension at 60° F. is 3 200 lb. over a normal span of 900 ft. In consequence of the breakages it was decided to fit vibration dampers, and a review of the existing types was undertaken. One of the determining factors influencing the choice was the fact that the lines were already erected and in commercial operation, and a damping device which took the minimum time to erect was therefore essential. The choice ultimately fell on the Stockbridge damper, which is illustrated in Fig. 10 of the paper. In general, it was decided to fit one damper per normal span of 900 ft., and additional dampers in special cases, i.e. navigable rivers of long span length, buildings, etc.

Following the erection of some 25 000 Stockbridge dampers in South-East and East England, careful observation has been kept of their performance, and examination of the conductors at suspension clamps is carried out periodically. Should excessive vibration still be observed in any span after the fitting of a damper, or the weights of the damper show signs of sagging, an additional damper is fitted. The sagging of the weights of a Stockbridge damper is a sure sign that it is being overloaded.

In the early inspections of the conductor a few cases were found where a number of strand breakages warranted the renewal of the conductor when opportunity offered. The following three cases illustrate the efficacy of the damper in arresting strand breakages.

Case "A."—Strand breakages at a suspension point on a primary transmission line were first found in July, 1933. Dampers were fitted during the summer of 1934, but it was decided to replace the conductor later. This was done in May, 1936, i.e. nearly 2 years later, but on examination of the conductor no further strands were found to be broken.

Case "B."—Strand breakages were found at several suspension points on a length of primary line in January, 1933. Dampers were fitted in May, 1934, and the length of conductor was replaced in September, 1936. No further strand breakages were found.

Case "C."—Similar to Case "B," but strand breakages were found in October, 1934, and dampers fitted in July, 1935. The conductor was replaced in January, 1936, and no further breakages found.

The inspection of 33-kV line conductor has not revealed any cases of strand breakage, and in view of the authors' statement in the paper it is perhaps significant that the type of conductor used consists of 7 steel strands of 0·062 in. diameter and 6 aluminium strands of 0·186 in. diameter, i.e. small-diameter steel and large-diameter aluminium. The stringing tension at 60° F. is 1 250 lb. over a normal span of 600 ft. In general, therefore, it was not deemed necessary to fit Stockbridge dampers to the Board's 33-kV transmission lines except in cases of special crossings.

It may be of interest to note that special equipment can be obtained for erecting Stockbridge dampers whilst lines are "alive," and this practice has been largely followed in America. The Board has not, however, adopted this method, but has carried out the erection of dampers during the periodic inspection of towers, insulators, and fittings.

The authors say "There is insufficient information available to show how weather conditions affect the performance of dampers—thus accumulations of ice or dirt inside the weight of the Stockbridge type or at the stops of the Hofmann lever damper might seriously impair their efficiency. . . ." My experience with the Stockbridge damper has been that that is not a serious matter, as the accumulation of dirt is very small. The behaviour of the Stockbridge damper under snow-loading conditions is, however, interesting. The loaded conductor will sag under the weight of accumulated snow, and, when the latter is discarded, whips up. The sudden upward movement of the conductor will often cause the flexible conductor of the damper to bend at right angles, leaving the weights hanging in a vertical position.

One important point to be watched in the erection of this type of damper on lines which have been in existence for some time is the necessity for cleaning the conductor at the point of attachment of the damper. Cases have

been known where the damper has been involved in a flashover, and a bad contact at the point of attachment may result in severe burning of the conductor. Despite this precaution, if there are indications that the damper has been involved it should always be removed and the conductor beneath its clamp inspected. A further point of importance when looking for strand breakages is that, as the authors point out, such breakages nearly always occur inside the clamp mouth. This means that the conductor must be raised from its clamp, as otherwise the clean-cut break in the strand will not be seen. Further, it has been my experience that breakages do occur in the inner layer of aluminium strands without breakages occurring in the outer layer. The outer layer must therefore be opened up so that the inner layer can be properly inspected.

The span of 3 060 ft. over the Thames presented a special case where steps to avoid conductor deterioration through vibration had to be taken. The conductors comprising the crossing are of a special type and consist of 7 cadmium-copper plus 84 phosphor-bronze wires, each 0·0856 in. in diameter. No signs of strand breakages had been observed, but Bate dampers consisting of 36-ft. lengths of the line conductor have been attached by means of self-releasing clamps at 4-ft. intervals, i.e. 18 ft. of supplementary conductor at either side of the suspension clamp. Routine inspection has indicated that these dampers are effective in preventing strand deterioration.

Mr. S. E. Clotworthy: I note that in the authors' view two dampers should be fitted per span, one at each end, while in Mr. Bolton's experience only one damper per span has been found necessary. I should like to know whether a vibration recorder has been fitted at the end of the span farther from the damper, and, if so, what it recorded. While a recorder fitted between the damper and the suspension clamp at its own end would show that the vibrations were damped out, one at the other end might indicate that the line was subject to certain vibration troubles.

The Stockbridge damper was evolved after a considerable number of tests, mainly to provide a remedy for the serious vibration troubles which were occurring in the United States some years ago. The main idea of those who developed it was to provide a remedy which would be simple to install and fairly cheap to produce.

It would be of interest to know whether investigations have been made as to the relative cost of the non-vibrating type of conductor and remedies using the ordinary, normal stranded type of conductor.

Mr. E. T. Painton: The subject matter of this paper is one which at one time caused us great anxiety, and it is as a result of researches such as have been carried out by the authors and others that we are now in a position to provide a complete cure for any line which is found to be suffering from the disease of vibration. But, while we have a complete cure, we have not enough information about the various factors which influence the formation of these vibrations. Some engineers feel that a reduction in tension will reduce the risk of vibration, and, following this suggestion, the proposal has been made that there should be a general reduction in the tension for which lines are designed. There are two very distinct objections to that. The first is that a reduction in the tension will be rather a costly matter, in that it will necessitate an increase in the height of the towers or a decrease in the span length, at a total cost which will possibly be more than the cost of fitting dampers. A much more important objection, however, is that no one knows by how much the tension should be reduced to achieve safety.

I believe that destructive vibration does not normally occur on spans less than about 400 ft. in length; it is only ir the case of the long spans of long-distance high-voltage lines that it is necessary to worry about it. One of the most important of the factors which influence the vibration of such lines is the configuration of the country. Vibration can occur only where there is a steady, low-velocity wind blowing on the conductors. If the wind is broken up by trees, hills, or houses, so that there is turbulence, the conductors cannot vibrate. If, for example, I were designing a line to go over Salisbury Plain I should be inclined to put in dampers when the line was first installed; but if I were putting up a line in North Wales or the Highlands of Scotland I think that I should omit the dampers, at any rate to start with.

There is no question that the Stockbridge damper is as good as, if not better than, any other, and its effectiveness has been proved by years of service; but it is not the cheapest, and as a cheaper alternative I would choose the Bate damper (shown in Fig. 7). The authors seem rather doubtful whether to call it a damper or a reinforcement, but I think that it is a true damper, because tests show that it definitely damps out vibrations. The Bate damper was one of the earliest to be designed; it has been in use in Australia for many years, and has given entire satisfaction. It has also been used in the Punjab, where, incidentally, it was designed quite independently of Bate by another member of The Institution.

In conclusion, I should like to suggest that the subject is worth further investigation and study from the practical point of view. It is desirable to collect data concerning lines which have given trouble, and to see whether it is possible to correlate the working conditions with the occurrence of such trouble.

Mr. W. E. Poole: The "nicking" of wires in conductors at suspension clamps is the most fruitful source of weakness, especially on lines constructed of aluminium, because of the soft nature of the metal. The first point which I should like to emphasize is that as the "nicking" occurs for the most part on the inner layer it would be much better to make conductors with one layer of aluminium only, instead of two. With regard to the clamp itself, it is customary to specify that suspension clamps shall grip the conductor and prevent it running through in case of breakage. While the authors were proceeding with their work on this subject I made some experiments with a view to producing a clamp which would normally grip the conductor lightly, but which would prevent running-through in case of breakage. I was successful in producing such a clamp, and I do not think that it would have cost more to produce in quantity than a clamp of the normal type. For my experiments we rigged up a piece of grid conductor with a suspension clamp in the middle. The tensioning was effected by a block and tackle and also a weight of about 4½ cwt. acting

over a pulley, which would come into operation when the conductor was cut at the other end. The weight was arranged to fall through a distance of 3–3½ ft., giving the clamp a sudden jerk. The apparatus gave a very good representation of the tug to which a suspension clamp on an ordinary primary line would be subjected in case of conductor breakage, and there was no slipping or running-through.

The authors have collected sufficient information to show that triangular conductors are less prone to vibration than the circular conductors which we normally use. This, I think, is what should be expected from a consideration of the way in which eddies act on conductors and produce vibrations. I suggest that it would be worth while to make practical experiments on a few miles of line fitted with triangular-section conductor.

In the non-vibrating type of conductor described in the paper the aluminium and the steel are tensioned separately, so that the aluminium is partly supported by the steel, just as it is in an ordinary steel-cored aluminium conductor. In the latter the aluminium becomes slackened after a few temperature cycles, and therefore we have, in effect, a similar set of circumstances to those which exist in the non-vibrating conductor, with the one difference that the space between the aluminium and the steel is very much less. The one actually vibrates, and the other is said not to. It would be interesting to know, therefore, what is the minimum spacing which will cause damping. When we know a little more about creep in aluminium, we may perhaps be able to devise some method by which in the ordinary type of conductor a sufficient gap may be produced between the aluminium and the steel for the whole to behave as a non-vibrating conductor.

"Galloping" is rather a rare phenomenon, but it does occur; and, although Mr. Painton referred to short spans as being fairly immune from vibration, I know of a case where this type of vibration trouble occurred on a set of spans of about 100 yd. each, in which an auxiliary cable was hung on a catenary wire. The spans were practically equal for a distance of 3–4 miles, and the line ran along a flat piece of country on the South Coast which was subject to the normal S.W. winds. In some 20–30 of these spans in succession the auxiliary cable looped itself over the catenary wire at about one-tenth of the span length from the far end as regards wind direction, and again looped itself up at about one-tenth of the length that was left, i.e. at points about 90 and 81 yd. respectively from one pole.

Mr. P. B. Frost: It has been suggested to me that the Post Office ought to be able to contribute something to this discussion, if only in the way of practical experience; but I should say at once that we do not use stranded line conductors—our experience of line construction is entirely concerned with solid conductors. It is true that when breakages occur on our lines they are generally to be found near the point of attachment; this is particularly the case with the smaller cadmium-copper wires, which more often than not break where they enter the sleeve that is used to joint or to terminate them. Vibrational fatigue at this point, where there is an abrupt change from a flexible conductor to a fairly stiff or rigid support, is probably the main cause of the trouble, but ordinary corrosion at that point is frequently a contributory cause.

With regard to dampers, we occasionally use a lead strip wound in a loose helix along the wire at a termination on a chimney, where humming noise is giving trouble to the occupants of the house. In a sense it is used there as a damper, but the intention is to damp out vibrations causing noise rather than vibrations likely to cause vibrational fatigue. In some instances we have even used a short length of light brass chain in the conductor at the termination. The circuit is, of course, bridged so that the chain is not depended upon to carry the current.

We do, on the other hand, use stranded conductors for wireless aerials. No vibration trouble has been experienced on the Rugby long-wave aerials or on the steel supporting triatics or the stays. The reason for there having been no trouble is probably the absence of rigid clamps. Each bay of the main aerial is ¼ mile in length (i.e. between masts) and the aerials are supported by a stranded steel cable a little under 1 in. in diameter, which at its ends is carried round a cable eye where an ordinary spliced joint is made. This steel cable carries ten "spiders" or spreaders in each bay, the spiders being formed like the spokes of a wheel; the spreaders are clamped to the steel wire at intervals of 40 yd., about ten to a span. Each spider has eight spokes, each spoke being 6 ft. in length. The interesting point is that the ends of the spiders terminate in knobs, which are slipped through loops formed in the 7/14 S.W.G. phosphor-bronze aerials by opening the strands to allow four wires to pass on one side and three on the other. The loops rest upon a shoulder and are kept tight by the tension, no binding or clamp of any kind being used. The ends of the phosphor-bronze strands are all gathered together and terminated round cable eyes with a simple tapering wrapped splice or joint.

The main stays, some of which are about 1 000 ft. in length, consist of large steel cables. They are made up of 151 No. 10 steel wires, of 110 tons per sq. in. breaking strength, and the bundle of parallel wires (unstranded) is introduced at the anchorage end into a conical nipple of about 1¾ in. diameter. The wires are splayed out with a small tapered plug, put under tension—as much as they will stand in that condition—and the nipple is filled with white metal. These terminations were tested to 125 tons before being put up, and may in an ordinary wind have to stand up to a tension of 30 to 40 tons. The method of terminating the steel wires, and two shackles, which afford freedom in directions at right angles to one another, provide a flexible termination which probably damps out the vibrations that would otherwise lead to fracture.

It is significant that the whole of this construction has been in use since 1924 (the aerials since 1925), and no repair work has been necessary. At the present moment, however, the aerial spiders are being replaced, as they were made of light steel, and corrosion has led to failure.

Mr. C. H. E. Ridpath: It is very hard to tell in advance whether dangerous vibrations of overhead-line conductors are liable to take place. For instance, I know of a line in Wales in which the spans are of 1 000 ft., and heavy clamps are in use. That line has been vibrating more or less continuously for about 12 years, and up to the present no trouble has been reported.

Amongst anti-vibratory devices, the Preiswerk non-

vibrating conductor is of interest. It is a pity that it involves more complicated erection work, owing to the separate tensioning of the aluminium and the steel and also because of the fact that the anchor clamps themselves are rather more complicated than those connected with ordinary steel-cored aluminium. Last April I had the opportunity when in Germany of seeing some of this conductor erected. The weather was cold and snowy, with a light wind, and generally the conditions were favourable for vibration. I put my ear against one of the steel towers and I could not even hear the sort of hum usually associated with overhead lines and telegraph lines; there was only a very light, high-pitched " chattering " sound, which could not be heard at any appreciable distance from the tower. The engineers in charge told me that this was caused by the steel core chattering lightly against the aluminium envelope of the conductor. It has been suggested that this movement might damage the inside area of the aluminium wires, but I think it must be remembered that the actual play is only 1 mm. Furthermore, I was told that some of this conductor had been opened up after 3–4 years of operation, and no wearing-away of the aluminium had been noticed.

Mr. H. W. B. Gardiner: The phenomenon of conductor vibration seems at first to be a fundamentally simple thing well suited to experimental study, but the closer one is brought into contact with service troubles, and the further one goes with experiments, the more and more abstruse does the whole problem seem. It is therefore a very definite achievement to carry such tests methodically to a stage where one may draw from the mass of data some definite conclusions of value, as the authors have done.

Referring now to their results, the effect of working tension is as would be expected, and it is interesting that lay ratio does not seem to play an important part. Have the authors any figures, obtained on an actual line, to confirm the experimental fact that the ratio of inner-strand to outer-strand failures is more than 2 : 1? I realize that as inner-strand breakages are not apparent until the conductor is dissected, such information is not easily obtained, but if it could be given it would be a valuable confirmation of the experimental facts.

I feel that the results given in Tables 3A and 3B are quite important in showing up the danger of generalizing regarding any desirable quality such as low clamp weight, inertia, or length; and how the actual merit of a particular clamp may depend not only on a combination of these but also on some other aspect of design which is not so easily definable.

It would be interesting to see how the ordinary trunnion clamp, used as a standard for most of the tests, compares with the other representative types referred to in Table 3A, and to check that its behaviour is not exceptional. I presume that in Table 3A the standard clamp is that referred to as Clamp No. 1. May we assume the test conditions for the results shown in Tables 1A and 3A to be identical?

When making tests to compare conductors of quite different construction, such as the ordinary and the locked-coil types, there is probably no simple alternative to vibrating each mechanically with the same amplitude, as the authors have done; but it should be remembered that the conductors may have quite different stiffness and internal friction, and therefore that under given conditions in service the amplitude attained by one conductor may not be the same as that of another. Experimental results at a fixed amplitude, whilst giving a comparison in some respects, do not necessarily give a perfectly true comparison of what may be expected under identical service conditions.

One outstanding effect not specially mentioned by the authors regarding the tests on " Ordinary " and " Balanced lay " conductors (Table 4) is the great reduction of inner-strand failures in the latter. I think this result is not merely fortuitous; it may possibly be due to some relief of torsional stresses; or to the fact that, with the directions and ratios of lay adopted, the strands of various layers of the balanced-lay construction were more nearly parallel, thus reducing the " nicking " effect associated with inner-strand breakages. I feel the latter to be possibly the more likely explanation. Perhaps the authors would give us particulars of the directions and ratios of lay for the two conductors, and let us have their views on this point.

Referring to Section (4), the authors divide mitigative measures into four groups. I should like to comment on Group (*c*), namely the suppression of vibration by the provision of dampers.

There is no doubt that the vibration usually experienced on overhead lines is a resonance effect; that is to say, the vibrational forces on a conductor have to occur at a frequency suitable to the mechanical constants of the line, and the amplitude has to build up progressively in order to produce vibrations of a dangerous order. Now the progressive increase of amplitude may be prevented, as the authors say, by damping, and if by this term we understand strictly the absorption or dissipation of energy (or the transformation of motional energy into heat energy) then the frictional damper of Fig. 17, and the fluid-piston type dampers of Figs. 12, 13, and 18, are all of this type.

The other devices mentioned under this heading, however, such as the Hofmann lever type (Fig. 11), the plate type (Fig. 14), and the spring-piston type (Figs. 15 and 16), appear to me to be not so much true dampers as " energy transformers " or " frequency-changers," and form a separate class, their action being such that if the vibrational acceleration becomes greater than that due to gravity, one part of the device has a motion which is different from the rest, and the resulting impact causes a force which is not now in phase with the motion of the conductor. Viewed in this way, I think it is clear why the performance of this class of device is, in general, independent of the natural frequency of the line. I think, too, that the " vibration-free " conductor shown in Fig. 3 may conceivably act in the same way, and might be included in this class of " energy transformers."

Normal conductor vibration is usually considered to produce a stationary wave, and given a perfect line clamp such vibration would produce practically no increase of stress at points of support; line failures due to vibration, if they did occur, would happen at the centre of the span, where the conductor would be subjected to greatest flexure. With " travelling " waves, however, considerable flexure would be caused at a point of support no

matter how perfect the clamp might be. In view of the fact that failures do take place at points of support, do the authors attach any importance to the possible effect of travelling waves?

In my own view, dancing or galloping of conductors is often an example of a travelling wave, and must therefore cause very severe stresses at the clamps, but as this form of vibration is relatively very rare its importance is not great.

Mr. G. W. Preston: The paper rightly emphasizes the vital part which the lack of hardness of a conductor material plays in determining its performance under conditions of vibration. There is no doubt that the ability of a conductor material to resist deformation or other mechanical damage is of fundamental importance from the point of view of fatigue failure.

In Section (3)(*b*) the authors say "The cross-sections found to be least liable to vibration are rather impracticable" and should be avoided. I am sure that that remark is intended to apply only to the larger lines, because there are many thousands of miles of 3-wire conductor of the smaller sizes in service and being erected every year which have proved eminently satisfactory, and which are known to be relatively free from vibration. It is interesting to note that 2-wire conductors have not proved entirely satisfactory. I know of one case in which a line with such conductors gave disastrous results. In conditions of high wind they developed a rotary dancing motion which became bigger and bigger as the velocity of the wind increased, and they had eventually to be replaced by 3-wire conductors.

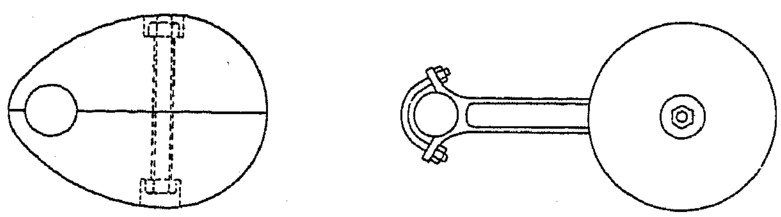

Fig. A

In Section (4)(*d*) the authors suggest that, to act as a reinforcement and to minimize reflection, aluminium tape might be wrapped round the conductor near the clamp. In one case of which I know long aluminium-tape binders were wound round the conductor but the ends came loose and the binder chafed the conductor very badly. The tape gradually wore through and little ringlets of it worked their way down the span. The same sort of thing is likely to occur with certain types of armour rods when the ends have a tendency to work loose and to damage the conductor by chafing.

The paper does not mention the torsional damper, of which two types are illustrated in Fig. A. The first consists of an egg-shaped mass of cast iron, and the second of an arm with two weights attached at the end. Both types of damper are clamped tightly to the conductor, so that the weights are supported horizontally. When a vibration is developed the weights cause a torque to be applied to the conductor which sets up inter-wire friction and so damps the vibration. Single dampers may be attached to the conductor in the vicinity of the tension clamps or they may be used in pairs, one on each side of the conductor, a few feet apart. It is claimed that the advantages of this type of damper are that it is simple in construction, has no moving parts, and causes the energy of the vibrations to be dissipated over a length of conductor instead of being absorbed all at one point, as in the case of some of the other types of damper shown in the paper.

Mr. H. Willott Taylor: We have just over 5 000 miles of e.h.t. lines erected, of which 200–300 have steel-cored aluminium conductors. We have had no vibration trouble at all with our copper conductors, on which no special damping precautions were taken. About 10 years ago, however, we used some No. 4 copperweld conductors tensioned very tightly, and in 6 months we had 9 failures. All the failures occurred at tension points and in spans where the conductors were tensioned at both ends. We have had no failures in 10 years with these conductors where carried on pin insulators and bound in with a stirrup-type binding in the top groove.

In the paper it is observed that more trouble is experienced at suspension points than at tension positions, and as the above experience seems to suggest that the latter are worse than where the conductor is bound into the top groove of a pin insulator, one would conclude that a pin-type line would give less vibration trouble than a suspension-insulator line. It is rather a pity, therefore, that some reference is not made in the paper to pin-insulator lines. The majority of lines in this country are of this type, and those undertakings which still use steel-cored aluminium conductors would doubtless be interested in the relative advantages of the pin- and suspension-insulator lines.

In view of the foregoing remarks, would a simple type of clamp, to which the conductor could be bound in the same way as in the top groove of a pin insulator, be a practical way of overcoming some of the troubles caused by the bolted type? Such an arrangement would almost totally eliminate any question of "nicking."

With regard to mitigative methods, I would recommend that, on new lines, no high-tensile composite conductor should be used if it can possibly be avoided.

Fig. 6 purports to show the differences between the various types of armour rods, but a great deal of extraneous matter which has nothing to do with armour rods is shown on some and not on others of the sketches, leading to unnecessary complication. I suggest that in order to facilitate comparison these diagrams could, with advantage, be redrawn.

Referring to Fig. 8, is the armour rod split in order to avoid any possibility of it carrying current, due to the contacts on the conductor under strain?

Mr. S. R. Siviour: Having seen a most wonderful example of the "dancing" or "galloping" of conductors myself, I hope that it will be a very rare occurrence. I saw the effect first in 1912, on two parallel 11-kV lines on which over a distance of nearly 2 miles the top and bottom conductors were dancing continuously for a period of some hours. The phenomenon was due to a combination of circumstances—ice-laden wires, temperature-changes, an oblique wind, and the particular configuration of the ground. Under such conditions similar experiences have been recorded in America. Next day we had to rebind every insulator, as all the binders were broken. The amplitude of the vibrations was of the order of 5–6 ft.

Mr. W. B. Buchanan (Canada) (*communicated*): The interest expressed by the authors in endeavouring to determine a normal working tension upon which certain known bending stresses could be superimposed by reason of æolian vibrations, the conductor still remaining unharmed, has been widespread. It has been stated that the incidence of vibration is below the economic limit of working tension. The problem then is to control or limit vibrations to such magnitude that certain maximum stresses should never be exceeded. In addition to the comments of the authors as to difficulties encountered, I suggest that this is not feasible, practically, for the following reason.

It has been shown by two independent methods that over a substantial part of its motion the loss due to the vibration of a conductor varies as the square of its amplitude. It is also accepted that energy input per loop, due to air eddies, increases with the amplitude of motion of the vibrating conductor. Assuming a tremor of any nature to start in such conductor, there would always be a probability that additional excitation would be derived at various locations in the span where the cross-component of wind velocity would vary from 30 % below to 30 % above the critical values, and " frequency-pulling," due to the motion of the conductor, might readily result in an increase in the power input much greater than the first power of the amplitude.

Supposing then, in this mechanical system, that conditions tend to cause the net power input to increase as the square of the amplitude and that this is also the rate of increase of loss of energy due to motion of the conductor, there would be no controlling factor to limit the amplitude of motion to any definite amount. Records taken by myself and my associates indicate that at times, not always, vibrations build up in amplitude very quickly, in such manner as to confirm the hypothesis of a regenerative action being present to some degree.

Limits are found in practice, of course, due to the falling-off in energy input from eddy currents of air when the amplitude of motion exceeds the diameter of the conductor, and to a rapid increase in losses at higher amplitude.

The practical utility of the foregoing argument would obviously be to indicate that the only sound method of dealing with vibration is to suppress it wherever it may be found to occur persistently at all. For such purposes, a variety of devices are available but they must be applied intelligently as some so-called damping devices are only mildly palliative in their damping effects.

The difficulties encountered in the authors' attempts to evaluate the stresses at the clamps are interesting because of the vital importance of this part of the problem. I suggest that the results of such a study probably never would be satisfactory without taking into account certain dynamic effects which can only be accounted for on a basis of travelling-wave theory. That such theory is applicable can be shown mathematically, and it has also been demonstrated by test that the sharpness of curvature at clamps (or such points of reflection) is very much greater than elsewhere in the span, even under the most uniform standing-wave conditions set up by the air currents. If this hypothesis be accepted, and I have no doubt as to its applicability, then it is clear that normal stringing tensions plus increments due to bending do not fully indicate the stresses to which a conductor is subjected at the mouth of a clamp. In addition, certain dynamic effects should be considered which may affect the magnitude of the tension component and also the effective stiffness of the conductor along its length in such a manner that its curvature becomes extremely complex and impossible to estimate.

The surface abrasions, which may be observed best in No. 4 of Fig. 1, appear to be very similar to some which I observed in cable that had been subjected to forced vibrations of high amplitude for approximately 50 million cycles. Abrasions such as those illustrated appeared at the mouth of the clamp and also between strands of adjacent layers throughout the length of conductor which was included between a pair of torsional dampers. In the former case apparently the longitudinal shear due to bending at the clamp exceeded the static friction coefficient, while in the latter the torsional stresses introduced contributed to the slipping between strands. Other sections of the cable showed no indication of any such motion.

These comments are made in support of the suggestion that in severe cases there must occur at or near the mouth of the clamp a partial collapse of the stiffness of the conductor. This evidence furnishes another reason why it is extremely difficult, probably practically impossible, to specify any definite limit to the amount of vibration that may be tolerated with impunity on any standard long-span construction.

Since it is pertinent to these considerations I should like to refer to a statement made in closing the discussion on my paper before the American Institute. The statement appears on page 780 of *Electrical Engineering* for July, 1935, and reads as follows: " . . . natural vibrations should be regarded as a ' symptom ' rather than as the principal disease itself; a symptom indicative of construction having high reflection and attenuation factors and hence particularly susceptible to damage from all travelling waves." Having in mind that so-called vibrations or standing waves are only special cases of travelling waves, hence included in the above category, I would reaffirm the above statement of opinion. The amplitude to which vibrations may be tolerated does not in practical work appear to be sufficiently under control; the mechanical reactions at the clamp are such as to tend to concentrate and aggravate the stress condition rather than to distribute and dissipate it, and the severity of the exposure with respect to æolian disturbance can only be conjectured to a very minor degree of accuracy.

Mr. N. Thornton (India) (*communicated*): The reason suggested by the authors for inner-layer failures being more numerous than outer-layer failures, namely that the inner-layer strands were subject to " nicking " on two sides whereas the outer-layer strands were subject to " nicking " only on the inner side, supports their conclusion that local " nicking " and " abrasion " accelerate fatigue fractures due to vibration. On the 132-kV lines in the Punjab, however, inner-layer failures have been extremely rare while outer-layer failures have been so numerous that it became necessary to dismantle and repair the cables on 100 miles of double-circuit line which had been erected only 4 years before.

The authors show that clamping stresses can cause local "nicking"; but when the cables on the Punjab lines were dismantled and the aluminium strands were removed from a number of scrapped lengths it was discovered that whilst "nicking" was continuous, it did not appear to be more pronounced at the clamps than in the spans. It is assumed, therefore, that this continuous "nicking" occurred during stringing, and it seems probable that it was caused by the inter-strand movements due to the fairly severe bending of the cable when running through the rollers. This would account for the fact of the "nicking" being continuous.

If the higher proportion of inner-strand failures given in Tables 1 and 2 is accepted as additional evidence proving that local "nicking" accelerates breakages, then it is suggested that the rarity of inner-strand failures on the Punjab lines is an indication that clamping stresses did not cause increased local "nicking."

The question arises, therefore, in view of the vast difference between erecting a short length of cable in the laboratory and erecting cables on long-span lines in the field, where the cables are liable to very severe "roller" stresses, whether the local "nicking" and "abrasion" observed by the authors are common phenomena in practice. It would be interesting to know whether the authors have investigated local "nicking" on existing transmission lines, whether continuous "nicking" was observed on the old cables used in their experiments, and, if so, whether clamping stresses had caused increased local "nicking."

Tapered armour rods of the type shown in Fig. 6(*b*) were installed in the first instance on the Punjab lines, and it is considered that these afforded protection against "hammering" as no evidence of this phenomenon was found.

It is suggested that the effect of wide variations in temperature on an S.C.A. cable should be considered in connection with aluminium-wire failures on account of the great difference in the modulus of elasticity and coefficient of expansion between aluminium and steel. At the time of stringing, the cable is tensioned by mechanical means and the strands are stretched and elongate uniformly throughout; but if, after stringing, the tension in the cable is increased by a drop in temperature, then, owing to its rate of contraction being approximately twice that of steel, the aluminium will be forced to take an abnormal proportion of the total stress. As the increased tension forces the cable as a whole to stretch, the aluminium will approach nearer to its yield point than the steel, on account of the greater contraction and the lower modulus of elasticity.

If the local increases in stress at the ends of the clamps, which are induced by the reaction of the clamps to the reverse bending which occurs with vibration, take place at a time when the temperature is much lower than when the cables were strung and the aluminium is approaching nearer to its yield point than is the steel, wire failures will be accelerated. It would also appear that if two S.C.A. cables of equal diameter and the same number of strands but with different proportions of aluminium and steel are subject to the same conditions of temperature-change and vibration, aluminium-wire failures will occur earlier in the cable with the smaller proportion of aluminium, as the abnormal stress thrown on to the aluminium when the temperature falls below that which prevailed at the time of stringing will have to be borne by a smaller sectional area of aluminium.

The shade temperature in the Punjab varies between 75° F. in the winter and 120° F. in the summer, and the lowest night temperatures are of the order of 30° F. Tests have shown that the sun temperature may be taken to be approximately 20 deg. F. higher than the shade temperature. The complete temperature-range may therefore be taken to be from 30° to 140° F., which is very large. The lines are most susceptible to vibration between the hours of sunset and sunrise in the cold weather, and it was discovered, on the 100 miles of double-circuit line repaired, that there was a definite correlation between the amounts of damage in the various sections and the temperatures prevailing when the cables were erected.

Broken wires were found on 50 % of the earth-wire clamps and 6 % of the conductor clamps on the cables which had been strung during the coldest part of the year, but on the section which had been strung during the hottest part of the year the figures were 93 % and 57 % on earth-wire and conductors respectively. Therefore, as all other characteristics were equal, it is assumed that wire failures developed more rapidly on the section strung during the hot weather in consequence of the greater variations between the temperature prevailing at the time of stringing and when vibration was present. It is further considered that temperature variations caused wire failures to develop more rapidly on the earth wire than on the conductors, on account of their different construction; the earth wire being 18/·0935 in. aluminium and 19/·0935 in. steel, and the conductors 30/·0935 in. aluminium and 7/·0935 in. steel.

It is concluded, therefore, that as there was no evidence of local "nicking" and "abrasion" or "hammering" on the Punjab lines the primary factor accelerating wire failures was the wide variation in temperature between the time of stringing and the time when vibrations were present; and it is considered that, on lines operating under a temperature range approximating to that of the Punjab, vibration must be suppressed if trouble is to be avoided.

The development of the non-vibrating conductor is most interesting, but it is surprising to learn that there were no signs of "hammering" or wear between the core and the sheath. Presumably this is due to the relatively large area over which the frequent impacts take place. In this connection it would be interesting to know how the non-vibrating conductor is gripped in the suspension clamps and what precautions are taken to avoid crushing of the cable. Also it would seem that complications must be introduced into the line splices unless the feature of relative movement between the steel and the aluminium is sacrificed.

In consequence of the tapered armour rods with which the Punjab lines were originally equipped not proving sufficient to prevent aluminium-wire failures, it was found necessary to employ vibration dampers. Many field tests were made to discover the most suitable form of vibration dampers for the lines in question, as a result of which it was ascertained from the charts drawn by vibration recorders that there was little to choose between

the efficiency of the Stockbridge damper and that of a damper of the Bate type of suitable length, which proved —for spans up to 1 100 ft.—to be 11 ft. on each side of the suspension clamps.

The Stockbridge damper had to be imported, whereas experiments proved that an efficient Bate-type damper could be made up from 25-ton 7/8 S.W.G. galvanized-steel stay wire, with the result that the cost of the latter was considerably less than the cost of an imported damper. Therefore, dampers of the Bate type were adopted. They have been in use for over 3 years, and recent inspections have revealed no further wire breakages. It is worthy of mention that it was found possible with suitable "hot line" tools to install Bate-type dampers without interruption of supply, although, actually, the necessity for so doing did not arise.

The design of reinforcement shown in Fig. 6(*c*), utilizing a chrome-leather strap, is attractive in that it avoids any auxiliary clamping spots on the conductor; but this design would be suitable only for use in climates of moderate temperature and even then it is possible that the chrome leather would need replacement or occasional treatment in order to preserve it. The inconvenience of such replacement or treatment requires no elaboration.

On the Punjab transmission system no vibration trouble was experienced at tension clamps, and this freedom from trouble was believed (as is suggested by the authors) to be due to the freedom of the clamp to follow the motion of the conductor.

The development of the spring-piston dampers is an interesting step forward, particularly if experience substantiates that their efficiency is independent of the line frequency. Again the question of cost must be considered, however, and it appears likely that such dampers would always be more costly than those of the Bate type.

It is questionable whether the advantages to be gained from the adoption of "eveners" or double-saddle clamps will warrant the additional cost usually involved by such special fittings.

With reference to the attempts made in New Zealand to reduce vibration in cadmium-copper telephone wires by wrapping fibrous materials round the wires near the points of supports (as shown in Fig. 22), such an idea could be applied to overhead cables in suspension clamps, but a disadvantage attached to such materials as cork is that, in consequence of their liability to compress, frequent renewals would probably be necessary. Also, if such dampers were employed on unequal spans or on a line running through hilly country, conductor creepage would be liable to occur. A further disadvantage associated with any type of reinforcement which totally encloses the cable is that if vibration trouble develops it cannot be detected until the reinforcement has been removed. Consequently routine inspections with such devices are not only a lengthy process requiring long shutdowns, but in themselves create the necessity for routine renewals, as there is obviously a limit to the number of times it is possible to refit, for example, such materials as cork or twisted armour rods.

Regarding the mitigative measures discussed in Section (5), the numbers of strand failures which occurred under the armour rods on the worst continuous section of damage on the Punjab transmission lines were as follows: Of 120 ground-wire clamps, 10 were found without breakages, 50 with 1 to 6, 48 with 6 to 12, and 12 with 13 to 18 broken aluminium strands. Of the 720 conductor clamps in the same section, 310 were without breakages, 369 with 1 to 5, and 41 with 6 to 12 broken strands; in no case was the number of broken strands in excess of 12. The strand failures occurred in all cases just outside the gripping portion of the clamps, which have a "transition radius of curvature" from the portion gripping the cable to the beginning of the bell mouthing.

If it were found necessary to add as many as three, or possibly four, Stockbridge dampers per span the cost of that method of vibration damping would become enormous. Even with two Stockbridge dampers per span, which the paper indicates are usually found sufficient, the cost is not likely to compare favourably with that of the Bate-type damper system.

Whilst festoons may be just as efficient as dampers, their presence must reduce line and tower clearances appreciably. Their appearance cannot be praised, and there is every reason to believe that they would be more costly than other simpler forms of dampers.

Too much emphasis cannot be placed upon the importance of simplicity and robustness of construction in either dampers or reinforcement, as such factors may be taken as the basis of the maintenance costs to be anticipated during the operation period. The more elaborate the design of the damping system the greater will be the maintenance charges to be expected.

In all, approximately 9 000 Bate-type dampers have been in use for over 3 years on some 250 miles of 132-kV lines in the Punjab, with spans varying from 500 to 3 850 ft. The initial cost of the damper was much less than that of any other type available, and expenditure on maintenance has been negligible, no repairs or renewals having been found necessary. The efficiency of these dampers may be judged from the fact that no vibration trouble has been observed since they were installed.

Messrs. E. W. W. Double and **W. D. Tuck** (*in reply*): We thank Mr. Bolton for giving us particulars of the Central Electricity Board's experience with the Stockbridge damper and for drawing attention to the necessity of cleaning the conductor to ensure good electrical contact before attachment. This point is liable to be overlooked.

The type of test about which Mr. Clotworthy inquired was made during a series of investigations with the Stockbridge damper and the Hofmann lever damper by Margoulies,* but only one test was carried out with a damper (a Hofmann) at one end of the span and a vibration recorder at the other. This showed that even at the "far" end vibrations were greatly reduced in amplitude—the percentage suppression obtained under these conditions would probably be sufficient for most cases, but we cannot say whether such a reduction always occurs. We were interested to learn, therefore, that the Central Electricity Board decided to fit only one damper on normal spans, and add a second in special cases, where vibration is severe or the single damper is found to be suffering from "drooping."

Our conclusion, that two dampers per span are

* See Bibliography, (24).

necessary but sufficient, is based on American and Continental experience and recommendations, and it will be interesting to see whether the Board's further experience will support it.

It is difficult to give Mr. Clotworthy a specific answer concerning the relative costs of an ordinary conductor fitted with dampers, etc., and of a " non-vibrating " conductor. We have no experience of the erection costs for the " non-vibrating " conductor in this country, and in Europe it has been erected under stringing regulations which differ from those of the Electricity Commissioners. The actual process of erection is more complicated and will increase costs to some extent, while the initial costs of the conductor and its clamps are higher than those of the normal type. Details of the erection of the " anti-vibration conductor " are given in paper No. 213 presented by G. Dassetto at the Paris H.T. Conference, 1939.

We agree with Mr. Painton that no one knows by how much the conductor tension should be reduced to achieve safety, or rather to reduce wire failure to negligible proportions. The problem is complicated by the tensile stress in the aluminium being affected by the actual construction of the conductor (i.e. ratio of aluminium to steel), and by the combined effects of the temperatures of stranding and of erection. In addition, as stated in Section 3(*b*), the numerous factors influencing vibration, together with " nicking," abrasion, hammering, clamp weight, etc., all affect wire failure, and it is not easy to isolate the effect of any one of these factors. All we can say with reasonable certainty about conductor tension is that a reduction thereof will reduce the risk of failure. Tests such as those recently reported by Pfender,* though supporting this general statement, do not indicate that there is any tension below which *no* failure will occur.

Configuration of the country is an important factor, but it is not our experience that the presence of trees, etc., will always prevent the formation of the steady winds necessary for vibration. All reinforcements, by minimizing reflection points, tend to reduce vibration, as well as stiffen the conductor at the clamp. (Dampers suppress vibration by virtue of their " out of phase " oscillations.) Since the Bate damper stiffens the conductor of the clamps we classified it under " reinforcements," though, as both Mr. Painton and Mr. Thornton state, it appears to damp out vibrations as effectively as dampers of the Stockbridge and Hofmann types, and is cheaper.

If the clamping pressure causing " nicking " can be reduced, the wire failure will also be minimized. Mr. Poole's experiments with his special clamp are therefore interesting, and would be well worth trying on a test span or in actual vibration tests. Field observations on triangular conductors (and with conductors of other cross-sections)† have already been made. These conductors were found to vibrate less than the circular type, and it was such observations that later led to the effect of conductor cross-section being dealt with more fully.

We have no data indicating what the minimum radial clearance between core and sheath of the " non-vibrating " conductor should be to ensure satisfactory damping; and although the load tends to be transferred from the aluminium to the steel in an ordinary S.C.A. conductor, and there are signs of a loosening of the aluminium sheath at low tensions, we doubt whether the " creep " of the aluminium would ever produce a " non-vibrating " conductor.

Mr. Gardiner asks for information upon the ratio of inner-layer failures to outer layer occurring in practice. Although inner-layer failures have been reported, we have no evidence to show that the number of such failures exceeds the number of outer-layer failures in practice. In the case of one line on which the conductors were changed, there were more outer than inner failures, and they occurred most often at suspension clamps. We should, in fact, be very much obliged if service engineers would forward us any relevant data they may possess.

The test conditions for Table 3A are the same as for 3B, and suspension clamp No. 1, weighing 10 lb. in 3A, was used for tests 2, 3, and 4, in Table 3B.

When testing conductors of different construction, some specific standard of comparison for laboratory tests of this nature must be adopted, and, as Mr. Gardiner states, we adopted the standard of vibrating the conductors at the same amplitude and loop length. This method neglects the effect of internal friction, but a factor such as this can only be taken into account (and a true comparison between the different types of conductor made) by a series of field tests, in which the conductors are allowed to vibrate under the action of the wind alone.

The construction of the " balanced-lay " conductor is interesting and perhaps should have been more fully dealt with in the original paper. Both the aluminium layers are laid up right-handed. Hence, as Mr. Gardiner suggests, the wires in each are more nearly parallel, there being fewer " crossing points " causing nicking between these layers than in the " ordinary " conductor. Though this affects both layers it might (together with such factors as the lower bending stresses in the inner layer) account for the smaller percentage of inner-layer failures which occurred (see Table 4) with the " balanced-lay " conductor.

We think the argument that some dampers may be regarded as " energy transformers " applies more to the plate type (Fig. 14) than to the others mentioned by Mr. Gardiner. The natural period of oscillation of most dampers is of course controlled by gravitational acceleration (g must enter into all the acceleration formulae). The plate damper, however, has no elastic parts which in combination with g tend to give it a specific natural frequency; since vibrational acceleration will in general be different from that due to gravity, the body of the damper (which will *oscillate* with the conductor) will have a different motion from that of the plates. This, as suggested by Mr. Gardiner, may account for its efficiency being independent of line frequency.

In practice, flexure and bending of the conductor at the clamps with either stationary or travelling waves cannot be avoided, because of the inertia of the clamps and the internal friction of the conductor. We have a ciné film of a vibrating conductor showing that both

* Paris H.T. Conference, 1939, Paper No. 224.
† See Bibliography, (8) and (38).

types of waves occur in service, travelling waves being quite common. The presence of "beats" in vibration records such as those obtained by Monroe and Templin* also indicates the occurrence of travelling waves, but we cannot say which type of wave is responsible for the larger number of wire failures.

We are interested to learn from Mr. Frost that no vibrational troubles have occurred with the stranded steel conductor used for aerials by the Post Office, and details of the clamping arrangements are therefore instructive. That failures sometimes occur with single-wire conductors near the point of attachment or the clamp is in line with general overhead-line experience.

Mr. Ridpath has mentioned one of the anomalies that we often meet in reviewing this vibration problem—i.e. vibration being reported for some years on a line of long spans and fitted with heavy clamps without wire failure having been detected. The exact details of erection and sagging, etc., are often not available and it would be helpful if we received further details of such cases. The data might throw light upon the safe maximum conductor tension already referred to.

Mr. Preston is right in assuming that our statement that "the cross-sections found least liable to vibration are rather impracticable" refers to the larger sizes of conductor. The statement also covers the square and fluted cross-sections tested by Davison, Ingles, and Martinoff† and by Maass‡. There are, of course, several thousands of miles of triangular conductor (i.e. consisting of three wires) of small cross-section (viz. the 0·05-sq. in. copper) existing and operating satisfactorily in this country.

The aluminium tape which we suggest could be used to protect the conductor need only be wrapped round that portion gripped by the clamp. In the few tests where we used this method of protection the tape extended about 1–2 in. spanwards from the clamp mouths, and we noticed no tendency for it to work loose or chafe the conductor. The torsional damper described by Mr. Preston has the great advantage of simplicity of construction. We have had no experience of it ourselves, but would like to know whether vibration of the damper tends to chafe or roll the outer layer strands about which it is clamped.

Mr. Taylor's recommendation that high-tensile composite conductor should be avoided on new lines is a little drastic, since there are some few thousand miles of it operating satisfactorily, although, it is true, dampers have been found necessary. It is surely a matter of costs at the time of erection, viz. high tension and dampers, or lower tension and increased tower heights, and/or more towers and shorter spans. In any case the cost of dampers is much less than that involved by a modification of tower heights or the number used per route-mile. It is unfortunate that the copperweld conductors were strung so tightly, but the resulting experience does show how useful it would be if the maximum "safe or critical tension" for a given conductor could be determined, either experimentally or from field data.

Pin insulators are usually employed on shorter spans than on those where suspension clamps are used, and on the whole with conductors of smaller diameter (some observers have found vibration more common on long than on short spans).* Such conductors often have only one layer of aluminium wires, and in such cases the mean stress in the aluminium is rather lower than in the larger conductors where two aluminium layers are employed. (The actual stress in the aluminium will of course depend upon the ratio of aluminium to steel.) A little while ago, however, we investigated failures occurring in S.C.A. conductors carried on pin insulators. The binding wire which had been wrapped directly over and round the conductor had bitten into it, thereby causing failure (i.e. a form of "nicking" had occurred).

Rather similar trouble occurred in Australia. In such cases failure can be reduced by protecting the conductor with tape or shims before applying the binding wire, or by laying the protected conductor in a suitable clip or clamp on the top of the insulator.

It might have been desirable to have produced standard drawings at an early stage, but we only reproduced such details where permission was given.†

Mr. Thornton found that the "nicking" of the wires continued well out into the spans, and was as strongly marked there as at the clamps where the clamping pressure is greatest. We have examined a few samples of dismantled S.C.A. conductor but have always found "nicking" to be severest at the clamp. Usually the "nicking" became very faint at distances beyond about 6 in. from the clamp mouths. We have, however, seen a few cases where signs of nicking occurred in the span, and in such instances the signs have extended for a length of 2–3 ft. The frequent formation of a node at such points might account for this (i.e. the bending and rubbing of the wires is greatest at a node), or the conductor might have been left swinging on the rollers for some time during stringing.

We cannot add more to Mr. Thornton's suggestion to account for the "nicking" being continuous in the conductors dismantled by him.

We agree that, other things being equal, the temperature of stringing will affect the relative distribution of tensile stress between the aluminium and steel core, but so also will the temperature at which the conductor is stranded in the factory. (The effect of temperature-changes upon the stresses in the aluminium and steel is at present being investigated by the E.R.A.) If two conductors each composed of the same number and size of wires but having different proportions of steel and aluminium are both erected so that under the maximum design conditions the conductor tension is half the breaking load (according to El.C. 53), then, apart from temperature-changes, the aluminium wires will have a greater stress in the conductor where the proportion of aluminium is smaller, and consequently would be more liable to fatigue failure. This would partially account for the higher percentage of failures occurring in the earth wire mentioned by Mr. Thornton, but under the special conditions cited a fall in temperature will accelerate failure.

It is valuable to have actual figures given of the number of failures found under the armour rods. Mr.

* See Bibliography, (26). † Ibid., (8). ‡ Ibid., (23).

* See Bibliography, (28).

† The armour rod shown in Fig. 8 and referred to by Mr. Taylor is split for mechanical reasons; in this instance it may be regarded as two separate armour rods.

Thornton does not state whether *all* these occurred *after* the reinforcements were fitted, but together with the investigation made by Deck* it shows that failures may continue after armour rods have been fitted. Though "dancing" or "galloping" of conductors is rare we were interested to learn that both Mr. Poole and Mr. Siviour have had experience of this phenomenon.

Mr. Buchanan emphasizes certain points which add to the difficulty of determining a "safe tensile stress" below which wire failure will be negligible, and also of evaluating the conductor stresses at the clamp mouth. Some of the factors influencing the magnitude of this "safe tensile stress," and also affecting wire failure, are given above in our reply to Mr. Painton, and we are inclined to agree that the problem of determining this stress for all types of conductor can hardly be solved with the information at our disposal. A survey of the many miles of overhead conductor now in service with details of the conductor size, construction, tension, its stringing conditions, years of service, and incidence of wire failure, might help to solve the problem. We doubt, however, whether it can be solved except on some such empirical basis.

We agree with the statement that normal stringing conditions plus increments due to bending (as determined by the amplitude of the vibrating loop) will not indicate the maximum stress in the conductor, since, as pointed out in Section 2*a*(*i*), there is sharp local bending of the conductor at the clamp mouths caused by the inertia, and the pivoting arrangement of the clamp employed. Even under the favourable experimental conditions described in Sections 2*a*(*ii*) and 2*c*(*ii*) it was difficult to measure accurately this local bending and to calculate the maximum bending stresses. Conclusions regarding the tensile and fatigue stresses necessary to cause wire failure and which are based upon such calculations should be applied with caution, because in any mathematical analysis of this nature it is not easy to allow for the effects of such important factors as "nicking," abrasion, and "hammering."

* See Bibliography, (10).

HEAT TREATMENT OF STEEL BY HIGH-FREQUENCY CURRENTS*

By G. BABAT and M. LOSINSKY.†

(*Paper first received 5th December, 1938, and in revised form 22nd June, 1939.*)

SUMMARY.

There has recently been a great increase in the use of high-frequency currents for heat treatment of metals, in such processes as surface hardening, cementation and alloying, welding, hot machining, and zonal tempering. The paper deals chiefly with surface hardening.

After giving an account of the advantages of the process and an estimate of the power requirements, the authors deal with the design of the heating coil in relation to the shape of the specimen, and describe a "model" method whereby the distribution of the electromagnetic field in the system may be investigated.

Curves are plotted from the authors' experimental results to show the distribution of temperature and current density over the specimen, and the variation of the power absorbed, during the heating period.

The effect of frequency upon the depth of penetration of eddy currents in steel is examined, and consideration is given to the relation between the depth of penetration and the shape of the surface being treated.

The paper concludes with a statement of the results which have been obtained from the application of the high-frequency method to the hardening of tools and gears.

(1) INTRODUCTION

Some considerable time has elapsed since high-frequency (h.f.) currents were first used in metallurgy for the purpose of melting metals. Recently there has been a great increase in the use of such currents in the various processes of heat treatment of metals,‡ including those mentioned below.

(a) Surface Hardening

The life of the majority of steel and cast-iron parts of modern machinery is limited mainly by wear caused by friction. Hardening of the peripheral layers of these parts, as distinct from the inner core, greatly increases their life. By treating such parts with h.f. currents it is possible to produce fine hardened layers at the surface, or at those areas of it which are subject to friction.

(b) Surface Cementation and Alloying

Currents circulating in the peripheral layers of a steel block increase the rate of penetration of carbon, molybdenum, chromium, etc., into the specimen. Moreover, as the core remains cold while the outer layers are heated, there is the possibility of achieving cementation and alloying without damaging the structure of the core. In this way the speed of the processes may be greatly accelerated.

(c) High-frequency Welding

Lengths of alloy-steel tubing may be welded together and high-grade joints obtained by heating the joints with h.f. currents.

(d) Hot Machining of Parts

The resistance of metals to shearing forces decreases as the temperature increases. Thus by heating the work with a special heating coil the cutting speed of lathes and milling machines may be increased many times. Hard steels may be easily machined in this way.

(e) Zonal Tempering

Various types of zonal tempering may be carried out by means of h.f. heating.

(2) GENERAL CONSIDERATIONS

The object of this paper is to discuss some of the electrical problems arising from the use of h.f. currents in processes involving peripheral heating. In the main the paper will deal with surface hardening.

This method possesses the following advantages:—

(*a*) As the heating extends to but a small percentage of the whole volume of the piece of metal which is being treated, defects (e.g. warping) such as are produced by the usual heating and hardening processes are almost completely avoided.

(*b*) High-frequency equipment for hardening can deal with large amounts of metal in a short time, because the heating of the peripheral layer of the specimen to the hardening temperature takes only a few seconds.

(*c*) The cost of this type of treatment is low.

(*d*) The working conditions are very much better than with other methods.

A detailed theory of inductive heating has been developed by Burch and Davis,‡ Ribaud, Northrup, and others. Theoretical analysis and practical experience alike show that over 90 % of the heat generated by eddy currents is concentrated in a peripheral layer of thickness

$$p = \frac{1}{2\pi}\sqrt{\left(\frac{\rho}{\mu f}\right)}$$

For carbon steel at 20° C.,

$$p_1 = \frac{20}{\sqrt{f}}\,\text{mm.}$$

* The Papers Committee invite written communications, for consideration with a view to publication, on papers published in the *Journal* without being read at a meeting. Communications (except those from abroad) should reach the Secretary of The Institution not later than one month after publication of the paper to which they relate.

† Svetlana Research Laboratory, Leningrad.

‡ E. F. NORTHRUP: "Practical Application of Induction Heating to Solid Materials," *Steel*, 6th and 20th March, 1933, pp. 21 and 23; also G. BABAT and M. LOSINSKY: "Surface Tempering of Steel by means of High-frequency Currents," *Révue Générale de l'Électricité*, 1938, vol. 44, p. 495; and G. BABAT and M. LOSINSKY: "Surface Hardening of Tools," *Stanki i Instrument* (*U.S.S.R.*), 1938 (No. 12) and 1939 (No. 6).

‡ "An Introduction to the Theory of Eddy-current Heating" (Benn, 1928).

while above the Curie point (800° C.)

$$p_2 = \frac{500}{\sqrt{f}} \text{ mm.}$$

To calculate the amount of power dissipated by eddy currents, it may be taken approximately that within the layer p the current density is constant and equal to

$$i_x = \frac{H_t}{4\pi p}$$

where H_t is the tangential component of the alternating magnetic flux at the surface of the specimen. Consequently the power dissipated by eddy currents will be

$$\Delta P = \frac{H_t^2 \rho}{16\pi^2 p}$$

For steel heated to the Curie point,

$$\Delta P = 1 \cdot 1 \times 10^{-6} H_t^2 \sqrt{f} \text{ watts per cm}^2$$

Surface hardening (especially hardening of thin layers) requires a power input of the order of 100 to 1 000 watts per cm². This suggests the necessity of using high frequencies, as only with frequencies of the order of 10^5 cycles per sec. may sufficient power be obtained with comparatively small values of magnetic field.*

Since with these frequencies the depth of penetration p is small compared with the dimensions of the specimen, the distribution of peripheral current-densities may be regarded as a problem of magnetostatics. The heating coil and the specimen may be considered to be made of material with zero magnetic permeability, so that the lines of magnetic force follow but do not cross their surfaces.

(3) METHODS OF CONFINING THE CURRENT TO THE HEATING COIL AND THE TREATED PART

In each individual case it is required to harden only some specific local surface areas of the given piece of metal—for example, in gears, the working surface; in dies, the cutting edges, etc. Surface hardening in most cases implies also zonal hardening.

The heating coil should be shaped so as to induce eddy currents only in the local surface areas which are intended to be heated. Consequently, the form of the coil will be determined by the shape of specific parts of the surface of the specimen. In designing equipment for surface hardening it is important to select the most suitable form of heating coil.

The simplest type of inductive heating system is shown in Fig. 1. The heating coil is a single infinite ribbon of width $2g$, located at a distance a from the plate. As there is no magnetic field in the interior of the plate, the eddy-current density at the surface of the plate at the distance x from the zero point is given by

$$i_{x0} = \frac{I_C}{p_a \cdot 2g} \cdot \frac{1}{\pi} \arctan \frac{2(a/g)}{(a/g)^2 + (x/g)^2 - 1}$$

* The use of high values of magnetic field is not practicable, because to create them very heavy currents are required, and considerable dynamic stresses arise between the heating coil and the specimen.

where I_C is the current in the heating coil. The current density in the coil is

$$i_C = \frac{I_C}{2gp_c}$$

Hence the ratio of the current densities in the plate and in the coil is

$$\frac{i_{x0}}{i_C} = \frac{p_C}{p_a} \cdot \frac{1}{\pi} \arctan \frac{2(a/g)}{(a/g)^2 + (x/g)^2 - 1}.$$

Let us denote

$$\frac{1}{\pi} \arctan \frac{2(a/g)}{(a/g)^2 + (x/g)^2 - 1} \text{ by } \phi\left(\frac{a}{g}, \frac{x}{g}\right)$$

In what follows, the quantity ϕ will be referred to as the current-distribution function. It is plotted against x/g in Fig. 1, for various values of a/g. The curves in this Figure show how greatly the current distribution in the

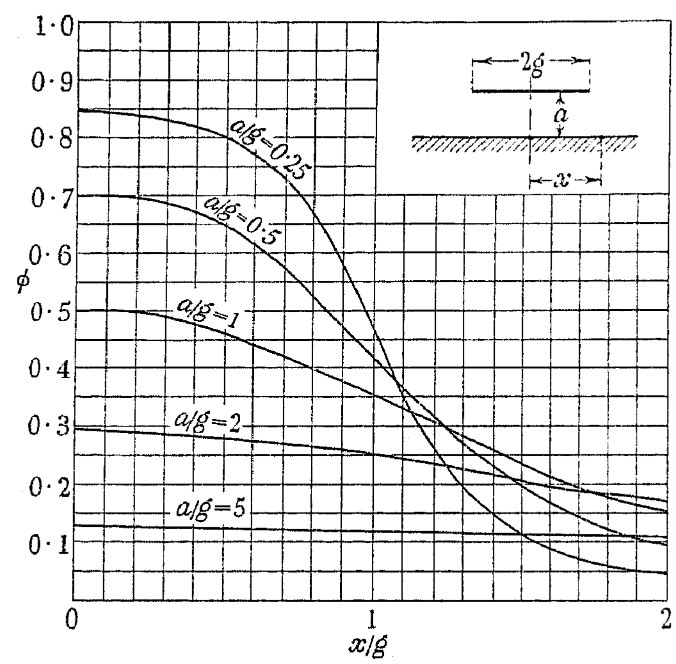

Fig. 1.—Distribution of peripheral current-density in an infinite plane plate with a single current-carrying ribbon above it.

plate is influenced by the ratio of the ribbon width to the plate-to-ribbon distance.

Using the curves of Fig. 1, and applying the superposition principle, we may find the function ϕ in cases where the heating coil consists of two ribbons, one above or in line with the other. This function is determined in Figs. 2 and 3 as the difference of distribution curves, each plotted for a single ribbon correspondingly located.

From Fig. 2 it may be seen that the maximum current density in the plate is obtained beneath the middle of the ribbon. Beyond the edge of the ribbon the current density falls very rapidly to zero and then becomes negative (in the central part of the plate the current flows in one direction, and in all other parts in the opposite direction). In Fig. 3, the current-distribution function is plotted for the case of a heating coil consisting of two ribbons in line. The curves show that variation in the distance between the ribbons ($2c$) influences relatively slightly the magnitude of the current density in the plate.

The latter is influenced to a much greater degree by variation of the plate-to-coil distance.

The distribution of the electromagnetic field in such a system as has been described may be calculated with some degree of accuracy, but only for the simplest forms of heating coil. G. Babat and W. S. Lukoshkow have developed a " model " method, using an electrolytic tank, whereby pictures of high-frequency magnetic fields may be recorded and the distribution of peripheral current density in samples having complicated shapes may be determined.

In this method the magnetic field is represented by the distribution of electric current around models of the heating coil and the specimen, both of which are made of dielectric (ebonite, paraffin wax, etc.) and placed in an electrolytic tank. If the coil and the specimen are small, it is desirable to make the dimensions of the models

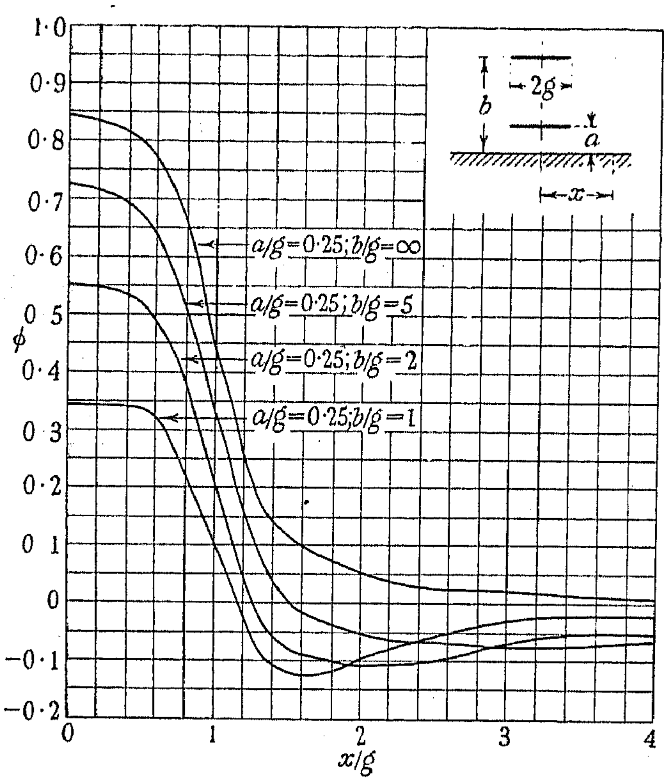

Fig. 2.—Distribution of peripheral current-density in an infinite plane plate for the case where the heating coil consists of two current-carrying ribbons one above the other.

larger than full size. The electrodes are arranged so as to correspond to the equipotential surfaces of the magnetic field.*

An alternating voltage is applied to the electrodes, and by means of a probe wire the distribution of current in the water around the models is determined. The electric field ϵ_t around the models will correspond to the strength of the magnetic field H_t. The surface density of eddy currents at the part of the specimen which is being heated will be $H_t/(4\pi)$.

It is important to emphasize here that when the depth of penetration of current in the material of the heating coil and of the specimen is much less than their dimensions, then neither the resisitivity ρ nor the permeability μ of the materials will affect the distribution of that current. The electrolytic-tank method of determining the distribution of magnetic field strength is applicable to either steel or copper specimens. The values of ρ and μ will affect only the depth of penetration and the amount

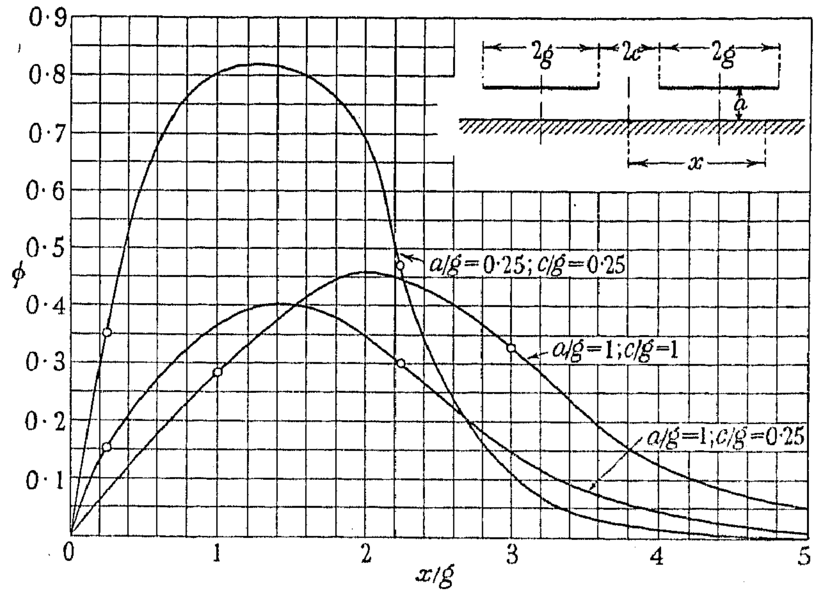

Fig. 3.—Distribution of peripheral current-density in an infinite plane plate for the case where the heating coil consists of two current-carrying ribbons at the same distance from the plate.

of heat generated by eddy currents in the surface layer of the metal.

Figs. 4, 5, and 6 show the distribution of the peripheral current-density as determined in the electrolytic tank. These Figures indicate that the configuration of the hardened layer depends upon the location and dimensions of the coil and the part which is being heated. Fig. 7 shows how the coil is arranged. In (i), the outer surface of a cylinder is being heated; and in (ii), the inner surface of a cylindrical bushing.

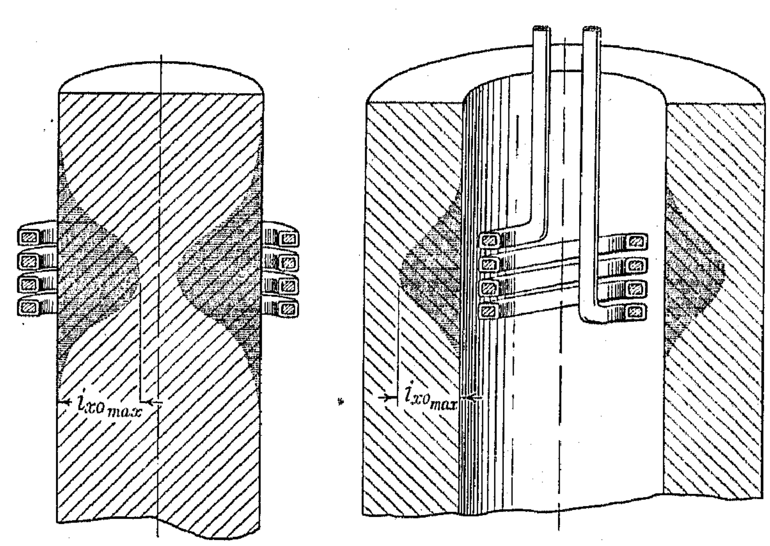

Fig. 4.—Distribution of peripheral current-density (i) in a cylinder of infinite length, and (ii) in a cylindrical bore.

A hardened layer of uniform thickness may be obtained over the surface of the specimen provided that the height of the heating coil approximately satisfies the condition

$$h_c = h_s - 2a$$

where h_c = height of heating coil, h_s = height of specimen, and a = distance between heating coil and specimen. Where the heating coil is of greater height [(2) and (3) in

* Specimens in which there is an axis of symmetry always have two equipotential surfaces, which take the form of planes. Specimens of complicated shape may have curvilinear equipotential surfaces. The " model " method is therefore more easily applied to symmetrical bodies than to others.

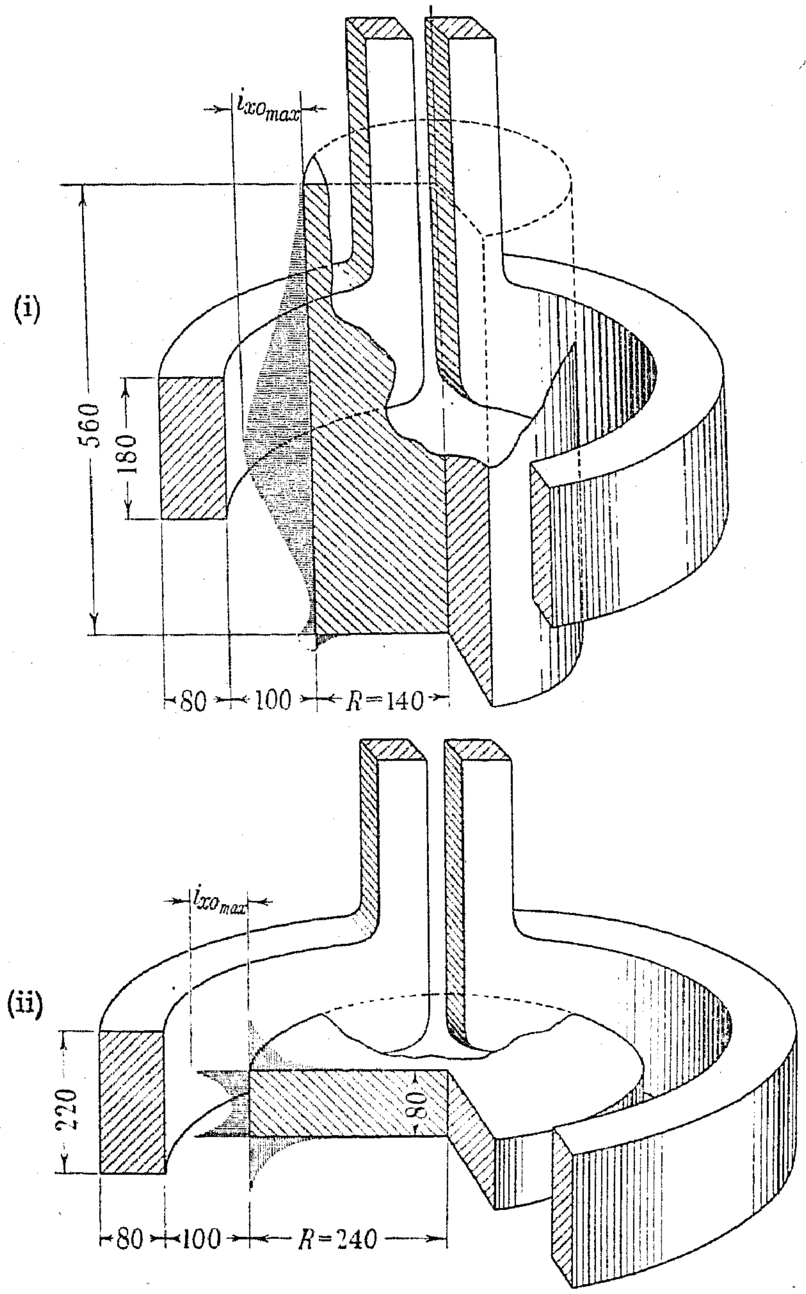

Fig. 5.—Examples of distribution of peripheral current-density in cylinders of finite length with different cylinder-to-coil size ratios.

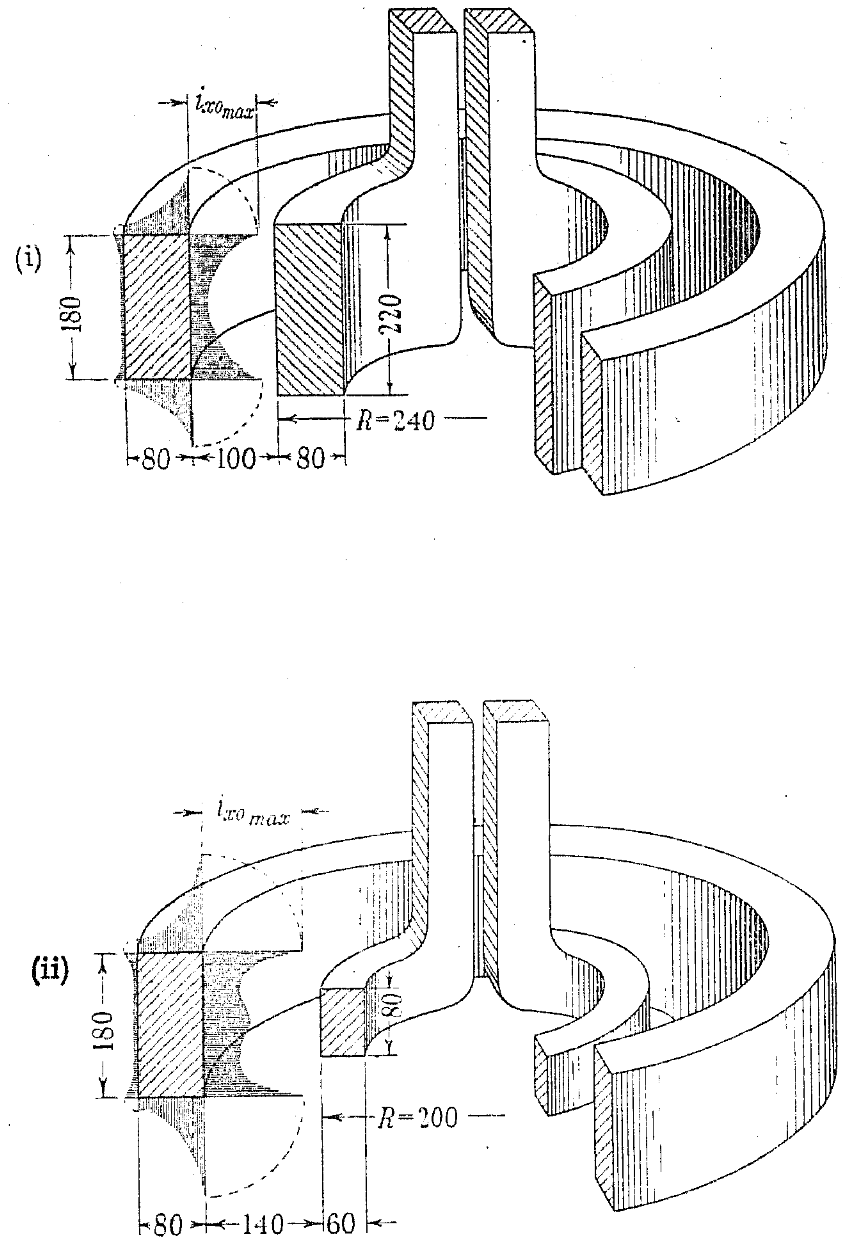

Fig. 6.—Examples of distribution of peripheral current-density in cylindrical bushings with different bushing-to-coil size ratios.

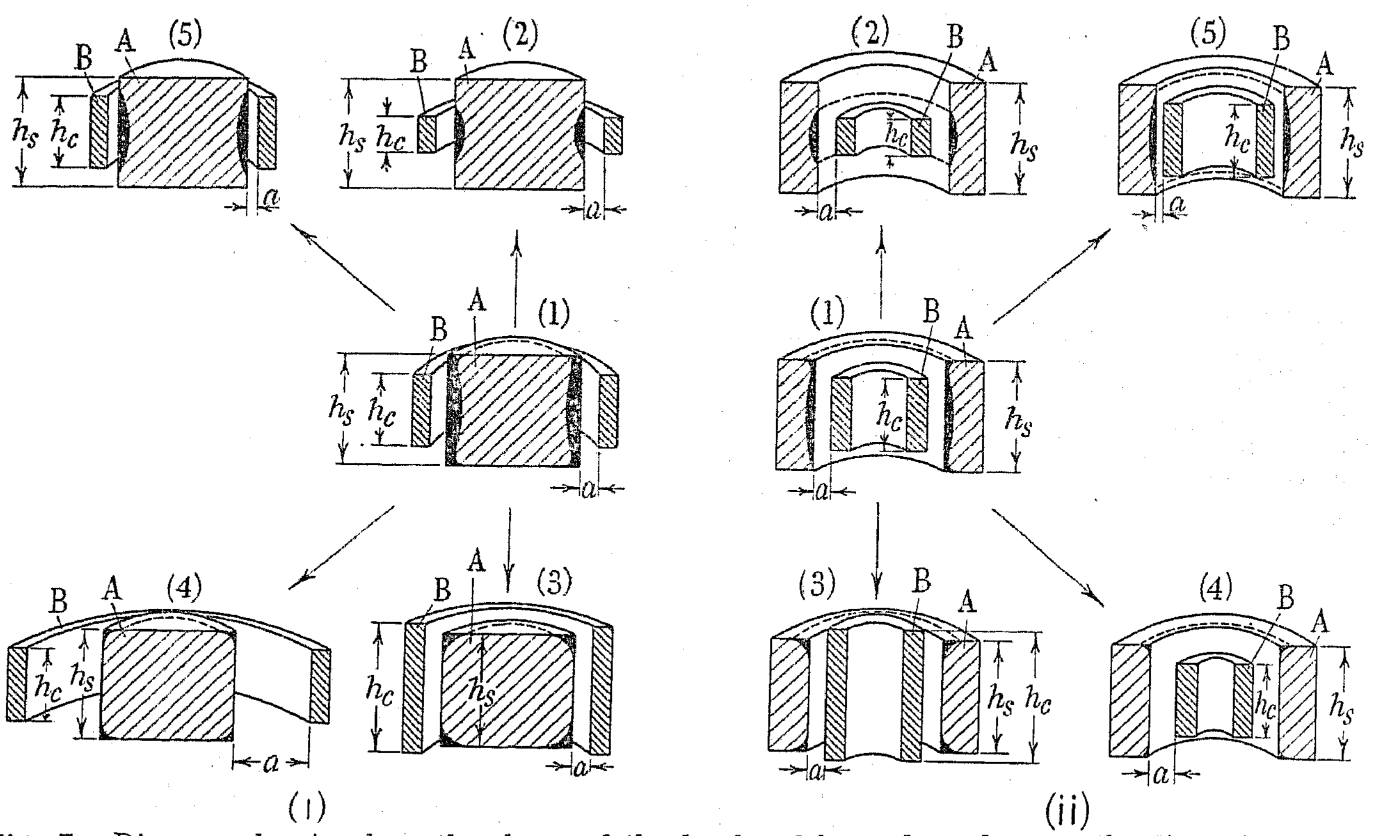

Fig. 7.—Diagram showing how the shape of the hardened layer depends upon the dimensions of the specimen (A) and of the heating coil (B).

Fig. 7], the ends of the specimen will be heated more intensely; and where the height of the coil is less, [(4) and (5) in Fig. 7], the middle part of the specimen will be heated more intensely.

Usually the distance from coil to specimen is 2–5 mm.; and therefore, in order to obtain a uniform thickness of hardened layer on the outer surface of a cylindrical specimen, the coil should be 5–10 mm. shorter than the specimen.

(4) SOME DETAILS OF THE PROCESS OF HEATING STEEL BY MEANS OF HIGH-FREQUENCY CURRENTS

The power dissipated in the part to be heated is given by

$$\Delta P = \frac{H_t^2 \rho}{16\pi^2 p}$$

Therefore, if the current in the heating coil (I_C) is constant,

$$P \equiv \sqrt{(\rho\mu)}$$

The value of ρ gradually increases with temperature, while μ slightly diminishes and then suddenly falls at

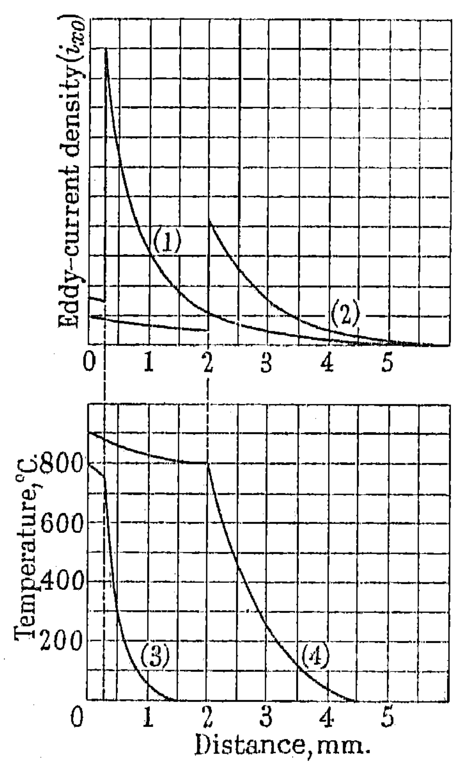

Fig. 8.—Distribution of current density and temperature over cross-section of specimen at various stages of the heating process.

(1), (3): 1 sec. after beginning of heating.
(2), (4): 2 sec. after beginning of heating.

721° C., when steel passes over to its non-magnetic modification (γ-steel). Above that temperature the permeability of steel is equal to unity. Hence the power absorbed by the specimen first increases with temperature and then suddenly diminishes. This peculiarity of the procedure of heating steel parts by eddy currents is of great importance, as it ensures a high grade of hardening and does not permit surface overheating.

If the surface layer is heated for hardening purposes by, for instance, autogen burners, the power input to the surface remains constant throughout the heating period. It is therefore very difficult to avoid overheating of the peripheral layer. On the other hand, when eddy-current heating is employed the power absorbed by the specimen abruptly diminishes above the point of magnetic transmutation. The heat is then conducted into the interior of the specimen, while the rate of rise of temperature at the surface diminishes.

Fig. 8 shows the distribution of temperature and current density over the cross-section of the specimen.

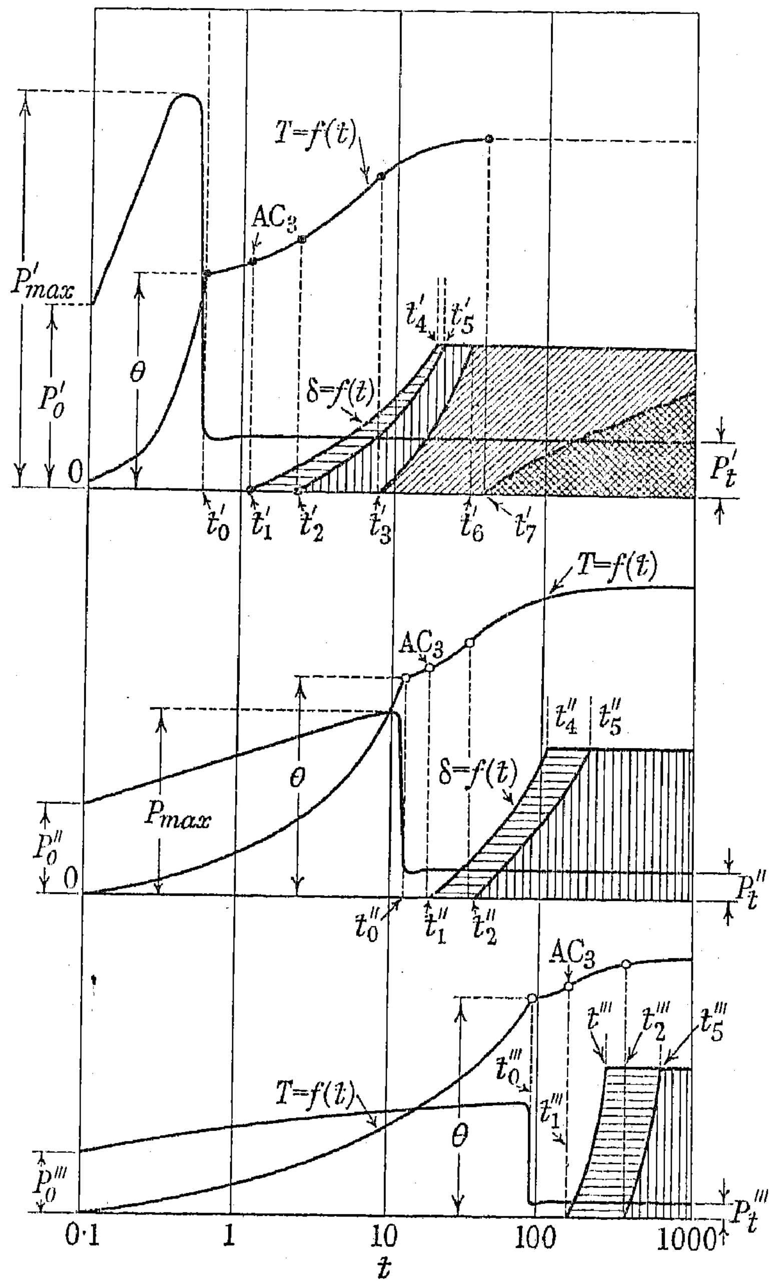

Fig. 9.—Variation, with time, of power absorbed by specimen, temperature of peripheral layer, and thickness of layer ready for hardening; high-frequency heating (10^5 to 10^7 cycles per sec.).

θ = Curie point.

As the temperature increases, the position of maximum current-density travels inwards and the current density at the surface diminishes. The thickness of the heated zone increases at a relatively high rate, while the temperature of the surface layer rises slowly.

Fig. 9 shows the variation of the power absorbed by the part during the heating period when the current in the heating coil (I_C) is maintained constant. The top (P', t') series of curves relates to large values of the initial power (P_0), the lower (P'', t'') series to medium values, and the bottom (P''', t''') series to small values.

At the instant t'_1 the surface layer of the specimen

reaches the temperature AC_3 at which the solid solution starts to form. The curve $\delta = f(t)$ illustrates how the process of formation of the solid solution is propagated into the material. At the instant t_2 the process of formation of the solid solution is complete, and further heating causes overheating of the material. Overheating starts at t_3, and thereafter the overheated zone is propagated into the interior of the specimen. Melting starts at t_4. In Fig. 9 the areas shaded with horizontal lines represent the zone of solid-solution formation, those shaded with vertical lines represent the zone ready for hardening, the area shaded with slanting lines represents the overheated zone, and the cross-hatched area represents the melted zone. Accordingly, the curves $t_1'$$t_4'$ and $t_2'$$t_5'$ indicate the start and finish respectively of solid-solution formation, and $t_3'$$t_6'$ indicates overheating.

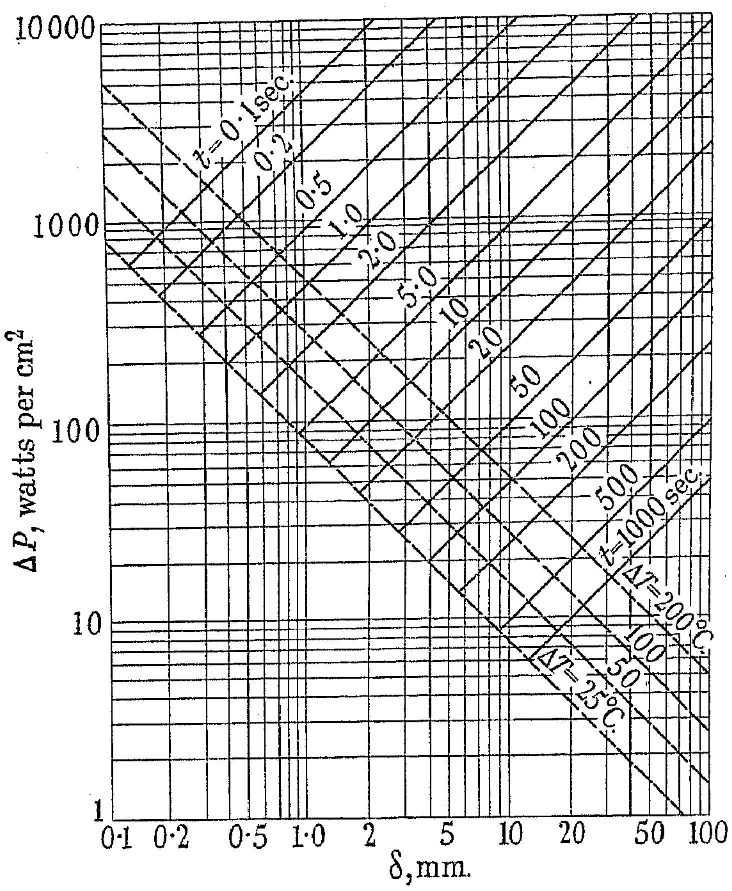

Fig. 10.—Diagram showing possible schedules of inductive heating for carbon steel (0·8 % C).

Fig. 10 shows the schedules of inductive heating for surface hardening, computed for carbon steel containing 0·8 % C. The abscissae denote the thickness, in millimetres, to be heated to the hardening temperature; and the ordinates the power, in watts, per square centimetre of the surface treated.

There are two fundamental cases to be considered: (*a*) When the heat is generated (approximately uniformly) in the whole zone heated, and the heating time is so short that the interchange of heat between the heated zone and the surrounding metal is negligible. (*b*) When the thickness where heat is generated is considerably less than that of the layer which is to be heated to the hardening temperature, and the heat is propagated primarily by thermal conduction. Between these two extremes there is a range of intermediate types of heating, involving both (*a*) and (*b*). The series of thick continuous straight lines in Fig. 10 refers to Case (*a*).

The amount of heat required to transform 1 cm³ of steel at room temperature to the γ-modification at 800° C. is 4 600 joules. Hence in this case the specific power is

$$\Delta P = 4\,600\frac{\delta}{t} \text{ watts per cm}^2$$

Where the heat is transmitted through the steel by conduction the specific power depends on the temperature drop in the layer heated. The thermal conduction of steel (0·8 % C) in the temperature range 600°–1 000° C. is 0·25 watt per deg. C. per cm. Let ΔT be the temperature drop in the layer; then in the steady state the flow of energy into the metal will be

$$\Delta P = 0{\cdot}25\frac{\Delta T}{\delta} \text{ watts per cm}^2$$

The series of thick broken lines in Fig. 10 represents $\Delta P = f(\delta)$ for various values of ΔT. The line $\Delta T = 100°$ is the limit for surface hardening. With smaller values of specific power input than those given by this line it would be impossible to harden a layer of given thickness.

The depth of penetration of eddy currents in steel heated above the Curie point is given by $500/\sqrt{f}$ mm., and in cold steel by $20/\sqrt{f}$ mm.

When the frequency of the current is such that the depth of penetration is considerably less than the thickness of the layer in preparation for hardening ($\delta\sqrt{f} > 1\,000$), conduction alone is important, and the corresponding heating schedule is represented by one of the broken lines such as are plotted in Fig. 10. When the frequency is lower ($\delta\sqrt{f} < 500$), the thermal capacity only is important, and the heating schedule is represented by a continuous line.

To obtain thicknesses of hardened layer of about 1 mm. it will be necessary to employ the thermal-capacity type of heating process, with short heating times (of the order of 1 sec.) and great power input. For thicknesses exceeding 5 mm. there is a very large choice of possible schedules. If a low frequency is employed the heating may be accomplished in a short time, but a higher power input is necessary. On the other hand, a high frequency means lower power input and a longer heating period. For a given power input, the area which can be treated in a given time is smaller with low-frequency than with high-frequency heating. The area which can be hardened in 1 sec. is determined by the power of the oscillator, and not by its frequency.

For the great majority of machinery parts 10^5 to 10^6 cycles per sec. may be taken as the optimal frequency range.

Fig. 10 gives an approximate theoretical value for the minimum power required to be dissipated on the surface of steel in the heating process. The power drawn from the line is equal to the value obtained from Fig. 10 divided by the efficiency of the heating coil and valve oscillator. In most cases the actual power consumption is 2–4 times greater than the value calculated from Fig. 10. For example, hardening to a thickness of 2–3 mm. requires 800–1 000 watts of installed power per square centimetre of the surface treated.

When a steel part is rapidly heated in a non-uniform

field—for instance, by means of a coil having many turns—a peculiar effect is observed: the degree of non-uniformity of the heating effect exceeds that of the field. In the case shown in Fig. 4, for example, the non-uniformity of the current distribution on the surface of the part does not exceed some tenths of 1 %, provided the clearance between the part and the coil is greater than the spacing between turns. When the part was heated through the temperature range 650°–750° C., however, its surface showed brighter and darker bands which together formed a mirror image of the heating coil.

This effect may be explained as being due to the instability of the process of transmitting power from the heating coil to the specimen, at temperatures below the Curie point. From Fig. 9 it may be seen that for temperatures up to 750° C. the power absorbed by the part heated increases with temperature. Therefore, as the temperature rises, any slight initial non-uniformity of the power distribution on the surface of the specimen tends to increase. Increasing the power absorbed raises

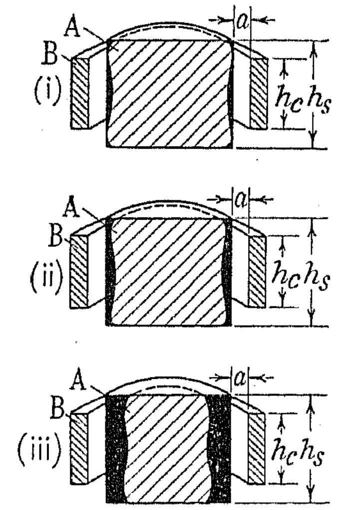

Fig. 11.—Diagram showing how the configuration of the hardened layer depends upon the power input per cm² and the heating time.

(i) $t = 0{\cdot}5$ sec., $\Delta P = 2$ kW per cm² (ii) $t = 3$ sec., $\Delta P = 0{\cdot}8$ kW per cm² (iii) $t = 15$ sec., $\Delta P = 0{\cdot}2$ kW per cm²

the temperature, causing an increase in the value of ρ and, consequently, a further increase of the power input to the areas heated to a higher temperature. Near the Curie point the power input decreases with rise of temperature, with the result that the temperature distribution on the surface of the specimen becomes more uniform, as areas heated to a higher temperature absorb less power than those heated to a lower.

For this same reason the configuration of the hardened layer depends upon its thickness. This is illustrated by Fig. 11. When the thickness of the layer is small as compared with its height, the smoothing effect of thermal conduction along the layer is absent, and in order to obtain a layer of uniform thickness the shape of the heating coil must be selected very carefully. Moreover, thin layers require high values of power input per unit area of surface. The thicker the layer to be hardened the more uniform is the temperature distribution (owing to thermal conduction), and the less is the configuration of the layer affected by the shape of the heating coil.

Fig. 12 shows the method of heating a steel cam in the field due to a cylindrical coil. As the coil is shorter than the cam, the current density should be uniform in the lateral surface of the cam.

It must, however, be taken into consideration that on convex surfaces the depth of penetration will be greater, and on concave ones less, than

$$\frac{1}{2\pi}\sqrt{\left(\frac{\rho}{\mu f}\right)}$$

Hence the eddy-current density will be less at the tooth points than in the tooth spaces. Also, as the heat generated is proportional to the square of the eddy-

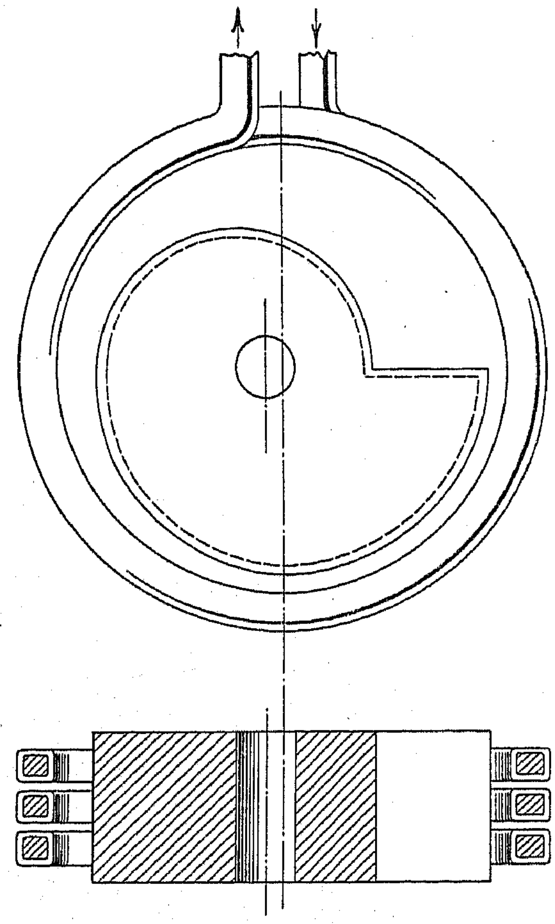

Fig. 12.—Method of heating a steel cam in the field due to a cylindrical heating coil.

current density, less heat will be dissipated at the points than in the spaces.

The higher the frequency, and therefore the less the depth of penetration p, the less will p be affected by the curvature of the surface. Therefore in order to make the heat distribution more uniform it is necessary to increase the frequency.

The more marked generation of heat in the tooth spaces is compensated by the conduction of heat into the interior of the specimen, which is better where the surface is concave than where it is convex. For this reason there are no particular difficulties in obtaining uniform thickness of the hardened layer in specimens having projections and cavities.

(5) DESIGN OF HEATING COILS

For the heating coil to have a high efficiency its breadth (denoted by $2g$ in Figs. 1–3) must be 10–20 times less than its length. (In cylindrical coils the length is $2\pi rn$, where r is the radius of the coil and n the number of turns.) Single-turn coils are therefore to be preferred

for hardening cylindrical parts where the height of the layer to be hardened is less than the diameter of the specimen. On the other hand, when heating such things as taps, reamers, etc., where the height of the layer to be heated is 2–3 times its diameter, it is preferable to use multi-turn coils. The relation between the configuration of the hardened layer and the heating-coil dimensions, illustrated in Fig. 7, also applies to the multi-turn coil.

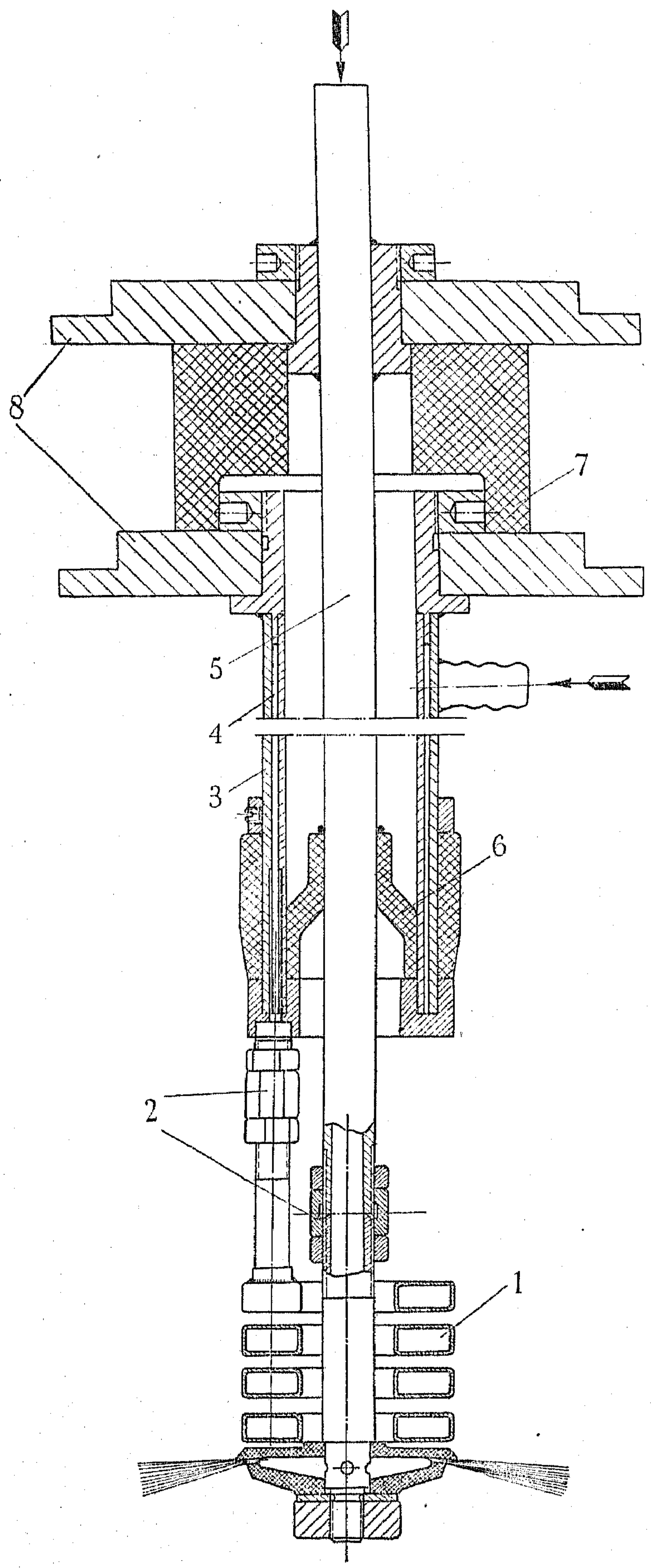

Fig. 16.—Section of a heating coil for hardening inner surfaces.

1. Heating coil. 2. Joints. 3. Water jacket. 4. Outer conductor tubing. 5. Inner conductor tubing. 6 and 7. Insulators. 8. Terminals.

It is not advisable to apply inductive heating to cylindrical and prismatic bodies of which the length is 10 or more times the diameter. Instead, the specimen should be connected to the terminals of a h.f. transformer.

Figs. 13, 14 and 15 (see Plate 1, facing page 168), and Fig. 16, show various heating-coil designs used for the hardening of tools and other steel parts. For specimens where there is a large surface area to be hardened, as, for instance, in rolling mills, the method of progressive hardening may be employed. In this method the heating coil is moved along the surface of the work, leaving behind a heated zone ready for hardening. The coil is followed by the arrangement employed for supplying the cooling liquid.

If t is the time necessary for a fixed coil to prepare the peripheral layer for hardening, and g is the width of the heating coil, the speed with which the coil should pass over the specimen is $u = g/t$.

Thus the more slowly the heating proceeds, the wider must be the heating coil to give the required speed of the hardening process. The efficiency of the progressive hardening method is less than that of the simultaneous method, because some of the heat is wasted by lateral radiation from the heated zone.

In order to obtain satisfactory results with progressive hardening, the condition must be satisfied that the width of the heated zone (as a first approximation it may be taken to be equal to the width of the heating coil) shall not be less than about 10 times the thickness of the layer to be hardened.

(6) RESULTS

Fig. 17 (Plate 1) shows cutters with tenons 3 mm. thick which have been hardened only from the outer side. The hardening of such cutters by the usual methods is accompanied by considerable warping. With inductive heating no deformation was observed after the hardening process, because the volume heated is in this case very small.

It is difficult to use heating coils for the heating of holes less than 20 mm. in diameter; yet dies are commonly employed for still smaller diameters (3 to 10 mm.). Surface hardening can be applied only to slotted dies, e.g. those shown in Fig. 18 (Plate 2). When a die which has no slots is placed in the heating-coil field, it absorbs heat only from the outside, while in a slotted die heating occurs both inside and outside. The heating time of the dies shown in Fig. 18 was 1·2 sec.

Fig. 19 (Plate 2) illustrates the heating of a grooving cutter in the field of a cylindrical coil. The photograph was taken 2·5 sec. after the h.f. current was switched on.

Investigation of the microstructure of all the tools shown in Figs. 17 and 18 revealed a very fine-grained martensite in the hardened zone and a smooth transition to the core structure. In hardening gears of larger dimensions each tooth has to be separately treated. A machine for hardening gears tooth by tooth is shown in Fig. 20 (Plate 2).

Plate 1

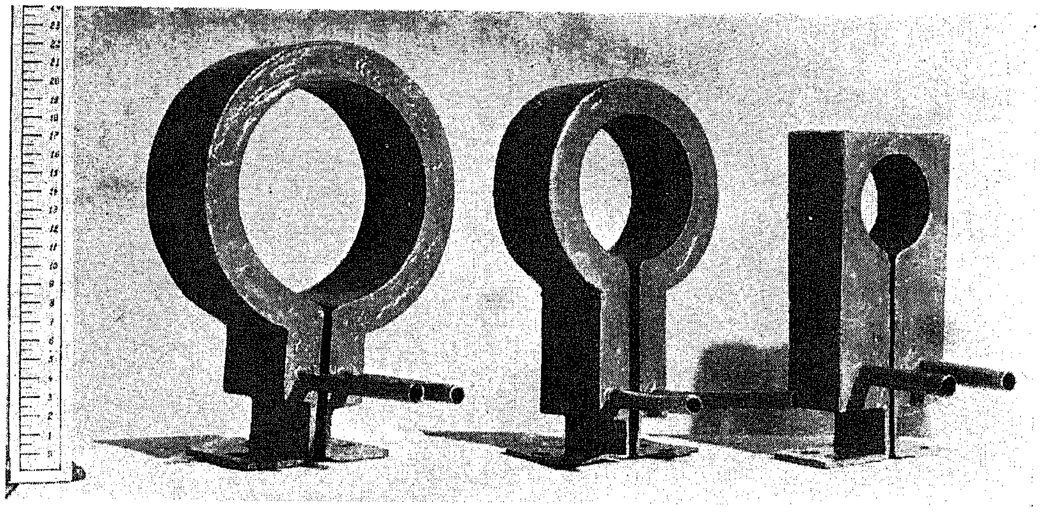

Fig. 13.—Heating coils for cylindrical specimens.

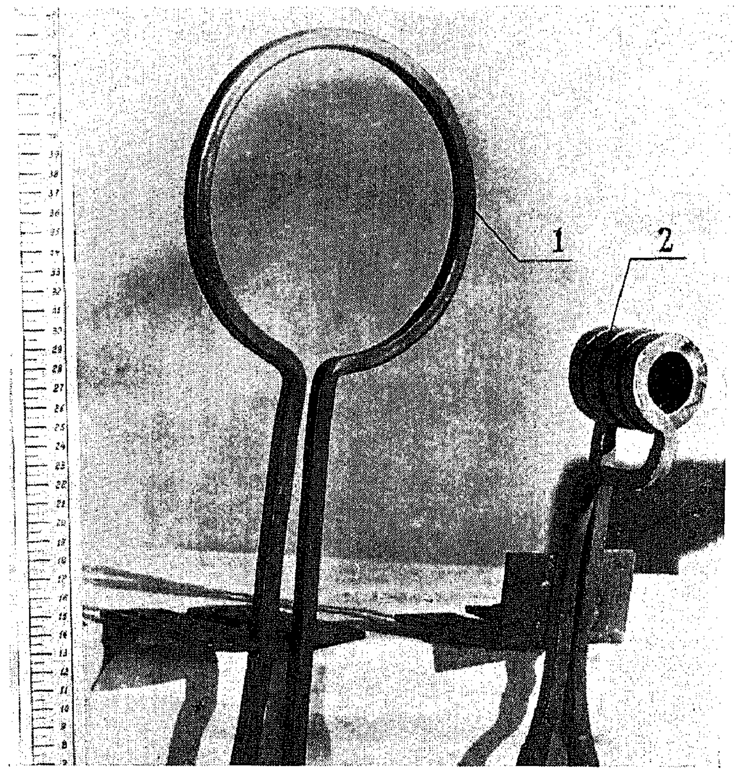

Fig. 14.—Heating coils for cutters (1) and taps (2).

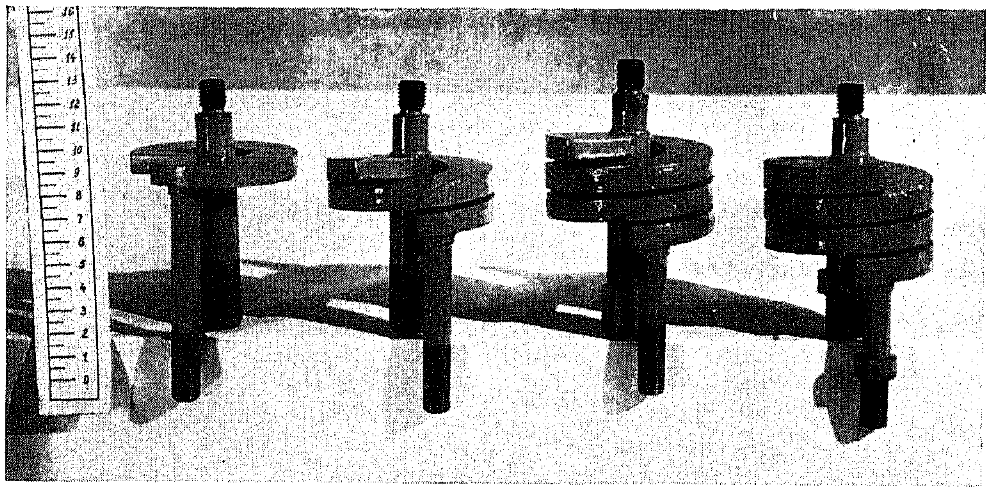

Fig. 15.—Heating coils for hardening inner surface of steel tubing.

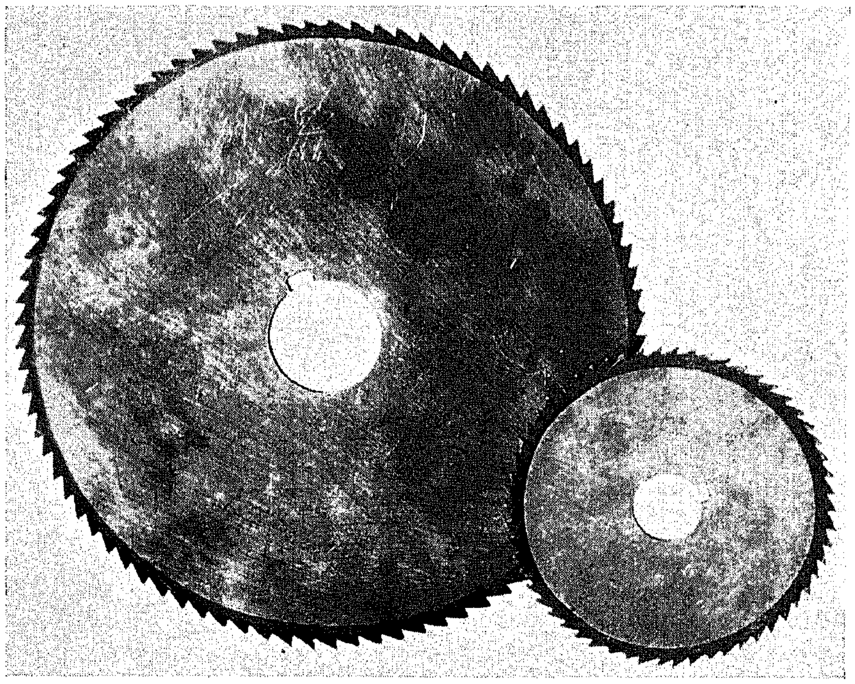

Fig. 17.—Macrostructure of cutters 160 mm. and 70 mm. in diameter, 3 mm. thick. Hardened layers 4 mm. thick. Hardness $R_c = 64\text{–}65$. Etched by Heine reagent.

Plate 2

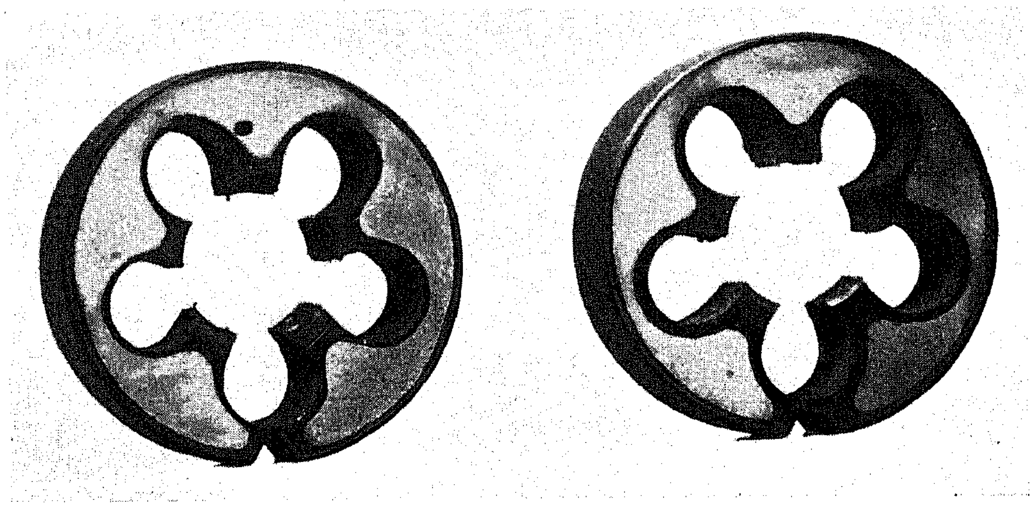

Fig. 18.—Macrostructure of 18 mm. dies hardened in the field of a cylindrical coil.

Material: carbon steel (1·2 % C).
Hardness: R_c = 64–65.
Etched by Heine reagent.

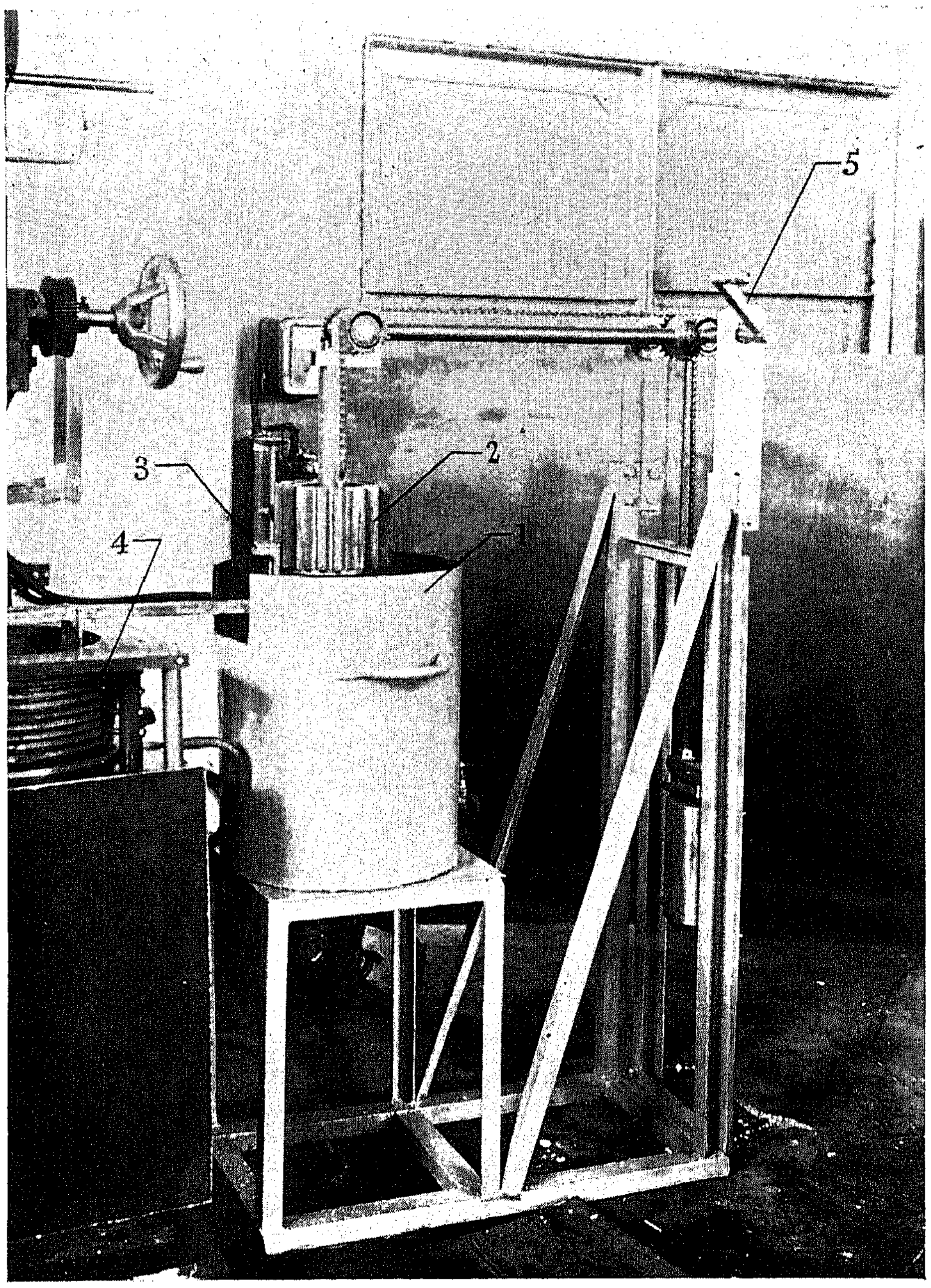

Fig. 20.—Hardening machine for gears.

1. Container for the cooling liquid. 2. Gear. 3. Heating coil. 4. High-frequency transformer (cover removed). 5. Handle for moving the gear.

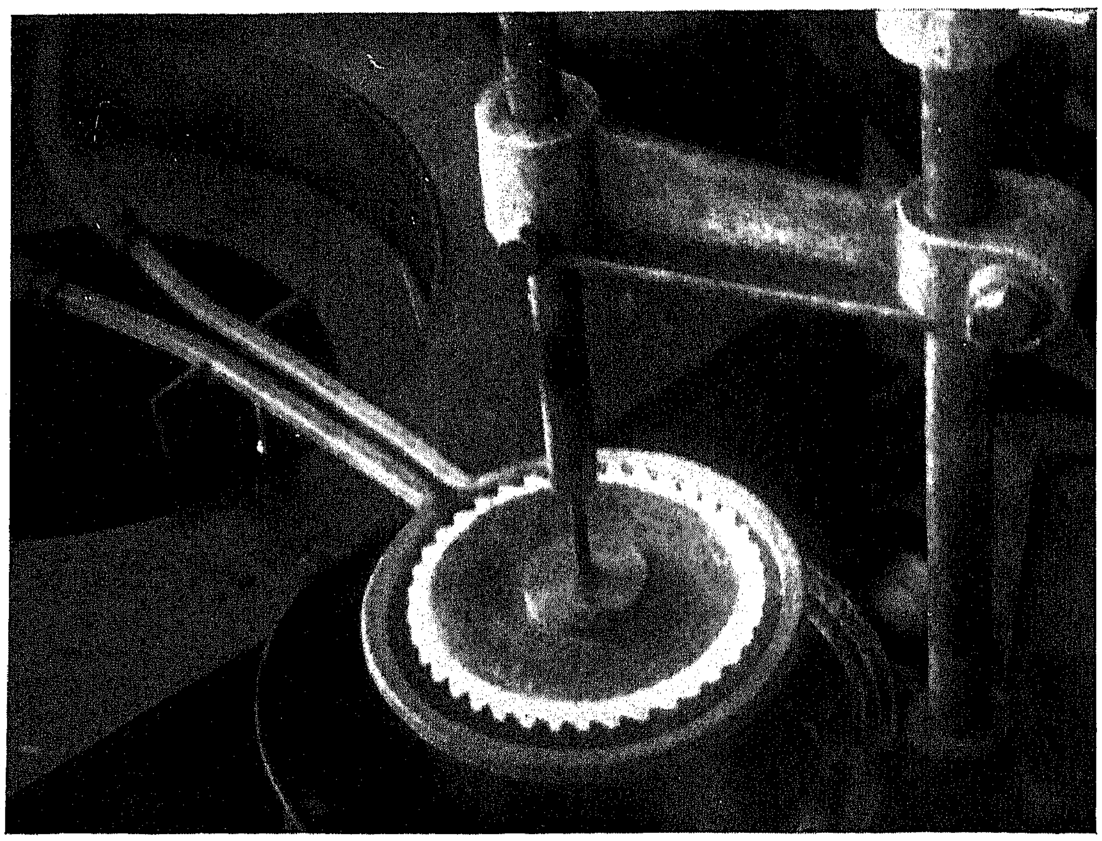

Fig. 19.—Heating a 160-mm. cutter in the field of a cylindrical coil. Frequency, 10^6 cycles per sec. Photograph taken 2·5 sec. after beginning of heating.

THE ELECTRIC AND MAGNETIC FIELDS OF A LINEAR RADIATOR CARRYING A PROGRESSIVE WAVE*

By F. M. COLEBROOK, B.Sc.

(*Paper first received* 13*th January, and in revised form* 29*th May*, 1939.)

SUMMARY

The vector- and scalar-potential method of calculating electric and magnetic fields is applied to a straight conductor carrying a progressive wave of current. It is shown that unless the conductor is assumed to be terminated by charges which satisfy the condition of electrical continuity at the ends, the calculated field has an anomalous and impossible character. Such charges need not be included in the case of closed circuits of conductors carrying progressive waves. The mutual cancellation in such cases of the anomalous features associated with each linear element is confirmed by analysis of a particular case. It is pointed out that a linear progressive wave radiator has only one axis of symmetry, i.e. the radiator itself, differing in this respect from a standing-wave radiator, which is symmetrical about the equatorial plane. The consequent asymmetry of the field from a progressive wave radiator is confirmed by analysis.

The field system associated with a progressive wave of current is regarded as a more fundamental conception than that due to a standing wave, since any standing-wave system of currents is resolvable into positive and negative travelling waves. This feature is illustrated by the synthesis of the travelling wave fields into the known form for a dipole standing-wave radiator.

The radiation resistance of a progressive-wave radiator is calculated by Pistolkors's method, and is shown to be equal to that for a standing-wave radiator when the length of radiator is an integral number of half wavelengths.

(1) INTRODUCTION

The author has recently had occasion to consider the design of a rhombic transmitting antenna for a particular purpose, and was thus led to study in some detail the nature of the field from a linear conductor carrying a progressive wave of current. In doing so he noted certain essential points of difference between conductors carrying progressive waves and conductors carrying standing waves. These have, no doubt, already been appreciated by the writers of the various papers on rhombic antennae which are now available (see Bibliography), but the present author has not found them explicitly dealt with in any of these papers. In view of the considerable and growing importance of the subject, it has been thought desirable to emphasize some of these difficulties and distinctive features in the present paper, which is analytical in character and is intended as a guide in future design and experimental work.

* Official Communication from the National Physical Laboratory.

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(2) ANALYTICAL BASIS

For the purpose of this paper, the method of analysis used by Pistolkors in his well-known paper on radiation resistance,† i.e. the derivation of the field from scalar and vector potentials, has some advantage over the more compact and elegant formulation given by Bechmann‡ in terms of the Hertzian vector.

Pistolkors starts with the formulae

$$\mathbf{E} = -\operatorname{grad}\psi - \frac{1}{c}\frac{d\mathbf{A}}{dt} \quad . \quad . \quad . \quad (1)$$

$$\mathbf{H} = \operatorname{curl}\mathbf{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

E and **H** being the electric and magnetic fields at a given point in space due to a system of charges and currents which give retarded scalar and vector potentials ψ and **A** at the point. The particular system to be studied in the first instance is that illustrated in Fig. 1, which represents a straight conductor of length $2l$ and of negligible cross-section, situated in free space. The conductor carries a current which is a function of time and of the distance along the conductor, its value being i at the instant t at the point distant z from the origin (the centre of the conductor).

The retarded vector potential **A** at the point P due to the current in the whole length of the conductor is given in absolute units by

$$\mathbf{A} = \mu c \int_{-l}^{l} \frac{[\mathbf{i}]}{r} dz \quad . \quad . \quad . \quad . \quad . \quad (3$$

where μ is the permeability of free space, c the velocity of light, and **i** a vector of magnitude i and direction parallel to the conductor. The vector **A** is therefore parallel to the conductor. The square brackets signify the usual retardation, i.e. in calculating **A** for the instant t the value of i to be used is that for the instant $t - r/c$.

There is a distribution of linear charge density σ along the conductor given by

$$\frac{\partial \sigma}{\partial t} = -\frac{\partial i}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the retarded scalar potential at P due to this charge distribution is given by

$$\psi_0 = \frac{1}{\kappa} \int_{-l}^{l} \frac{[\sigma]}{r} dz \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

† See Bibliography, (8).

‡ *Ibid.*, (3).

where κ is the permittivity of free space, i.e.

$$c^2 = \frac{1}{\kappa\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Initial calculations for a linear radiator carrying a progressive wave, based on the above values of **A** and ψ_0, lead to certain anomalous results; in particular, a radial component of electric intensity even at great distances from the radiator. Applied to systems in which such linear radiators were formed into closed circuits, however, the calculations lead to apparently quite correct

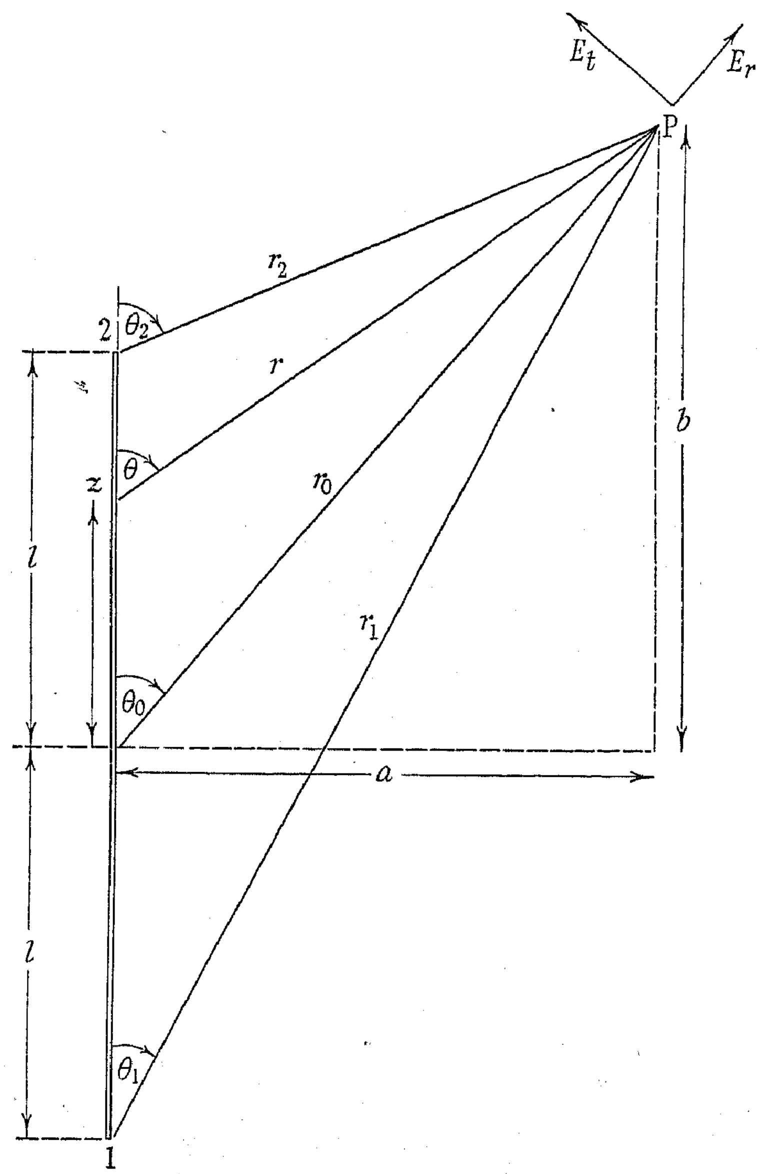

Fig. 1

results. It was suggested to the author by a colleague (Mr. W. Ross) that the explanation probably lay in the incomplete formulation of the scalar potential. This author is also indebted to Mr. I. H. Cole of the Bawdsey Research Station for calling his attention to a paper by J. Groszkopf* in which this point is very explicitly dealt with.

Pistolkors was concerned with standing-wave radiators, having therefore zero current at the open ends. For such systems equation (5) completely represents the retarded scalar potential. In a linear radiator carrying a progressive wave, however, the amplitude of the current will be substantially uniform along its length. A single linear radiator carrying such a current, as shown in Fig. 1, is not therefore a physically self-consistent conception, and it is not surprising that anomalous results were obtained. A closed circuit of such conductors is, on the other hand, a physically self-consistent system. As Groszkopf points out, the single radiator can equally be made self-consistent by assuming it to be terminated by charges q_1 and q_2 at the ends 1 and 2, given by

$$\left.\begin{aligned} q_1 &= -\int i_1 dt \\ q_2 &= \int i_2 dt \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

i_1 and i_2 being the values of the current at the ends 1 and 2. These charges will give rise to retarded potentials ψ_1 and ψ_2 given by

$$\psi = \frac{1}{\kappa}\frac{[q]}{r}; \text{ suffixes 1, 2} \quad . \quad . \quad . \quad (8)$$

and the total scalar potential is therefore

$$\psi = \psi_0 + \psi_1 + \psi_2 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

as defined by (5) and (8).

The significance of these additional terms in the determination of the field will be shown later.

Thus far, the only simplifying approximation made is that the diameter of the wire is negligible compared with all the other lengths involved.

It will now be assumed that the current distribution takes the form of an undamped progressive wave which, for simplicity, will be expressed in the exponential form

$$i = I_0 e^{j(\omega t + \alpha)} e^{-jmz} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where $m = 2\pi/\lambda_c$, λ_c being the wavelength of the current wave along the conductor. In practice, however, the conductor will have a certain ohmic resistance, with a consequent small attenuation of the current wave. An even larger factor is the attenuation due to the radiation from the conductor, which, as will be shown later, exists for a progressive wave just as it does for a standing wave. (The statement to the contrary by Everett and Byrne in 1929* was made prior to the introduction and use of progressive-wave transmitting-aerial structures and would presumably no longer be maintained by them. There appears to be no valid theoretical justification for it.) Thus, for an exact representation of a physically realizable system, it would be necessary to assume a current distribution of the form

$$i = I_0 e^{j(\omega t + \alpha)} e^{-(n + jm)z} \quad . \quad . \quad . \quad (11)$$

The resulting integrals cannot, however, be expressed in terms of known and tabulated functions. As a first approximation, therefore, it is necessary to neglect the effect of attenuation, reserving it for later consideration. It is likely to have a relatively small effect on amplitude rather than on the general character of the field.

One further point should be noted at this stage. The assumed current distribution

$$i = Ie^{-jmz}$$

where

$$I = I_0 e^{j(\omega t + \alpha)}$$

* See Bibliography, (13).

* See Bibliography, (10).

represents a progressive wave of current travelling up the conductor from 1 to 2, i.e. in the positive direction of z. Such a system has only one axis of symmetry, namely the conductor itself. A similar conductor carrying a standing wave can, as is well known, be regarded as carrying equal and opposite progressive waves, and its field will therefore be symmetrical with respect to the equatorial plane in direction and amplitude. No such symmetry can be expected for the progressive-wave radiator and does not, in fact, exist. The point is sufficiently obvious when stated in this way, but the author fell into error in one instance through overlooking it. Further, in all combinations of linear conductors carrying progressive waves it is the direction of the wave, rather than that of the current, which matters. Consider, for example, the rectangular structure shown in Figs. 2(*a*) and 2(*b*), in which progressive waves are obtained by a suitable choice of the terminating impedance. A consistent sign convention for the direction of the current would be as shown by the arrows in Fig. 2(*a*). The wave

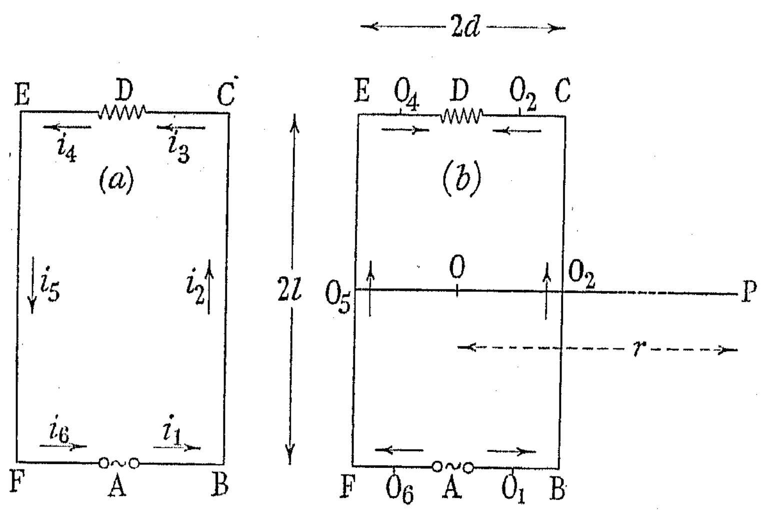

Fig. 2

directions, however, are as shown in Fig. 2(*b*), and it is found that the field formulae depend on the relation between the direction of the current and the direction of the travel of the current wave.

For the assumed current distribution, it is easy to show that

$$[\mathrm{i}] = \mathbf{I}e^{-j\left(\frac{\omega r}{c} + mz\right)} \quad . \quad . \quad . \quad . \quad (12)$$

where $\mathbf{I}$ is a vector of magnitude $I = I_0 e^{j(\omega t + \alpha)}$ and direction z. Also

$$[\sigma] = \frac{m}{\omega} I e^{-j\left(\frac{\omega r}{c} + mz\right)} \quad . \quad . \quad . \quad (13)$$

It is now necessary to make the further simplifying assumption that the wavelength λ_c along the conductor is the same as that in free space, $\lambda = 2\pi c/\omega$. This assumption is known to be very approximately correct and is, moreover, consistent with the assumed absence of attenuation. It is desirable, however, that all such assumptions should be clearly stated and appreciated. This gives

$$m = \frac{2\pi}{\lambda} = \frac{\omega}{c} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and the expressions for the current and charge density simplify to

$$[\mathrm{i}] = \mathbf{I}e^{-jm(r+z)} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and

$$[\sigma] = \frac{1}{c} I e^{-jm(r+z)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Further (see Appendix 2)

$$[q_1] = -\int [i_1] dt$$

$$= -\frac{1}{j\omega} I e^{-jm(r_1 - l)} \quad . \quad . \quad . \quad . \quad (17)$$

and

$$[q_2] = \int [i_2] dt$$

$$= \frac{1}{j\omega} I e^{-jm(r_2 + l)} \quad . \quad . \quad . \quad . \quad (18)$$

For the above values of i and σ, it is easily shown that in practical units,

$$\psi_0 = 30I \, . \, F \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and

$$\frac{1}{c}\frac{d\mathbf{A}}{dt} = jm30I \, . \, F \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where

$$F = \int_{-l}^{l} \frac{e^{-jm(r+z)}}{r} dz \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The detail of this integration is contained in one of the papers by Bechmann, and is outlined in Appendix I for convenience of reference.

(3) THE CO-ORDINATE SYSTEM

The most convenient co-ordinate system is spherical-polar, with the origin at the centre of the radiator, the co-ordinates r_0 and θ_0 of P being as shown in Fig. 1. However, the r and θ shown for the point z in Fig. 1 are not required after integration and it will therefore be permissible and convenient to drop the zero suffixes in r_0 and θ_0 in the formulae for the fields at P. On account of axial symmetry the system is effectively two-dimensional and the ϕ co-ordinate of P is not required.

(4) THE ELECTRIC AND MAGNETIC FIELDS

It will be convenient to distinguish between the component of the total electric field at the point P which is due to the distributed current and charges of the linear radiator, and the component which arises from the fictitious terminating charges introduced in order to make the system physically consistent, for, as will be shown later, only the former need be taken into account in dealing with closed-circuit structures of linear radiators. For convenience, the terminal-charge component will be distinguished by a suffix, q, i.e. the two main components will be written E and E_q.

For distant points, the most convenient resolution of the electric field is into radial and tangential components E_r and E_t; $(E_q)_r$ and $(E_q)_t$, as shown in Fig. 1. For some purposes, e.g. determination of radiation resistance by Pistolkors's method, a more convenient resolution is into components parallel and perpendicular to the radiator, i.e. E_z and E_x, etc.

The vector **H** is everywhere perpendicular to the plane defined by r and the radiator, and will otherwise be specified by its magnitude H.

An outline of the evaluation of these various components is given in Appendices 2 and 3. The results are:—

$$E_r = \frac{-30I}{r}\left\{\frac{r_2 - l}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + l}{r_1}e^{-jm(r_1-l)}\right\} \quad (22)$$

$$E_t = \frac{30I}{r}\cot\theta\left\{\frac{r_2 - l + r/\cos\theta}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + l + r/\cos\theta}{r_1}e^{-jm(r_1-l)}\right\} \quad . \quad (23)$$

$$(E_q)_r = 30I\left\{\left(1 - \frac{j}{mr_2}\right)\left(\frac{r - l\cos\theta}{r_2}\right)\frac{e^{-jm(r_2+l)}}{r_2} - \left(1 - \frac{j}{mr_1}\right)\left(\frac{r + l\cos\theta}{r_1}\right)\frac{e^{-jm(r_1-l)}}{r_1}\right\} \quad . \quad (24)$$

$$(E_q)_t = -30Il\sin\theta\left\{\frac{e^{-jm(r_2+l)}}{r_2^2} - \frac{e^{-jm(r_1-l)}}{r_1^2}\right\} \quad . \quad (25)$$

for the radial and tangential components of E, while for the parallel and perpendicular components

$$E_x = \frac{-30I}{r\sin\theta}\left\{\frac{r_2 + r\cos\theta - l}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + r\cos\theta + l}{r_1}e^{-jm(r_1-l)}\right\} \quad . \quad (26)$$

$$E_z = 30I\left\{\frac{e^{-jm(r_2+l)}}{r_2} - \frac{e^{-jm(r_1-l)}}{r_1}\right\} \quad . \quad . \quad . \quad . \quad (27)$$

$$(E_q)_x = 30I\,.\,r\sin\theta\left\{\left(1 - \frac{j}{mr_2}\right)\frac{e^{-jm(r_2+l)}}{r_2^2} - \left(1 - \frac{j}{mr_1}\right)\frac{e^{-jm(r_1-l)}}{r_1^2}\right\} \quad . \quad (28)$$

$$(E_q)_z = 30I\left\{(r\cos - l)\left(1 - \frac{j}{mr_2}\right)\frac{e^{-jm(r_2+l)}}{r_2^2} - (r\cos\theta + l)\left(1 - \frac{j}{mr_1}\right)\frac{e^{-jm(r_1-l)}}{r_1^2}\right\}. \quad (29)$$

Finally,

$$H = -E_x \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

These formulae suffice for the complete determination of the field at any point in space due to any closed-circuit configuration of linear conductors carrying progressive waves, and also for the field due to any single conductor carrying standing waves, since the latter can always be resolved into its progressive-wave components. In this sense they are more fundamental than the usual formulae for standing-wave radiators.

(5) THE FORMULAE FOR DISTANT NON-AXIAL POINTS

In nearly all cases of practical interest, the point P is so distant that l is very small compared with r, and certain simplifications can be made in the above formulae. They must, however, be introduced with some care. It is, for instance, permissible to put

$$\frac{1}{r_1} = \frac{1}{r_2} = \frac{1}{r}$$

since this only involves the neglect of terms proportional to $1/r^2$ compared with terms proportional to $1/r$, but for the exponential indices a closer approximation must be used, i.e.

$$r_2 + l \simeq r + l(1 - \cos\theta)$$

and

$$r_1 - l \simeq r - l(1 - \cos\theta)$$

Then, neglecting terms in $1/r^2$ (i.e. the induction field) compared with those in $1/r$ (i.e. the radiation field)

$$E_r = \sin\left\{ml(1 - \cos\theta)\right\}\frac{e^{-jmr}}{r}60jI \quad . \quad . \quad (31)$$

$$E_t = -E_r\,.\cot\theta/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$(E_q)_t = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

$$(E_q)_r = -E_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

for radial and tangential components, and

$$E_z = -E_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

$$E_x = E_r\cot\theta/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$(E_q)_z = -E_r\cos\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

$$(E_q)_x = -E_r\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

for parallel and perpendicular components. Also

$$H = -E_r\cot\theta/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

$$= E_t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

The formulae are not valid for axial points, for which separate consideration is required (see Section 7).

(6) THE SIGNIFICANCE OF THE TERMINAL CHARGES

The common factor e^{-jmr}/r, where $m = 2\pi/\lambda$, shows that the field at distant points is inversely proportional to the distance, a well-known characteristic of the radiation field from a point source. Further, the factor e^{-jmr}, which determines phase, is such that the direction of maximum rate of change of phase is radial, i.e. the wave is spherical at distant points and the direction of propagation is radial. It will be seen, however, that if the terminal charges had been omitted the wave would apparently have had a component of electric intensity in the direction of propagation, and, moreover, the magnetic field, instead of being equal to the electric field, would have been equal to the tangential component. The formulae show that this anomalous radial component is due to the omission of the terminal charges. In fact, when the terminal charges are included, they contribute, obviously, nothing to the magnetic field, and to the electric field a radial component which exactly cancels the anomalous component, the resultant total field having all the characteristic features of a regular spherical wave in free space.

If the formulae are applied to a closed structure, for example that shown in Figs. 2(*a*) and 2(*b*), the terminal charges are not required to make the system physically

consistent, and there will therefore be a radial component of field for each linear radiator. On the other hand, the essential character of the system will not be changed if the circuit be opened at the ends of each linear element and completed electrically by assuming the appropriate equal and opposite charges to exist on the opposite sides of each gap. The calculated fields at distant points from each such terminated linear radiator will be free from the anomalous radial component. It follows from this that in calculating the distant field for a closed circuit carrying a progressive wave, the resultant of all the radial components at distant points will necessarily be zero, and there is in fact no need to calculate them, except as a means of checking the accuracy of the calculations. This is illustrated in Section (11) for the structure illustrated in Figs. 2(*a*) and 2(*b*).

(7) THE FIELD AT AXIAL POINTS

As stated in Section (6), it is not necessary to calculate the radial components of the electric field at distant points due to any system of progressive-wave radiators forming a closed circuit, except as a means of checking the formulae employed. In making any such calculations, however, it is necessary to note that a linear progressive-wave radiator differs from a standing-wave radiator with respect to the field at axial points.

Referring to (22) and (23) we have, for axial points above the radiator (i.e. $\theta = 0$),

$$r = r_2 + l = r_1 - l \quad . \quad . \quad . \quad . \quad (41)$$

and, on making these substitutions,

$$E_r = \frac{l}{r^2 - l^2} e^{-jmr} 60I \quad . \quad . \quad . \quad . \quad (42)$$

Thus the radial component is very small and vanishes as $1/r^2$. It is obvious on grounds of axial symmetry that the tangential component E_t must be zero. The result of direct substitution is actually indeterminate, but is easily shown to be zero by the usual methods.

For axial points below the radiator, however, i.e. for $\theta = \pi$, a different result is obtained. We have in this case

$$r = r_2 - l = r_1 + l \quad . \quad . \quad . \quad . \quad (43)$$

The tangential component E_t is zero as in the previous case, but the radial component becomes

$$E_r = -30Ie^{-jmr}\left\{\frac{e^{-j2ml}}{r+l} - \frac{e^{j2ml}}{r-l}\right\} \quad . \quad . \quad (44)$$

and if l is small compared with r this becomes

$$E_r = \sin 2ml \cdot \frac{e^{-jmr}}{r} \cdot 60jI \quad . \quad . \quad . \quad (45)$$

Thus there is an appreciable radial component of order $1/r$ from the back end of a linear progressive-wave radiator, a notable point of difference from the standing-wave radiator and one liable to be overlooked. An example of this is given in Section (12), where the field from a rectangular progressive-wave aerial structure is calculated. It is there shown that the field at P (Fig. 2*b*) due to the vertical members BC and FE contains a radial component of order $1/r$. The parts AF and CD of the horizontal members are, however, in a back-end-on position with respect to the point concerned, and it is found that the radial components from these opposite halves of the top and bottom exactly cancel the radial component due to the vertical members, giving a regular resultant field with no radial component.

(8) THE ASYMMETRY OF THE FIELD FROM PROGRESSIVE-WAVE RADIATORS

For distant points r, θ and r, $\pi - \theta$, which are symmetrical about the equatorial plane of a linear progressive-wave radiator, the tangential components of the electric field are respectively, from (31) and (32),

$$E_t(r, \theta) = -\sin\left\{ml(1 - \cos\theta)\right\}\cot\theta/2\frac{e^{-jmr}}{r}60jI \quad . \quad (46)$$

and

$$E_t(r, \pi - \theta) = -\sin\left\{ml(1 + \cos\theta)\right\}\tan\theta/2\frac{e^{-jmr}}{r}60jI \quad (47)$$

Thus the field is not symmetrical about the equatorial plane. A progressive-wave radiator has in fact only one axis of symmetry, i.e. the radiator itself. The difference already noted with respect to the axial points in the forward and backward directions is the extreme case of this asymmetry.

(9) FORMULAE FOR A REVERSED OR DOWNWARD WAVE OR CURRENT

The formulae in Section (4) refer to the system shown in Fig. 1 and to an upward wave of current, i.e. in the direction 1 — 2 represented by

$$i = I_0 e^{j(\omega t + \alpha)} e^{-jmz} \quad . \quad . \quad . \quad . \quad (48)$$

For a current having the same phase and direction at the centre, but with the wave direction downwards, the current distribution would be

$$i' = I_0 e^{j(\omega t + \alpha)} e^{+jmz} \quad . \quad . \quad . \quad . \quad (49)$$

For the first, the components E_r and E_t are, as already shown in Section (4),

$$E_r = \frac{-30I}{r}\left\{\frac{r_2 - l}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + l}{r_1}e^{-jm(r_1-l)}\right\} \quad . \quad (22)$$

$$E_t = \frac{30I}{r}\cot\theta\left\{\frac{r_2 - l + r/\cos\theta}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + l + r/\cos\theta}{r_1}e^{-jm(r_1-l)}\right\} \quad . \quad (23)$$

For the second, the same method of calculation as is outlined in Appendices 1 and 2, or, alternatively, the interchange of r_1 and r_2, substitution of $\theta + \pi$ for θ, and reversal of the sign of I, gives

$$E_r' = \frac{-30I}{r}\left\{\frac{r_2 + l}{r_2}e^{-jm(r_2-l)} - \frac{r_1 - l}{r_1}e^{-jm(r_1+l)}\right\} \quad . \quad (50)$$

and

$$E_t' = \frac{30I\cot\theta}{r}\left\{\frac{r_2 + l - r/\cos\theta}{r_2}e^{-jm(r_2-l)} - \frac{r_1 - l - r/\cos\theta}{r_1}e^{-jm(r_1+l)}\right\} \quad . \quad (51)$$

(10) THE SYNTHESIS OF A STANDING-WAVE RADIATOR

If both the forward and reversed wave [(48) and (49)] are carried simultaneously by the conductor, the total current at the point z is given by

$$i + i' = I_0 e^{j(\omega t+\alpha)}\left\{e^{-jmz} + e^{jmz}\right\} \quad . \quad . \quad (52)$$

$$= 2I_0 e^{j(\omega t+\alpha)} \cos mz \quad . \quad . \quad . \quad . \quad (53)$$

which is the exponential form of expression for a standing wave of maximum amplitude $2I_0$ at the centre and $2I_0 \cos ml$ at the ends. If in addition $\cos ml = 0$, i.e. for example, $2l = \lambda/2$, we have a half-wave dipole with zero current at the ends, i.e. a physically possible system. Thus the fields for a half-wave dipole can be derived by superposition of (22) and (50), and (23) and (51). Thus, for a current distribution on a half-wave dipole (i.e. $2l = \lambda/2$)

$$I_0 e^{j(\omega t+\alpha)} \cos mz \quad . \quad . \quad . \quad . \quad . \quad (54)$$

we have

$$\frac{E_r + E_r'}{2} = 30I \cdot \frac{l}{r} \cdot j\left(\frac{e^{-jmr_2}}{r_2} - \frac{e^{-jmr_1}}{r_1}\right) \quad . \quad (55)$$

and

$$\frac{E_t + E_t'}{2} = -30I \cdot \frac{\cot\theta}{r} \cdot j\left\{l\left(\frac{e^{-jmr_2}}{r_2} - \frac{e^{-jmr_1}}{r_1}\right) + \frac{r}{\cos\theta}\left(\frac{e^{-jmr_2}}{r_2} + \frac{e^{-jmr_1}}{r_1}\right)\right\} \quad . \quad (56)$$

It is easily shown that for distant points, and neglecting terms in $1/r^2$, the radial component vanishes and the tangential component is

$$-\frac{\cos(\pi/2\cos\theta)}{\sin\theta} \cdot \frac{e^{-jmr}}{r} 60I \quad . \quad . \quad . \quad (57)$$

Axial points need separate consideration, as before, and are found to give zero fields to the order $1/r^2$.

Other standing-wave distributions can, of course, be obtained by suitable combinations of the component progressive waves, as illustrated in this example. The process is not merely academic. The aerial system described in Ladner's paper* is one example of its practical application.

(11) APPLICATION OF FORMULAE TO A CLOSED-CIRCUIT RADIATOR

As a simple example of the application of the linear radiator formulae of Section (5) to a closed-circuit structure, consider the field at a distant point P due to the rectangular progressive-wave radiator shown in Fig. 2(*b*).

Let the current at A, the point of excitation, be

$$I = I_0 e^{j(\omega t+\alpha)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

Let I_n be the current at the point O_n. In the formulae of Section (5), the direction of the travel of the wave along the radiator is taken as the positive direction. With this convention, we have

$$\left.\begin{aligned} I_1 &= -I_6 = e^{-jmd/2}I \\ I_2 &= -I_5 = e^{-jm(l+d)}I \\ I_3 &= -I_4 = e^{-jm(2l+3d/2)}I \end{aligned}\right\} \quad . \quad . \quad (59)$$

* See Bibliography, (11).

Let $(E_t)_n$ be the tangential component of E due to the linear element with centre O_n. Then, from Section (8), only the vertical elements will contribute to E_t and P, and, from (32),

$$(E_t)_1 = -\sin ml \frac{e^{-jm(r-d)}}{r-d} e^{-jm(l+d)} 60jI \quad . \quad . \quad (60)$$

and

$$(E_t)_5 = \sin ml \frac{e^{-jm(r+d)}}{r+d} e^{-jm(l+d)} 60jI \quad . \quad . \quad (61)$$

By the simple addition of these terms, and neglecting terms in $1/r^2$ compared with terms in $1/r$,

$$E_t = \frac{\sin ml \sin md}{r} \cdot e^{-jm(l+d)} e^{-jmr} 120I \quad . \quad (62)$$

or, considering amplitudes only,

$$\hat{E}_t = \frac{120 \sin ml \sin md}{r} \hat{I} \quad . \quad . \quad . \quad (63)$$

Consider now the radial components. From (31)

$$(E_r)_2 = \sin ml \frac{e^{-jm(r-d)}}{r-d} e^{-jm(l+d)} 60jI \quad . \quad . \quad (64)$$

$$(E_r)_5 = -\sin ml \frac{e^{-jm(r+d)}}{r+d} e^{-jm(l+d)} 60jI \quad . \quad (65)$$

and, neglecting terms in $1/r_2$,

$$(E_r)_2 + (E_r)_5 = \frac{-\sin ml \sin md}{r} \cdot e^{-jm(l+d)} \cdot e^{-jmr} 120I \quad (66)$$

However, as pointed out in Section (7), the elements AF and CD are in a back-end-on relation to P, and each will contribute a radial component of order $1/r$ to the field at P. From (45)

$$(E_r)_6 = -\sin md \frac{e^{-jm(r+d/2)}}{r+d/2} e^{-jmd/2} 60jI \quad . \quad . \quad (67)$$

and

$$(E_r)_3 = \sin md \frac{e^{-jm(r-d/2)}}{r-d/2} e^{-jm(2l+3d/2)} 60jI \quad . \quad (68)$$

Therefore

$$(E_r)_6 + (E_r)_3 = \frac{\sin md \sin ml}{r} e^{-jm(l+d)} e^{-jmr} 120I \quad . \quad (69)$$

(neglecting terms in $1/r^2$ as before). Thus the radial components from the end-on elements just exactly cancel those from the vertical members, and the resultant field is wholly tangential and in the plane of the wave-front.

It is not intended in the present paper to multiply instances or to consider the complicated question of the actual design of structures for given types of directivity. This subject has already been dealt with in various more comprehensive papers, e.g. that by Foster.* The present paper is concerned more with certain distinctive features of the radiation from progressive-wave systems, and the above elementary example is included only to bring out the rather interesting point about the asymmetry of such radiators, particularly in relation to axial points.

A practical point of some interest, however, is the comparison between similar conductors carrying standing and progressive waves respectively, from the point of

* See Bibliography, (2).

view of radiation resistance. This is considered in the next Section.

(12) THE RADIATION RESISTANCE OF A LINEAR PROGRESSIVE-WAVE RADIATOR

From the formula in (26) for the component of the electric field parallel to the radiator, it is a comparatively simple matter to determine the self-radiation resistance of a linear progressive-wave radiator forming part of a closed circuit by the method of Pistolkors.* The evaluation is given in outline only since the theoretical basis of the method is very lucidly described in Pistolkors's paper.

For the current distribution

$$i = I_0 e^{j(\omega t+\alpha)} e^{-jmz}$$
$$= I e^{-jmz} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

in the radiator shown in Fig. 1, the field-component E_z, parallel to the radiator at the point P is, as shown in Section (4),

$$E_z = 30I\left\{\frac{e^{-jm(r_2+l)}}{r_2} - \frac{e^{-jm(r_1-l)}}{r_1}\right\} \quad . \quad . \quad (71)$$

and if the point P is situated on the radiator itself

$$r_1 + r_2 = 2l \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

and

$$E_z = -30Ie^{-jml}\left\{\frac{e^{jm(r_1-2l)}}{r_1 - 2l} + \frac{e^{-jm(r_1-2l)}}{r_1}\right\} \quad . \quad (73)$$

Putting

$$\left.\begin{aligned} i &= (a_1 + jb_1)I \\ E_z &= (a_2 + jb_2)I \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (74)$$

and

the mean power contribution dW for the element dz is

$$dW = (a_1a_2 + b_1b_2)\frac{I_0^2}{2}dz \quad . \quad . \quad . \quad (75)$$

From (74), in conjunction with (70) and (71),

$$\left.\begin{aligned} a_1 &= \cos mz \\ b_1 &= -\sin mz \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (76)$$

$$a_2 = \frac{1}{2l - r_1}\cos m(r_1 - 3l) - \frac{1}{r_1}\cos m(r_1 - l)$$

$$b_2 = \frac{1}{l - r_1}\sin m(r_1 - 3l) + \frac{1}{r_1}\sin m(r_1 - l) \quad . \quad (77)$$

Therefore

$$W = 30\frac{I_0^2}{2}\int_{-l}^{l}\left\{\frac{\cos m(r_1 - 3l + z)}{2l - r_1} - \frac{\cos m(r_1 - l - z)}{r_1}\right\}dz \quad . \quad (78)$$

Note: The temptation to put $r_1 = z$ must be avoided at this stage, since r_1 is an intrinsically positive quantity and z is not. The more complicated substitution

$$r_1 = |z|$$

is avoided by a shift of origin.

* See Bibliography, (8).

Putting

$$x = z + l$$

and

$$r_1 = x$$

we have

$$W = 30\frac{I_0^2}{2}\int_0^{2l}\left(\frac{\cos 2m(2l - x)}{2l - x} - \frac{1}{x}\right)dx \quad . \quad . \quad (79)$$

This is not integrable as it stands, but putting

$$2l - x = y \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

in the first integrand only we have

$$\int_0^{2l}\frac{\cos 2m(2l - x)}{2l - x}dx = \int_0^{2l}\frac{\cos 2my}{y}dy \quad . \quad . \quad (81)$$

$$= \int_0^{2l}\frac{\cos 2mx}{x}dx \quad . \quad . \quad . \quad . \quad (82)$$

Therefore

$$W = 30\frac{I_0^2}{2}\int_0^{2l}\frac{\cos 2mx - 1}{x}dx \quad . \quad . \quad . \quad . \quad (83)$$

$$= -30\frac{I_0^2}{2}\int_0^{4ml}\frac{1 - \cos u}{u}du \quad . \quad . \quad . \quad . \quad (84)$$

$$= -30\frac{I^2}{2}F(4ml) = -30\frac{I_0^2}{2}F\left(\frac{8\pi l}{\lambda}\right) \quad . \quad . \quad (85)$$

where $\quad F(x) = 0{\cdot}577 + \log_e x - Ci(x) \quad . \quad . \quad (86)$

the numeral $0{\cdot}577$ being Euler's constant.

Defining the radiation resistance R as $|W|/\frac{1}{2}I_0^2$

$$R = 30F\left(\frac{8\pi l}{\lambda}\right)$$

By the same method, it can be shown that the "self" radiation resistance of a length $2l$ of a conductor of unspecified total length having a standing-wave current distribution

$$i = I_0 \sin mx e^{j(\omega t+\alpha)} \quad . \quad . \quad . \quad . \quad (87)$$

x being measured upward from the bottom of the conductor, is given by

$$R = 30\left\{\frac{1}{2}\left(1 + \cos^2\frac{4\pi l}{\lambda}\right)F\left(\frac{8\pi l}{\lambda}\right) - \frac{1}{4}\sin\frac{8\pi l}{\lambda}Si\left(\frac{8\pi l}{\lambda}\right)\right\} \quad . \quad . \quad (88)$$

In this case $2l$ can only be the total length of the conductor if it is so related to λ as to give zero current at the top open end, i.e. if $2l$ is a whole number of half wavelengths. For all such values of $2l$

$$\frac{1}{2}\left(1 + \cos^2\frac{4\pi l}{\lambda}\right) = 1$$

and

$$\sin\frac{8\pi l}{\lambda} = 0$$

and

$$R = 30F\left(\frac{8\pi l}{\lambda}\right)$$

R being in this case the total radiation resistance, since it refers to the whole length of the conductor.

Thus we have the interesting result that the radiation resistance of a straight conductor of length $2l$ in free space is the same whether the current distribution be a standing wave of a given maximum amplitude or a progressive wave of the same amplitude, the length $2l$ being consistent with a standing wave distribution, i.e. a whole number of half wavelengths. For example, if $2l$ is half a wavelength, we have

$$\begin{aligned} \text{Euler's constant} &= 0\cdot577 \\ \log_e 2\pi &= 1\cdot835 \\ -C_i(2\pi) &= 0\cdot028 \\ \hline & \ 2\cdot440 \end{aligned}$$

and $$R = 30 \times 2\cdot440 = 73\cdot2$$

(13) ACKNOWLEDGMENTS

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APPENDIX 1

The integral

$$F = \int_{-l}^{l} \frac{e^{-jm(r+z)}}{r} dz \quad . \quad . \quad . \quad . \quad (89)$$

where $$r^2 = a^2 + (b - z)^2 \quad . \quad . \quad . \quad . \quad (90)$$

figures so frequently in calculations relating to aerials that it is worth while to include here an outline of its evaluation.

Referring to Fig. 1,

$$r = a/\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

$$b - z = a\cot\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

Therefore $$r + z = b + a\tan\tfrac{1}{2}\theta \quad . \quad . \quad . \quad . \quad (93)$$

also $$dz = r d\theta/\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

and $$F = e^{-jmb}\int_{\theta 1}^{\theta 2} \frac{e^{-jma\tan\frac{1}{2}\theta}}{\sin\theta} d\theta \quad . \quad . \quad (95)$$

and putting $$-ma\tan\tfrac{1}{2}\theta = u \quad . \quad . \quad . \quad . \quad (96)$$

$$d\theta/\sin\theta = du/u \quad . \quad . \quad . \quad . \quad . \quad (97)$$

Therefore $$F = e^{-jmb}\int_{u_1}^{u_2} \frac{e^{ju}}{u} du \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

$$= e^{-jmb}\left\{Ei(u_2) - Ei(u_1)\right\} \quad . \quad . \quad . \quad (99)$$

where $$Ei(x) = Ci(x) + jSi(x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

$$u_2 = -ma\tan\tfrac{1}{2}\theta_2 = -m(r_2 - b + l) \quad . \quad (101)$$

and $$u_1 = -ma\tan\tfrac{1}{2}\theta_1 = -m(r_1 - b - l) \quad . \quad (102)$$

APPENDIX 2

It was shown in Section (2) that the scalar potential ψ and the time differential of the vector potential are both expressible in terms of the integral F, i.e.

$$\psi = 30I \,.\, F \quad . \quad . \quad . \quad . \quad (103)$$

and $$\frac{1}{c}\frac{d\mathbf{A}}{dt} = jm30\mathbf{I} \,.\, F \quad . \quad . \quad . \quad . \quad (104)$$

where $\mathbf{I}$ is a vector of magnitude $I = I_0 e^{j(\omega t + \alpha)}$

Since $$E = -\operatorname{grad}\psi - \frac{1}{c}\frac{d\mathbf{A}}{dt} \quad . \quad . \quad . \quad (105)$$

we shall require the differential of F with respect to the co-ordinates of P.

Let η be any co-ordinate of P. Then it can be shown that

$$\frac{\partial F}{\partial \eta} = -jm\frac{\partial b}{\partial \eta}\cdot F + e^{-jmb}\left\{\frac{\partial u_2}{\partial \eta}\cdot\frac{e^{ju_2}}{u_2} - \frac{\partial u_1}{\partial \eta}\frac{e^{ju_1}}{u_1}\right\} \quad (106)$$

For example, suppose we wish to evaluate E_t, the tangential component of E. In this case the relevant co-ordinates of P are r and θ (the r_0 and θ_0 of Fig. 1, but with the suffix omitted for convenience, since no ambiguity is involved).

The component of $\frac{1}{c}\frac{d\mathbf{A}}{dt}$ is, from (104),

$$-\left(\frac{1}{c}\frac{d\mathbf{A}}{dt}\right)_t = -jm\,.\,30F\,.\,I\sin\theta \quad . \quad (107)$$

and since the differential elements are dr and $-rd\theta$, the component of $-\text{grad}\,\psi$ is given by

$$-(\text{grad}\,\psi)_t = -30I(\text{grad}\,F)_t = -30I\left(-\frac{1}{r}\frac{\partial F}{\partial \theta}\right) \quad (108)$$

$$= 30I\cdot\frac{1}{r}\left\{-jmF\frac{\partial b}{\partial \theta} + e^{-jmb}\frac{e^{ju_2}}{u_2}\frac{\partial u_2}{\partial \theta} - \frac{e^{ju_1}}{u_1}\frac{\partial u_1}{\partial \theta}\right\} \quad (109)$$

Since $\qquad b = r\cos\theta$

$$\frac{\partial b}{\partial \theta} = -r\sin\theta \quad . \quad . \quad . \quad . \quad (110)$$

and the first term of (109) merely cancels the contribution from the vector potential [see (107)].

Since $\qquad u_2 = -m(r_2 - b + l) \quad . \quad . \quad . \quad (111)$

and $\qquad r_2^2 = r^2 + l^2 - 2rl\cos\theta$
$\qquad\qquad = r^2 + l^2 - 2lb \quad . \quad . \quad . \quad . \quad (112)$

it can be shown that

$$\frac{1}{u_2}\frac{\partial u_2}{\partial \theta} = \frac{r\sin\theta}{r_2}\,\frac{r_2 + l}{r_2 + b + l} \quad . \quad . \quad (113)$$

and by using the relationship

$$a^2 = r^2\sin^2\theta = (r_2 - b + l)(r_2 + b - l) \quad . \quad (114)$$

$$\frac{1}{u_2}\frac{\partial u_2}{\partial \theta} = \frac{1}{r_2}\,\frac{r + (r_2 - l)\cos\theta}{\sin\theta}$$

Similarly $\qquad \dfrac{1}{u_1}\dfrac{\partial u_1}{\partial \theta} = \dfrac{1}{r_1}\,\dfrac{r + (r_1 + l)\cos\theta}{\sin\theta} \quad . \quad . \quad (115)$

Therefore, inserting these in (109) and multiplying through by e^{-jmb}

$$E_t = \frac{30I\cot\theta}{r}\left\{\frac{r_2 - l + r/\cos\theta}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + l + r/\cos\theta}{r_1}e^{-jm(r_1-l)}\right\} \quad . \quad (116)$$

The similar evaluation of E_r involves $\partial F/\partial r$. Since

$$\frac{\partial b}{\partial r} = \cos\theta \quad . \quad . \quad . \quad . \quad . \quad (117)$$

the contribution from the vector potential is cancelled out by part of that from the scalar potential as before, and further it can be shown that

$$\frac{1}{u_2}\frac{\partial u_2}{\partial r} = \frac{1}{r}\cdot\frac{r_2 - l}{r_2}$$

and $\qquad \dfrac{1}{u_1}\dfrac{\partial u_1}{\partial r} = \dfrac{1}{r}\cdot\dfrac{r_1 + l}{r_1} \quad . \quad . \quad . \quad (118)$

whence

$$E_r = \frac{-30I}{r}\left\{\frac{r_2 - l}{r_2}\cdot e^{-jm(r_2+l)} - \frac{r_1 + l}{r_1}e^{-jm(r_1-l)}\right\} \quad (119)$$

The determination of the parallel and perpendicular components E_z and E_x follows similar lines and need not be given in detail.

APPENDIX 3

Referring to Fig. 1, suppose the current at the ends of the radiator to give rise to terminal charges q_1 and q_2,

where
$$\left.\begin{aligned} q_1 &= -\int i_1 dt \\ q_2 &= \int i_2 dt \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

Since $\qquad i = I_0 e^{j(\omega t + d)}e^{-jmz} \quad . \quad . \quad . \quad . \quad (121)$

$$i_1 = I_0 e^{j(\omega t+d)}e^{jml} \quad . \quad . \quad . \quad . \quad (122)$$

$$q_1 = -\frac{1}{j\omega}I_0 e^{j(\omega t+d)}e^{jml} \quad . \quad . \quad . \quad (123)$$

and $\qquad [q_1] = -\dfrac{1}{j\omega}I_0 e^{j(\omega t+d)}e^{-jm(r_1-l)}$

$$= -\frac{1}{j\omega}Ie^{-jm(r_1-l)} \quad . \quad . \quad . \quad . \quad (124)$$

Similarly $\qquad [q_2] = \dfrac{1}{j\omega}Ie^{-jm(r_2+l)} \quad . \quad . \quad . \quad . \quad (125)$

The field $\mathbf{E}_q$ at P due to these hypothetical charges is given by

$$\mathbf{E}_q = -\text{grad}\,\psi_1 - \text{grad}\,\psi_2 \quad . \quad . \quad (126)$$

where, converting to practical units,

$$\left.\begin{aligned} \psi_1 &= -30I\frac{1}{jm}\,\frac{e^{-jm(r_1-l)}}{r_1} \\ \text{and}\qquad \psi_2 &= 30I\frac{1}{jm}\,\frac{e^{-jm(r_2+l)}}{r_2} \end{aligned}\right\} \quad . \quad . \quad (127)$$

Thus, for the radial component $(E_q)_r$ of the field at P due to these charges

$$(E_q)_r = -\frac{\partial\psi_1}{\partial r} - \frac{\partial\psi_2}{\partial r} \quad . \quad . \quad . \quad (128)$$

and since $\qquad r_2^2 = r^2 + l^2 - 2rl\cos\theta$

and $\qquad r_1^2 = r^2 + l^2 + 2rl\cos\theta \quad . \quad . \quad . \quad (129)$

$$\left.\begin{aligned}\frac{\partial r_2}{\partial r} &= \frac{r - l\cos\theta}{r_2} \\ \text{and} \qquad \frac{\partial r_1}{\partial r} &= \frac{r + l\cos\theta}{r_1}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (130)$$

Therefore

$$\left.\begin{aligned}-\frac{\partial\psi_1}{\partial r} &= -\,30I\frac{r + l\cos\theta}{r_1}\cdot\frac{e^{-jm(r_1-l)}}{r_1}\cdot\frac{(jm+1/r_1)}{jm} \\ \text{and} \qquad -\frac{\partial\psi_2}{\partial r} &= \quad 30I\frac{r - l\cos\theta}{r_2}\cdot\frac{e^{-jm(r_2+l)}}{r_2}\cdot\frac{(jm+1/r_2)}{jm}\end{aligned}\right\} \quad (131)$$

and
$$\begin{aligned}(E_q)_r &= -\left(\frac{\partial\psi_1}{\partial r} + \frac{\partial\psi_2}{\partial r}\right) \\ &= 30I\left\{\left(1 - \frac{j}{mr_2}\right)\left(\frac{r - l\cos\theta}{r_2}\right)\frac{e^{-jm(r_2+l)}}{r_2} - \left(1 - \frac{j}{mr_1}\right)\left(\frac{r + l\cos\theta}{r_1}\right)\frac{e^{-jm(r_1-l)}}{r_1}\right\} \quad (132)\end{aligned}$$

A similar evaluation of $\frac{1}{r}\frac{\partial\psi}{\partial\theta}$ leads to

$$(E_q)_t = -30\ Il\sin\theta\left\{\frac{e^{-jm(r_2+l)}}{r_2^2} - \frac{e^{-jm(r_1-l)}}{r_1^2}\right\} \quad (133)$$

A NOTE ON SELF-INDUCTION*

By Professor ALFRED O'RAHILLY, M.A., B.Sc., Ph.D., D.Litt.†

(*Paper first received* 28*th January, and in revised form* 22*nd May,* 1939.)

SUMMARY

Apart from any complications which may be involved in particular applications, the subject of self-induction and magnetic flux is elementary; and nothing very new can be said about it. But the logical sequence of the argument is not always appreciated, nor is it generally realized that the usual treatment involves the rather artificial reduction of the current in the wire-circuit to two current-filaments. Moreover, there have been serious misunderstandings concerning the exact meaning of "rate of change of flux." A clear exposition of the argument, illustrated by reference to a rectangular circuit, is here attempted.

(1) THE DEFINITION OF L

Self-induction is merely the mutual induction of the various current-filaments into which the total current traversing a wire may be taken as divided. Using electromagnetic–magnetic units, we take each filament as a current $i = udS$, where dS is the infinitesimal cross-section and u is the current density, which—except in the case of very rapidly alternating currents—we can take to be the same for each of the filaments composing the cross-section. That is, $u = I/S$, where I is the total current and S the cross-section of the wire.

Ampere's law of equivalence was deduced from experiments on the mutual actions of currents in *different* circuits (or circuits and magnets), for which purpose we can regard each of the (total) currents as a filament. But in order to deal with the phenomenon of self-induction we must apply the law to each of the filamentary circuits in a given wire-circuit. Now it is a well-known result that the flux (Φ) through a magnetic shell (contour s) due to another shell (strength i', contour s') is $\Phi \equiv mi'$, where

$$m = \iint l^{-1} ds ds' \cos\gamma \quad . \quad . \quad . \quad . \quad (1)$$

Here γ is the angle between the elements ds and ds' at the interdistance l, and the integrals are taken along the two contours or filaments. The mutual potential energy of the two shells is

$$w = -i\Phi = -mii'$$

That is, the work done in a small displacement is $-dw$. The mathematically convenient idea of a "shell" here disappears and we have the physically more direct formula connecting the potential energy (p.e.) with elements of the current-filaments.

To find the mutual p.e. of a number of current-filaments, let us use the notation

$$q_{12} \equiv l_{12}^{-1} \cos\gamma_{12} \equiv q_{21}$$

We have

$$\begin{aligned}
&\Sigma i_1 i_2 q_{12} ds_1 ds_2 \\
&= i_1 ds_1(i_2 q_{12} ds_2 + i_3 q_{13} ds_3 + \ldots + i_n q_{1n} ds_n) \\
&\quad + i_2 ds_2(i_3 q_{23} ds_3 + \ldots + i_n q_{2n} ds_n) \\
&\quad + \ldots \\
&\quad + i_{n-1} ds_{n-1}(i_n q_{n-1} ds_n) \\
&= \tfrac{1}{2} i_1 ds_1(i_2 q_{12} ds_2 + \ldots + i_n q_{1n} ds_n) \\
&\quad + \tfrac{1}{2} i_2 ds_2(i_1 q_{21} ds_1 + \ldots + i_n q_{2n} ds_n) \\
&\quad + \ldots \\
&\quad + \tfrac{1}{2} i_{n-1} ds_{n-1}(i_1 q_{n-1,1} ds_1 + \ldots + i_n q_{n-1,n} ds_n).
\end{aligned}$$

Hence the mutual p.e. of the shells or filaments is

$$\begin{aligned} W &= -\Sigma i_1 i_2 \iint q_{12} ds_1 ds_2 \\ &= -\tfrac{1}{2}\Sigma i_1 \Phi_1 \end{aligned}$$

where

$$\Phi_1 = i_2 m_{12} + i_3 m_{13} + \ldots + i_n m_{1n}$$

is the flux through shell No. 1 due to all the other shells. This explains the occurrence of the factor $\frac{1}{2}$, which is sometimes puzzling to students. We may express the result as follows:—

$$W = -\tfrac{1}{2}\Sigma\Phi i = -\tfrac{1}{2}\Sigma\Sigma mii' \quad . \quad . \quad . \quad (2)$$

where m is defined by (1). Here W is the thermodynamic potential or the free energy, which is such that the work generated in a virtual displacement (at constant temperature and current-intensity) is $-\delta W$. It is usual to employ minus this quantity ($W_e \equiv -W$) and to call W_e "the electrokinetic energy." Hence (2) is written

$$W_e = \tfrac{1}{2}\Sigma i\Phi = \tfrac{1}{2}\Sigma\Sigma mii' \quad . \quad . \quad . \quad (3)$$

Since $i = IdS/S$ and $i' = IdS'/S$, this is equivalent to

$$W_e = \tfrac{1}{2}LI^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

$$L \equiv \iint mdS\frac{dS'}{S^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

the summations over the filaments being replaced by integrals over the cross-sections. This is the fundamental formula for the self-induction of a circuit of constant cross-section.

(2) L FOR A RECTANGULAR CIRCUIT

Though the result is well known, it is worth while to reproduce the calculation of L for a rectangular circuit,

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in order to elucidate the steps of the argument. Let the lengths of the sides be a and b and the cross-section circular of radius r, where r/a and r/b are assumed to be small fractions. Fig. 1 shows two of the filamentary circuits, separated by a distance d which for clearness is exaggerated in the Figure. From (3) we have, for the sides AB and A′B′ (with $\cos\gamma = 1$),

$$
\begin{aligned}
M_1 &= \int_0^a dx \int_0^a dx' \left[d^2 + (x - x')^2\right]^{-\frac{1}{2}} \\
&= 2\left\{d - \sqrt{(d^2 + a^2)} + a \log\left[a + \sqrt{(d^2 + a^2)}\right] - a \log d\right\} \quad . \quad . \quad (5a)
\end{aligned}
$$

The expression for M_1, we may observe, coincides with that for the mutual potential of two rods uniformly charged to unit density. Since d/a is assumed to be small, we have, very approximately,

$$M_1 = 2a\left(\log \frac{2a}{d} - 1\right)$$

In accordance with (5), this results in the following

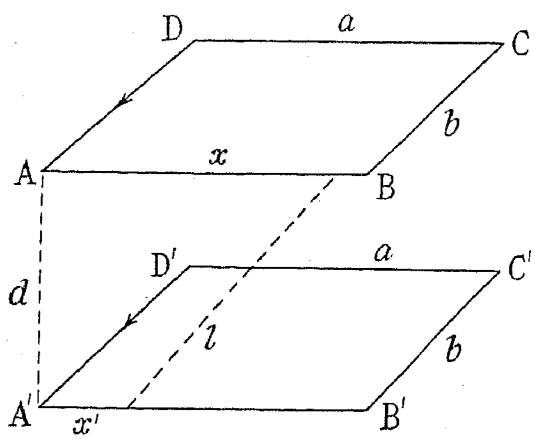


Fig. 1

contribution to the coefficient of self-induction, when we sum up for all such pairs of filaments along these two sides of the circuit:—

$$
\begin{aligned}
L_1 &= \iint M_1 dS \frac{dS'}{S^2} \\
&= 2a(\log 2a - 1) - \frac{2a}{S^2}\iint dS dS' \, \log d
\end{aligned}
$$

Let δ be what is called the "geometric mean distance" of points in the cross-section. That is,

$$\log \delta \equiv \iint \frac{dS dS' \log d}{S^2}$$

Then
$$L_1 = 2a\left(\log \frac{2a}{\delta} - 1\right) \quad . \quad . \quad . \quad . \quad (6)$$

That is, this portion of the coefficient is the same as the coefficient of mutual induction of two filaments δ apart. In the case of a circular cross-section it is a well-known mathematical result that $\delta = re^{-\frac{1}{4}}$, so that

$$L_1 = 2a\left(\log \frac{2a}{r} - \frac{3}{4}\right) \quad . \quad . \quad . \quad . \quad (7)$$

Next consider the filaments AB and C′D′ (Fig. 1). We obtain M_2 from ($5a$) by changing the sign (since the currents are now in opposite directions) and by substituting $(d^2 + b^2)$ for d^2. And, since d^2/b^2 and d^2/a^2 are negligible, we can write

$$M_2 = -\left\{b - \sqrt{(a^2 + b^2)} + a \log\left[a + \sqrt{(a^2 + b^2)}\right] - a \log b\right\} \quad . \quad . \quad . \quad (8)$$

Since M_2 is independent of d, equation (5) gives

$$L_2 = M_2$$

Similarly L_3 for BC, B′C′ and L_4 for BC, D′A′ are obtained from L_1 and L_2 by interchanging a and b. Adding these, and doubling for the whole circuit, we obtain*

$$
\begin{aligned}
L = 8&\left[\sqrt{(a^2 + b^2)} - (a + b)\right] \\
&+ 4a\left\{\log 2 + \log(b/\delta) - \log\left[a + \sqrt{(a^2 + b^2)}\right] + \log a\right\} \\
&+ 4b\left\{\log 2 + \log(a/\delta) - \log\left[b + \sqrt{(a^2 + b^2)}\right] + \log b\right\} \quad . \quad . \quad . \quad (9)
\end{aligned}
$$

Putting $a + b = c$ (semi-perimeter), $\sqrt{(a^2 + b^2)} = d$ (diagonal), $\delta = re^{-\frac{1}{4}}$, and changing to the base 10 for the logarithms, we see that (9) becomes Mascart's formula, namely

$$
\begin{aligned}
L = 8d - 7c + 9{\cdot}210&\left[c \log_{10}(2ab/r)\right. \\
&\left. - a \log_{10}(a + d) - b \log_{10}(b + d)\right]
\end{aligned}
$$

If in (9) we put $a/b \equiv p$, we have

$$
\begin{aligned}
\frac{L}{4b} = 2&\left[\sqrt{(1 + p^2)} - 1\right] \\
&+ p\left\{\log(2p) - \log\left[p + \sqrt{(1 + p^2)}\right] + \log(b/\delta)\right\} \\
&+ \log 2 - \log\left[1 + \sqrt{(1 + p^2)}\right] + \log(a/\delta) \quad . \quad . \quad . \quad . \quad (10)
\end{aligned}
$$

For a very long rectangle $p \to 0$, and

$$
\begin{aligned}
L &= 4b \log \frac{a}{\delta} \\
&= 4b\left(\log \frac{a}{r} + \frac{1}{4}\right) \quad . \quad . \quad . \quad (11)
\end{aligned}
$$

For a square, $p = 1$, and

$$\frac{L}{8a} = \log \frac{a}{\delta} + \log 2 - \log(1 + \sqrt{2}) + \sqrt{2} - 1 \quad . \quad (12)$$

The only reason for repeating this well-known calculation is to emphasize that, in this case as in others, the self-induction of the circuit is expressed as the mutual induction of two filaments which approximately coincide with the wire-circuit and are separated (inside the wire) by the distance δ, which is small relatively to a or b. The argument is seen to be general, provided the circuit is polygonal or has a large radius of curvature. For example, in the case of a circular wire (coil-radius a, cross-section radius r, with r^2/a^2 negligible), we find

$$M = 4\pi a\left(\log \frac{8a}{\delta} - 2\right)$$

* If we took the two rectangular filaments to be in the same plane, one at a distance δ inside the other, we should obtain $\log\left(\frac{a}{\delta} - 1\right)$ and $\log\left(\frac{b}{\delta} - 1\right)$ instead of $\log(a/\delta)$ and $\log(b/\delta)$. Since, *ex hypothesi*, these quantities (a/δ and b/δ) are very large, the expressions are practically identical.

Hence

$$L = 4\pi a\left(\log\frac{8a}{r} - \frac{7}{4}\right)$$

We can now simplify the general formula (5) and express the self-induction of a wire-circuit, in accordance with (1), as

$$L = \iint \frac{(\mathbf{ds}\mathbf{ds}')}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

the bracket denoting the scalar product. Here the integrals are taken over *two* filamentary circuits within the wire, separated by the geometric mean distance δ, which, for a circular cross-section, is $re^{-\frac{1}{4}}$. And the energy given by (4) is now seen to be *half* that due to a current I in *each* of the two filamentary circuits. The highly artificial—though mathematically convenient—nature of this substitution is not generally adverted to.

(3) MAGNETIC FLUX

There is no difficulty in defining the flux through a filamentary circuit; it is the surface-integral of the external field through any surface bounded by the circuit. But, when once we realize that an ordinary current has to be regarded as composed of a number of filamentary currents, it is not so easy to define the self-flux of the circuit. If Φ is the flux through a filament (cross-section dS), due to all the other filaments composing the current, " the flux through the wire-circuit " is *defined* to be the mean magnetic flux

$$\Phi_m \equiv \frac{1}{2}\int \Phi \frac{dS}{S}$$

The factor $\frac{1}{2}$ here occurs for the same reason as it appears in formula (2), and this could be explicitly shown by a similar argument. Since $dS/S = i/I$, we have, from (3) and (4),

$$\Phi_m = \frac{1}{2}\sum \frac{\Phi i}{I} = LI \quad . \quad . \quad . \quad . \quad (14)$$

This justifies the common definition of self-induction as the linkage per unit current. But it implies the prior definition of what is not at all self-evident, namely " linkage," i.e. the magnetic flux not through a surface bounded by a single filament but " through the wire-circuit," which includes innumerable filaments.

If, however, we first accept the reduction of the ordinary current to two representative filaments (separated by the geometric mean distance) each carrying the current I, we can start by defining the flux as $\Phi_m = LI$. That is, from (13),

$$\Phi = I\int_s\int_{s'}\frac{(\mathbf{ds}\mathbf{ds}')}{l}$$

$$= \int_{s'}(\mathbf{A}\mathbf{ds}')$$

where the vector-potential $\mathbf{A}$ is defined by

$$\mathbf{A} = I\int_s \frac{\mathbf{ds}}{l}$$

By an application of Stokes's theorem, this gives

$$\Phi = \int_{S'}(\mathbf{H}\mathbf{dS}') \quad . \quad . \quad . \quad . \quad (15)$$

where $\mathbf{H} = \text{curl }\mathbf{A}$ is the magnetic intensity due to the circuit s. Thus we can express the flux as a surface-integral of magnetic force. But observe that $\mathbf{dS}'$ is the vector-element of area of a surface bounded by the other circuit s' (and therefore has no connection with our previous dS', which referred to the cross-section of the wire). Hence, while the magnetic intensity or force is taken as due to one representative filament (current I in s), the area over which the flux or surface-integral is taken is bounded not by this filament but by the other (s').

Let us apply formula (15) to find the flux (Φ_m) through

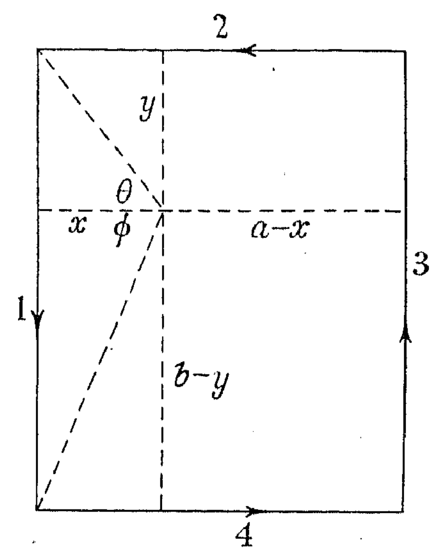


Fig. 2

the rectangular wire-circuit already considered. Using the Biot–Savart formula

$$dH = \frac{Ids\sin\theta}{l^2}$$

we easily find that the magnetic force at P due to side No. 1 (Fig. 2) is up through the paper and equal to

$$H_1 = I\frac{\sin\theta + \sin\phi}{x}$$

$$= \frac{I}{x}\left\{\frac{y}{\sqrt{(x^2+y^2)}} + \frac{b-y}{\sqrt{[x^2+(b-y)^2]}}\right\}$$

Similarly, for the other sides,

$$H_2 = \frac{I}{y}\left\{\frac{x}{\sqrt{(x^2+y^2)}} + \frac{a-x}{\sqrt{[(a-x)^2+y^2]}}\right\}$$

$$H_3 = \frac{I}{a-x}\left\{\frac{y}{\sqrt{[(a-x)^2+y^2]}} + \frac{b-y}{\sqrt{[(a-x)^2+(b-y)^2]}}\right\}$$

$$H_4 = \frac{I}{b-y}\left\{\frac{x}{\sqrt{[x^2+(b-y)^2]}} + \frac{a-x}{\sqrt{[(a-x)^2+(b-y)^2]}}\right\}$$

The total magnetic force at P is

$$H = H_1 + H_2 + H_3 + H_4 \quad . \quad . \quad (16)$$

The flux is

$$\Phi_m = \int H dS'$$

$$= \int_\delta^{a-\delta} dx \int_\delta^{b-\delta} dy H \quad . \quad . \quad . \quad . \quad (17)$$

On effecting the integrations, we find $\Phi_m = LI$, where L is given by (9). We have thus verified the identity of the two alternative methods for calculating the flux.

For the case of two long leads (a/b very small), the calculation is quite brief. For then we can take the magnetic intensity at P due to the left-hand wire to be

$$H = \frac{I}{x}(1 + \sin\theta)$$

The flux through the strip bdx is

$$d\Phi_m = \frac{Idx}{x}\int_0^b \left[1 + \frac{y}{\sqrt{(x^2 + y^2)}}\right]dy$$

$$= \frac{Idx}{x}\left[y + \sqrt{(x^2 + y^2)}\right]_0^b$$

$$\rightarrow 2Ib\frac{dx}{x}$$

Doubling this, to take account of both wires, we have

$$\frac{\Phi_m}{I} = 4b\int_\delta^{a-\delta}\frac{dx}{x}$$

$$= 4b \log\left(\frac{a}{\delta} - 1\right)$$

$$\rightarrow 4b \log\frac{a}{\delta}$$

which is formula (11).

(4) LOCALIZED ENERGY

We have seen that the so-called electrokinetic (or magnetic) energy can be expressed as

$$W_e = \tfrac{1}{2}\Sigma_1\Phi_1 i_1$$

where Φ_1 is the flux through filament No. 1 due to all the other filamentary currents. Since we are assuming a uniform current-distribution, we can put

$$i\mathbf{ds} = \mathbf{u}d\tau$$

where $d\tau$ is the element of volume and $ids = u$. Hence

$$\Phi_1 i_1 = i_1\int(\mathbf{A}_1\mathbf{ds}_1) = \int(\mathbf{A}_1\mathbf{u}_1)d\tau,$$

or

$$W_e = \tfrac{1}{2}\Sigma_1\int(\mathbf{A}_1\mathbf{u}_1)d\tau$$

In the limit, for filaments with infinitely small cross-sections, the contribution to the vector-potential which each filament generates of itself vanishes. So we can use $\mathbf{A}$ the *complete* vector-potential (not merely $\mathbf{A}_1$ due to the other filaments). That is, we can take

$$W_e = \tfrac{1}{2}\int_\infty(\mathbf{Au})d\tau$$

the summation being included in the integral, which we can now take over all space since $\mathbf{u} = 0$ outside the circuit. In the case of stationary volume-currents,

$$\text{curl } \mathbf{H} = 4\pi\mathbf{u}$$

Hence, using well-known vector-formulae,

$$\begin{aligned} 4\pi(\mathbf{Au}) &= (\mathbf{A} \text{ curl } \mathbf{H}) \\ &= (\mathbf{H} \text{ curl } \mathbf{A}) + \text{div } V\mathbf{HA} \\ &= H^2 + \text{div } V\mathbf{HA} \end{aligned}$$

When we integrate over all space, the " div " term gives (by Green's theorem) a surface-integral which vanishes. Hence we obtain the usual expression

$$W_e = \int_\infty H^2\frac{d\tau}{8\pi}$$

This result is, of course, well known. We have merely shown how, in the case of self-induction, we are mathematically justified in treating the energy of a current-carrying wire-circuit as continuously distributed through space.

This expression for the energy provides a simple proof of the formula $\delta = re^{-\frac{1}{4}}$. Consider the rectangular circuit. We can take the energy *outside* the wire as $\frac{1}{2}I\Phi_m$, where Φ_m is the flux due to an axial filamentary current I. That is,

$$\Phi_m = \int_r^{a-r} dx \int_r^{b-r} dy H$$

Comparing it with (17), we see that this gives $\Phi_m = L'I$, where L' is given by (9) with r substituted for δ. To this energy $\frac{1}{2}L'I^2$ we must now add the energy *inside* the wire, which we can take to be the same as if the elements in the opposite sides of the rectangle were infinitely far apart, so that at an axial distance x inside the wire $H = 2Ix/r^2$. (This expresses our assumption of uniform current-distribution over a cross-section.) Hence, for the two wires of length a, the energy is

$$2\int H^2\frac{d\tau}{8\pi} = \frac{I^2a}{\pi r^4}\int_0^r x^2 2Mx dx = \tfrac{1}{2}I^2 a$$

Therefore the energy inside the wires is $\frac{1}{2}L''I^2$, where $L'' = a + b$. Hence the total coefficient of self-induction is

$$L = L' + a + b$$

Here L is given by (9) and L' is the same expression with r substituted for δ. That is,

$$-4(a+b)\log\delta = -4(a+b)\log r + a + b,$$

or $\delta = re^{-\frac{1}{4}}$. It is also obvious from our proof that the formula

$$L = L' + \tfrac{1}{2}l_p$$

where l_p is the length of the perimeter, holds for any wire, circular in cross-section, which is polygonal or has a large radius of curvature; provided, of course, that our general assumption of uniform distribution is valid.

(5) THE RATE OF CHANGE OF FLUX

With a slight change of notation, formula (15) for the flux is

$$\Phi_m = \int H_n dS$$

where H_n is the component of H normal to the element of surface dS (formerly called dS'). Consider the variation of this integral in time dt. We first have the variation due to a change in the field (H) alone, keeping to the same surface S. This is given by

$$\partial\Phi_m = \int \partial H_n dS$$

We must further calculate the change $\delta\Phi_m$ due to the motion of S (velocity $\mathbf{v}$ at any point). Let S' be the position of the surface at time $t' = t + dt$. Close up the two open surfaces S and S' by adding the small strip of

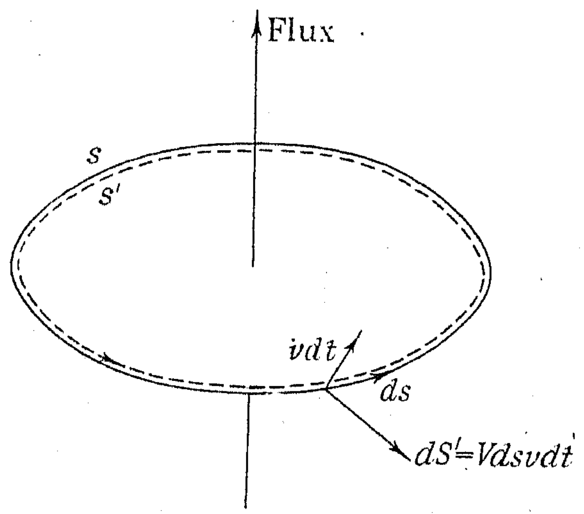


Fig. 3

surface described by the bounding curve, each element of which traces the vector-element* of area $\mathbf{dS}' = dt V\mathbf{ds}'\mathbf{v}$ (as in Fig. 3). Applying Green's theorem to this whole surface and the included volume,

$$\int_{S'} H_n dS - \int_S H_n dS + \int H_n dS' = \int d\tau \text{ div } \mathbf{H}$$

But div $\mathbf{H} = 0$. Hence

$$\delta\Phi_m \equiv \int_{S'} H_n dS - \int_S H_n dS$$
$$= - \int H_n dS'$$
$$= - \delta\Phi'_m$$

where $\delta\Phi'_m$ is the flux cut through by the bounding curve. We therefore have the total rate of change:—

$$\frac{d\Phi_m}{dt} = \frac{\partial\Phi_m}{\partial t} + \frac{\delta\Phi_m}{\delta t} \quad . \quad . \quad . \quad . \quad (18)$$

* Hamilton's notation. $V\mathbf{ab}$ is used to denote the vector product.

where, since $\mathbf{ds}'/dt = V\mathbf{dsv}$,

$$\frac{\delta\Phi_m}{\delta t} = - \frac{\delta\Phi'_m}{\delta t}$$
$$= - \int (\mathbf{H} V\mathbf{dsv})$$
$$= - \int (\mathbf{ds} V\mathbf{vH}) \quad . \quad . \quad . \quad . \quad (19)$$

taken over the closed contour.

The difficulty is to apply these formulae to the case of *self*-induction, once we realize that this latter is due to the mutual induction of filamentary currents. For while $d\Phi_m$ has a clear meaning, its decomposition into $\partial\Phi_m$ and $\delta\Phi_m$ becomes artificial. If we replace the wire-circuit by two representative filaments (s and s') at a distance δ —which is what we ordinarily do when we calculate L— this decomposition means that we break up the motion into two parts: (i) s' changes into the new position, s remaining the same; (ii) then s changes while s' remains fixed. Suppose the representative filaments are originally in the position which we shall designate as $1 + 2$ (Fig 4),

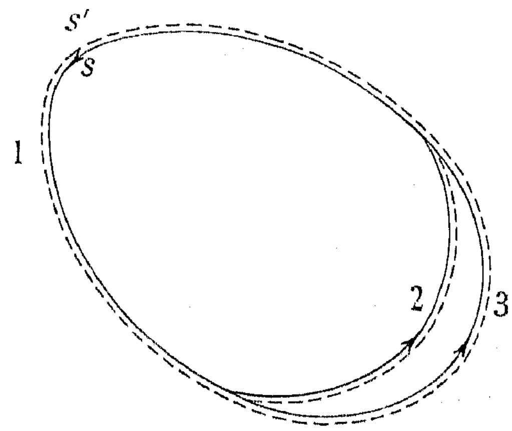


Fig. 4

while at the end of time dt they are in the position $1 + 3$, i.e. the segment 2 of the circuit has become displaced into 3. Adopt the notation

$$L \equiv \oint\oint \frac{(\mathbf{dsds}')}{l}$$
$$\equiv (1 + 2, 1 + 2)$$

meaning that the integrals are taken over s and s' in the original positions denoted by $1 + 2$.

(*a*) In calculating $\partial\Phi_m$: the "inducing" circuit s' changes from $1 + 2$ to $1 + 3$, while s remains $1 + 2$. Hence

$$\frac{\partial\Phi_m}{I} = \partial L = (1 + 2, 1 + 3) - (1 + 2, 1 + 2)$$
$$= (1 + 2, 3 - 2).$$

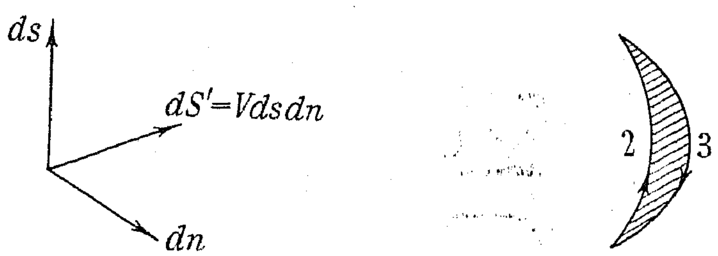
The "-2" implies that $\mathbf{ds}'$ along 2 is taken in the direction opposite to that of the arrow marked in the Figure. Another way of putting this is to say that $(\Phi_m + \partial\Phi_m)$ is the flux of H, due to I around s' in the position $1 + 3$, over an area bounded by s in the position $1 + 2$.

(*b*) In calculating $\delta\Phi_m$: the circuit s' remains* in the position $1 + 2$, while the "induced" circuit s changes from $1 + 2$ to $1 + 3$. Hence

$$\begin{aligned}\frac{\delta\Phi_m}{I} = \delta L &= (1 + 3,\ 1 + 2) - (1 + 2,\ 1 + 2) \\ &= (3 - 2,\ 1 + 2).\end{aligned}$$

Otherwise: $(\Phi_m + \delta\Phi_m)$ is the flux of H, due to I round s' in the position $1 + 2$, over an area bounded by s in the position $1 + 3$.

(*c*) Let us now calculate $\delta\Phi'_m$, the flux cut by the part 2 as it moves into 3. This is equal to the flux of $\mathbf{H} = \text{curl}\ \mathbf{A}$ through the shaded area [Fig. 5(*b*)]. And, by Stokes's theorem, this is the circulation (or line-integral) of $\mathbf{A}$ round the contour. To determine the direction of the circulation, observe that $\mathbf{dS}'$ points down through the paper [Fig. 5(*a*)]. Hence the circulation is



(*a*) (*b*)

Fig. 5

clockwise, i.e. 2–3 as shown by the arrows in Fig. 5(*b*). That is

$$\delta\Phi'_m = \int_{2-3}(\mathbf{A}\mathbf{ds})$$

where
$$\mathbf{A} = I\int_{1+2}\frac{ds'}{l}$$

Or
$$\begin{aligned}\delta\Phi'_m &= I(2 - 3,\ 1 + 2) \\ &= -I(3 - 2,\ 1 + 2) \\ &= -\delta\Phi_m\end{aligned}$$

as already proved.

(*d*) We shall also require $\delta\Phi''_m$, the flux cut through by the moving part 2—though this time not the flux due to the whole circuit $(1 + 2)$ but only the flux due to the non-moving part (1). That is, we take

$$\mathbf{A} = I\int_1\frac{\mathbf{ds}'}{l}$$

Thus
$$\begin{aligned}\delta\Phi''_m &= \int_{2-3}(\mathbf{A}\mathbf{ds}) \\ &= I(2 - 3,\ 1).\end{aligned}$$

But it is easy to see that

$$(2 - 3,\ 1) = (1,\ 2 - 3).$$

For, just as in obtaining formula (8), when the distances between the corresponding elements are finite, the practically infinitesimal distance δ becomes negligible. It is therefore immaterial whether we are dealing with s or s' in the position 1 when the other elements are situated in the circuit $2 - 3$; and vice versa.

Now
$$\begin{aligned}\frac{d\Phi_m}{I} &= (1 + 3,\ 1 + 3) - (1 + 2,\ 1 + 2) \\ &= (1,\ 1) + (1,\ 3) + (3,\ 1) + (3,\ 3) \\ &\quad - \left[(1,\ 1) + (1,\ 2) + (2,\ 1) + (2,\ 2)\right] \\ &= (1,\ 3 - 2) + (3 - 2,\ 1) + (3,\ 3) - (2,\ 2).\end{aligned}$$

The last two terms, referring to the infinitesimally distant arcs 3 and 2, cancel each other out. Hence

$$\begin{aligned}d\Phi_m &= -2I(2 - 3,\ 1) \\ &= -2\delta\Phi''_m\end{aligned}$$

That is,*

$$\frac{d\Phi_m}{dt} = -\frac{2\delta\Phi''_m}{\delta t} \quad . \quad . \quad . \quad . \quad (20)$$

It will be observed that our proof is general, and that the result really depends on geometrical considerations. We also infer that

$$\begin{aligned}\partial\Phi_m &= \tfrac{1}{2}d\Phi_m - dk \\ \delta\Phi_m &= \tfrac{1}{2}d\Phi_m + dk\end{aligned}$$

where
$$dk = I(2,\ 2 - 3) = -I(2 - 3,\ 2) \quad . \quad . \quad (21)$$

We shall presently find that, for the moving cross-piece of a rectangular circuit,

$$dk = 2Ivdt\left(\frac{a}{\delta} - \sqrt{2}\right)$$

If $\mathbf{H}$ is the field due to 1 only (and not to 2),

$$\begin{aligned}\frac{\delta\Phi''_m}{\delta t} &= \int\left(\frac{\mathbf{H}\mathbf{dS}'}{\delta t}\right) \\ &= \int(\mathbf{H}V\mathbf{ds}\mathbf{v}) \\ &= -\int\left(\frac{\mathbf{v}\mathbf{dF}}{I}\right)\end{aligned}$$

where
$$\mathbf{dF} \equiv IV\mathbf{ds}\mathbf{H} \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Hence
$$\begin{aligned}\int(\mathbf{v}\mathbf{dF}) &= -I\frac{\delta\Phi''_m}{\delta t} \\ &= \tfrac{1}{2}I\frac{d\Phi_m}{dt} \quad . \quad . \quad . \quad . \quad (23)\end{aligned}$$

Let us now apply these results to the rectangular circuit shown in Fig. 6, in which the lower cross-piece is movable. From formula (9) we have

$$\begin{aligned}\frac{d\Phi_m}{dt} &= v\frac{d\Phi_m}{db} = Iv\frac{dL}{db} \\ &= 4Iv\left\{\log\frac{a}{\delta} + \log 2 - \log\left[1 + \sqrt{(1 + p^2)}\right]\right. \\ &\qquad \left. + \sqrt{(1 + p^2)} - 1\right\} \quad . \quad (24)\end{aligned}$$

where $p \equiv a/b$.

* We can take the position of s' to be $1 + 2$ instead of $1 + 3$; for, the variation being infinitesimal, we can treat its separate components independently.

* This exposition corrects and supersedes the proof given in the author's "Electromagnetics" (Longmans, 1938), p. 574.
The time-element is called dt or δt indifferently. The symbols employed in the author's book have been modified in some instances to bring them into line with the practice adopted for the *Journal*.

Next let us calculate $\partial\Phi_m/\partial t$ as defined above. The flux $(\Phi_m + \partial\Phi_m)$ is that due to the current I around s' in the new position (i.e. in the rectangle with sides a and $b' = b + vdt$), taken over the area enclosed by s (represented by the dotted line in Fig. 6). Thus

$$\frac{\partial\Phi_m}{\partial t} = v\int dS\frac{\partial H}{\partial b}$$
$$= v\int_\delta^{a-\delta} dx \int_\delta^{b-\delta} dy\frac{\partial H}{\partial b}$$

Now H is given by (16), so that

$$\frac{\partial H}{\partial b} = -\frac{Ix}{(b-y)^2\sqrt{[x^2+(b-y)^2]}} - \frac{I(a-x)}{(b-y)^2\sqrt{[(a-x)^2+(b-y)^2]}}$$

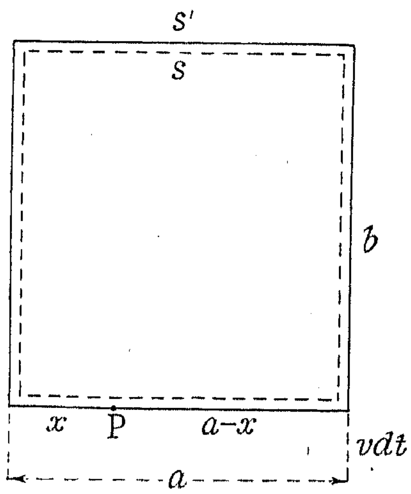


Fig. 6

On calculating the integrals, we find

$$\frac{\partial\Phi_m}{\partial t} = 2Iv\left\{\log\frac{a}{\delta} + \log 2 - \log\left[(1+\sqrt{(1+p^2)}\right] + \sqrt{(1+p^2)} - 1 + \sqrt{2} - \frac{a}{\delta}\right\} \quad . \quad (25)$$

Similarly, following the definition of $\delta\Phi_m/\delta t$, we find

$$\frac{\delta\Phi_m}{\delta t} = 2Iv\left\{\log\frac{a}{\delta} + \log 2 - \log\left[1+\sqrt{(1+p^2)}\right] + \sqrt{(1+p^2)} - 1 - \sqrt{2} + \frac{a}{\delta}\right\} \quad . \quad (26)$$

Comparing formulae (24), (25), and (26), we immediately verify that*

$$\frac{d\Phi_m}{dt} = \frac{\partial\Phi_m}{\partial t} + \frac{\delta\Phi_m}{\delta t}$$

The area swept out by the element dx of the cross-piece in time dt is

$$dS' = |V\mathbf{dx}\mathbf{db}| = -dxdb = -dxvdt$$

the negative sign meaning that the area-vector is pointing down through the paper. Hence

$$-\frac{\delta\Phi'_m}{\delta t} = -\int H\frac{dS'}{dt} = v\int_\delta^{a-\delta} Hdx \quad . \quad . \quad . \quad (27)$$

The magnetic force at P (Fig. 6) due to the two sides of length b is

$$H = \frac{Ib}{x\sqrt{(x^2+b^2)}} + \frac{Ib}{(a-x)\sqrt{[(a-x)^2+b^2]}}$$

Performing the integration in (27), we obtain, for the contribution to $-\delta\Phi'_m/\delta t$,

$$2Iv\left\{\log\frac{a}{\delta} + \log 2 - \log\left[(1+\sqrt{(1+p^2)})\right]\right\} \quad (28)$$

The field due to the upper cross-piece is

$$H = \frac{Ix}{b\sqrt{(x^2+b^2)}} + \frac{I(a-x)}{b\sqrt{[(a-x)^2+b^2]}}$$

On inserting this in (27) and integrating, we obtain the further contribution

$$2Iv\left[\sqrt{(1+p^2)} - 1\right] \quad . \quad . \quad . \quad . \quad (29)$$

We still have to add the contribution due to the other filament in the movable cross-piece itself. For this

$$H = \frac{Ix}{\delta\sqrt{(x^2+\delta^2)}} + \frac{I(a-x)}{\delta\sqrt{[(a-x)^2+\delta^2]}}$$

This contributes to (27) the quantity

$$2Iv\left(\frac{a}{\delta} - \sqrt{2}\right) \quad . \quad . \quad . \quad . \quad (30)$$

Adding (28), (29), and (30), and comparing with (26), we obtain

$$-\frac{\delta\Phi'_m}{\delta t} = 2Iv\left\{\log\frac{a}{\delta} + \log 2 - \log\left[1+\sqrt{(1+p^2)}\right] + \sqrt{(1+p^2)} - 1 - \sqrt{2} + \frac{a}{\delta}\right\}$$
$$= +\frac{\delta\Phi'_m}{\delta t}$$

We have thus verified formula (19).

By omitting the terms (30) we can write down the expression for $-\delta\Phi''_m/\delta t$. Adding (28) and (29), and comparing the result with (24), we find

$$-\frac{\delta\Phi''_m}{\delta t} = 2Iv\left\{\log\frac{a}{\delta} + \log 2 - \log\left[1+\sqrt{(1+p^2)}\right] + \sqrt{(1+p^2)} - 1\right\}$$
$$= \frac{1}{2}\frac{d\Phi_m}{dt}$$

We have thus verified formula (20). This also involves the verification of (23), namely

$$Fv = \frac{1}{2}I\frac{d\Phi_m}{dt} \quad . \quad . \quad . \quad . \quad (31)$$

where

$$F = I\int_\delta^{a-\delta} Hdx$$

* R. Rühle, for the case of an expanding square, has given elaborate calculations of $-\delta\Phi_m/\delta t$ (which he calls *Ruheschwund*) and of $-\delta\Phi_m/\delta t$ or $+\delta\Phi'_m/\delta t$ (which he calls *Bewegungschwund*). See *Elektrotechnische Zeitschrift*, 1933, **54**, pp. 796, 843.

the effect of the "inducing" filament in the moving cross-piece being omitted in calculating H.

(6) THE FORCE ON THE CROSS-PIECE

It can easily be shown* that the work done in a small displacement is $-dW$ (or $+dW_e$) at constant I. That is, the work is $\frac{1}{2}I^2dL$ or $\frac{1}{2}Id\Phi_m$. Hence, for the rectangular circuit, the force on the movable cross-piece is

$$F = \frac{1}{2}I\frac{d\Phi_m}{db}$$

Or the rate of working is

$$Fv = \frac{1}{2}I\frac{d\Phi_m}{dt} \quad . \quad . \quad . \quad . \quad (32)$$

Hence, from (24),

$$\frac{F}{2I^2} = \log\frac{a}{\delta} + \log 2 - \log\left[1 + \sqrt{(1+p^2)}\right] + \sqrt{(1+p^2)} - 1 \quad . \quad (33)$$

This result can also be verified directly by integrating the force between the current-elements.†

Instead of (33), Dunton‡ gives

$$\frac{F}{2I^2} = \log\frac{a}{r} + \log 2 - \log\left[1 + \sqrt{(1+p^2)}\right]$$

He has erred by equating δ and r and by omitting the force due to the top cross-piece, which contributes the extra term $\left[\sqrt{(1+p^2)} - 1\right]$ in formula (33). In the case of a rectangle, for which $p \equiv a/b \to 0$, Kopec§ gives, for the force on the cross-piece,

$$\frac{F}{I^2} = \log\frac{a}{r} + \frac{1}{4}$$

This is only half the correct result.

Comparing (32) and (23), we see that the force coincides with F as previously defined in (22). We have thus verified the general formula

$$\mathbf{F} = I\int V\mathbf{dsH}$$

for the mechanical force. In the present case, formula (23) becomes

$$Fv = -I\frac{\delta\Phi''_m}{\delta t}$$

In calculating H and $\delta\Phi''_m/\delta t$ the magnetic intensity due to the other representative filament in the cross-piece is not included. And we can at once see that this procedure is correct, if we abandon the rather artificial reduction of the actual metallic circuit to two representative filamentary circuits. The forces are then seen to be due to the interactions of the elementary current-streams passing through the wire. And obviously the mutual interactions of the current-filaments in the cross-piece itself cancel out and so cannot contribute to the resultant force on the cross-piece.

* Cf. the author's "Electromagnetics," p. 127. † *Ibid.*, p. 111.

‡ *Journal of Scientific Instruments*, 1927, **4**, p. 444. Dunton gives a different but still incorrect expression on the next page of the same paper.

§ *Elektrotechnik und Maschinenbau*, 1925, **43**, p. 658. Kopec confuses db and ds.

(7) THE INDUCED E.M.F.

The induced e.m.f., when part of the circuit is moving, is

$$V = -\frac{d\Phi_m}{dt}$$

Since the current is assumed to be kept constant, there must be an extra rate of input of energy (from, for example, the battery), which—neglecting loss in resistance—is

$$-IV = I\frac{d\Phi_m}{dt} = 2Fv$$

Hence energy is being supplied at twice the rate at which ponderomotive forces are doing work. If dW_e is the mechanical work in a virtual displacement and dU is the change in internal energy, the equation of energy is

$$Id\Phi_m = dW_e + dU$$

Now it can be shown* that

$$U = -W = +\tfrac{1}{2}I^2L = \tfrac{1}{2}I\Phi_m$$

so that $dU = dW_e$. Hence half the supply is spent in increasing the internal energy of the system.

From the standpoint of the electron theory there is nothing mysterious about what, from the thermodynamical point of view, is called the internal energy. For it can be shown† that the force exerted on a moving charge by a circuit (s) is

$$\mathbf{P} = \nabla\psi + V\mathbf{vH} - \frac{\partial\mathbf{A}}{\partial t} \quad . \quad . \quad . \quad (34)$$

We can divide $\mathbf{H}$ into $\mathbf{H}_1$ due to the portion 1 (all except the moving portion or cross-piece) and $\mathbf{H}_2$ due to the part of s in the moving portion (called 2 in our previous notation). Taking the integral over the circuit s', we have

$$\int(\mathbf{P}\mathbf{ds}') = \int(\mathbf{ds}'V\mathbf{vH}_1) + \int(\mathbf{ds}'V\mathbf{vH}_2) - \frac{\partial}{\partial t}\int(\mathbf{A}\mathbf{ds}') \quad . \quad (35)$$

The left-hand side is V, the e.m.f. of induction. From (22) we see that the first term on the right-hand side is

$$-\int\frac{\mathbf{v}d\mathbf{F}}{I}$$

The last term is $-\partial\Phi_m/\partial t$. And the second term, when multiplied by dt, is

$$\int(\mathbf{H}_2\mathbf{dS}')$$

where $\mathbf{dS}' = dtV\mathbf{dsv}$. That is, by Stokes's theorem, it is the circulation of $\mathbf{A}_2$ in the circuit $(2-3)$, i.e. it is equal to $I(2-3,\ 2)$ or to $-dk$ by formula (21). Thus the second and third terms on the right-hand side of (35) give

$$-\frac{\partial\Phi_m}{\partial t} - \frac{dk}{dt} = -\tfrac{1}{2}\frac{d\Phi_m}{dt}$$

* "Electromagnetics," p. 130.

† *Ibid.*, pp. 549, 553, where the equation is given in electrostatic-magnetic units.

Accordingly, when we multiply both sides of (35) by $-I$, we obtain

$$I\frac{d\Phi_m}{dt} \equiv -IV = \int(\mathbf{vdF}) + \tfrac{1}{2}I\frac{d\Phi_m}{dt}$$

Hence we see once more that, owing to the special form of the force-formula (34), that portion of the rate of working which we designate as mechanical is half the total.

Our investigation has confirmed the correctness of the expression $V = -d\Phi_m/dt$ for the e.m.f. of induction. Assuming the distinction between $\delta\Phi_m$ and $\delta\Phi''_m$ (ignored in current expositions), we can thus express, for the case of self-induction with which alone we are here concerned, the " cutting rule " which is still commonly applied to this case:—

$$V = -\frac{\delta\Phi''_m}{\delta t}$$

Our exposition has shown two errors in this formula: (i) It gives the wrong sign.* (ii) It gives only half the correct result.

The cutting rule has been wrongly attributed to Maxwell. In his " Treatise " (§ 541) he says: " The total e.m.f. acting round a circuit at any instant is measured by the rate of decrease of the number of lines of magnetic force which pass through it." Later on (§§ 598–599) he gives an expression for **P** formally identical with that given by the electron theory (formula 34 above), and he also gives

$$V = \oint(\mathbf{Pds})$$

which, as we have shown, is equivalent to

$$V = -d\Phi_m/dt.$$

* This is concealed in the usual formulation of the cutting rule, because the surface-integral or flux is invariably taken with reversed sign.

ELECTRIC ILLUMINATION*

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INTRODUCTION

The 3-year period 1936–39 has seen the increasing influence of the electric discharge lamp on lighting technique. The "breakaway" from filament-lamp lighting mentioned in the last review† has by no means constituted a landslide. Having regard to the proportion of filament to discharge lamps sold, the use of the latter is as yet on a very limited scale indeed. Nevertheless, the technical achievements over this period have been noteworthy, and the relatively high cost of electric discharge lamps must probably be regarded as the chief factor in hindering their more extensive use. Every lamp still requires its own series choke-coil (or the equivalent), and the lamps themselves, although having longer lives than filament sources, cost several times more. They are, however, about three times more efficient, and the economic balance sheet per 1000 hours' running is usually in favour of the discharge lamp. Nevertheless, the purchaser is found to give more heed to first cost of a lamp than to overall running expense, and the utility company does not always find that devices which effect economies in electricity consumption are to its economic advantage. There are of course other factors such as colour, form or novelty, which have weighed some on one side and some on the other, and the upshot has been a steady, though not rapid, increase in the adoption of electric discharge lamps for those services for which they have special attraction. The developments to be recorded emanate mainly from Europe, but the United States have just shown (1938–39) a remarkable step forward in the large-scale production and marketing of 15-watt to 40-watt low-voltage luminescent discharge tubes, in spite of those adverse considerations just mentioned which have so far prevented a similar popularity in England or on the Continent.

LAMPS

The author proposes now to review types of electric lamps which call for notice.

Filament lamps have shown but little change since 1935. It appears as though the coiled-coil filament marks the last major advance which is to occur in efficiency. It is true that the use of krypton in place of argon has been found capable of enhancing efficiency somewhat, because krypton, in a greater measure than the present argon, reduces the loss of energy from the filament resulting from conduction, so that a higher proportion of the total energy supplied to the filament is available for producing light. Owing to its heavier atom, krypton is also claimed to permit filaments to run at a slightly higher temperature for the same degree of tungsten volatilization, i.e. filament life. Improvements have been made by this means in small miners' lamp bulbs, the light of which has been increased by some 20 % without too serious an increase in bulb cost. Where, however, lamp bulbs are larger, as for general lighting service lamps, the cost of the krypton appears to necessitate a serious increase in cost of the lamp. This has to be weighed against the further gain of efficiency (of the order of 10 %) which might be achieved when krypton is used with coiled-coil filaments.

The use of luminescent materials to add light and improve the colour of mercury discharge lamps was mentioned in the last review. Much attention has been concentrated on this technique, which bids fair to prove a major development.

The powder technique is not so favourable when used with hot concentrated discharge sources as it is with the "cold" tubular type of lamps. At present the powders suffer if exposed to the temperatures which prevail in the neighbourhood of the inner component of the high-pressure mercury lamps. In order therefore to secure as far as possible the colour-improving properties of luminescent powders, the latter must be coated on to the inner surface of the outer glass envelope, and this must then be sufficiently large to ensure a temperature not greater than about 150° C. By these means, as well as by changes in the inners, high-pressure mercury lamps have been marketed which, though not perfect, give a sufficiently improved colour rendering for advantageous use in industrial and open-air lighting.

With the tubular low-pressure type of discharge lamp the walls are sufficiently cool for the luminescent powders to be coated directly on to them, and the radiation from the discharge then impinges on the powder without the interposition of any glass. The stimulation by ultra-violet rays is thus relatively intense and a large contribution of light results from the luminescent material. In practical low-pressure mercury-discharge tubing, giving white light, the luminescent powder increases the light (and therefore the efficiency) some 7 times. The luminescent material effectually corrects the colour of the light, which can apparently be modulated as desired. The first tubular sources to use luminescent powders were of the long high-voltage types. They were introduced in 1934, but in 1938 have appeared short mains-voltage luminiscent lamps consuming from 15 to 40 watts. These lamps do not at present compete in any outstanding way with tungsten lighting on the ground of economy, for the cost of the lamps and of their auxiliary gear counter-balances the greater light efficiency. Their popularity arises from the same reasons which made the high-voltage tubular lamp attractive, i.e. low brilliancy, well-chosen hue of light, tubular form, and low heat. With these lamps we can for the first time obtain almost any colour or any shade of daylight from sunlight to cold north sky-light at good efficiencies.

* A review of progress.
† *Journal I.E.E.*, 1936, **78**, p. 171.

Sodium lamps have held their own, in popularity, with mercury lamps, and their running efficiency has been increased by about 15 %. The rivalry between mercury and sodium in general effectiveness continues acute and is usually decided in each instance in accordance with the preference of the purchaser for one colour or the other.

The use of quartz as a material for the envelopes of lamps has been common practice for many years for ultra-violet lamps, but it has been introduced for normal lighting lamps with interesting effects. A capillary discharge in a quartz tube only 4 mm. diameter and 10 mm. long can dissipate 40 watts in air, but if the tube is cooled by water this dissipation can be raised 10 times with an increase of brilliancy—though not of efficiency. The quartz air-cooled capillary lamp ensures about the same efficiency (about 40 lumens per watt) for low wattages as is given by 250-watt and 400-watt high-pressure mercury lamps in glass envelopes. 125-watt and 80-watt lamps of this type have consequently been introduced into the general lighting service group of lamps.

Apart from the use of quartz in lamps for general lighting, this material constitutes the key to really high-intrinsic-brilliancy lamps for projection purposes. Such lamps, using mercury vapour at from 15 to 150 atmospheres pressure, yield brilliancies (in candles per mm^2) from 10 to 50 times that of the ordinary tungsten filament —brilliancies which approach, but do not quite attain, that of the high-current-density arc. Such quartz projection lamps are either water- or air-cooled. The 500-watt size in the former has a small column about 12 mm. long and 1 mm. diameter—whilst the 500-watt air-cooled type approximates more closely to a point—and consists of an arc about 5 mm. long and 2·5 mm. wide between electrodes, at the centre of a small quartz bulb approximately 4 cm. diameter. Such a source is so concentrated that optical projection apparatus which is not optically worked is often not perfect enough to take advantage of the small dimensions. It would appear that a step forward is now due in the perfection of mirrors and the like used for projection with such sources.

UTILIZATION

No new lamp has yet arrived on the market which for domestic lighting will compete in overall cost with the low-wattage filament lamp. But where cost is not the main consideration the advantage of tubular forms of light is now recognized. The popularity of these has increased, as also has the provision in living rooms of entirely decorative devices which depend for their effectiveness on the strong and aesthetic illumination of some object of intrinsic charm or dignity.

The tubular type of lighting is also appearing in factories for places where its distributed character is of special advantage to the process concerned. But for utility factory lighting the chief change lies in the adoption of either high-pressure mercury or sodium lighting in points placed well up. By this means high illumination levels are attainable at relatively low running cost. Particularly has this been the case in the high factory buildings put up for munition production.

The lighting of shops, shop windows and offices, has been helped in efficiency and in effect by lucidaries using combined tungsten and mercury lamps in such a manner that the light from them is mixed before it reaches the objects to be lighted.

The general level of illumination provided tends still to rise—that is to say, more establishments employ the higher levels above 20 foot-candles which had already been shown for many industrial processes to be economically sound.

THE "BLACKOUT"

The creation of optimum conditions for seeing with a minimum of light has been forced on illuminating engineers by war-time "blackout" conditions. Looked at technically this is not a retrograde condition, for so long as plenty of light is available the incentive to study the science and art of good seeing in its absence does not operate. It is realized that under these conditions the fundamental desideratum is to aid seeing when the human eye is adapted for virtual darkness. The tendency in street lighting has been to bring up the levels of light to such a point that the eye is operating in a condition well removed from dark adaptation. When, however, as in the blackout, the eye finds itself dark-adapted, many aids to vision are possible which would otherwise and in most situations be unwanted. Such aids to vision are (*a*) the complete elimination of light sources which can be seen near the line of vision, and in any case the reduction of the intensity of any light to a level which is no more than a glimmer; (*b*) the enhancement of normal contrasts by the use of large surfaces of low brightness, which may of course consist of whitewashed areas; and (*c*) the employment of safe automobile or hand-torch beams for back reflection from light surfaces or from reflectors fixed to surfaces. The improvement of appropriate highway markings to assist seeing in the virtual absence of artificial light has shown marked progress and will probably remain of value even when normal lighting on highways is restored.

STREET LIGHTING

The interim report of the Departmental Committee on street lighting was mentioned in the last progress review. This Committee brought out its final report in August, 1937. The recommendations do not deviate from those of the interim report but amplify and add to them. They confirm, for instance, the 25 ft. height and 150 ft. maximum spacing and add valuable provisions for the correct location of lighting standards. This report holds the field as the main authoritative guidance for lighting authorities throughout the country. Most new installations conform with it so far as concerns height and spacing of sources and quantity of light, particularly in the case of trunk roads taken over by the Ministry of Transport, for the lighting of which the Government contribute a grant. The value of the provisions of the report are being realized for the placing of light in the most advantageous position for ensuring adequate brightness of the carriageway and of other backgrounds at the critical points on highways. Many modern installations now exist where the ideal of uniform and symmetrical placing of lights has been discarded in favour of the ideal of placing lights solely for the purpose of good seeing from the driver's view-point.

It is being realized that each lamp in an installation makes its individual and unique contribution to the

bright background of the highway and its surroundings against which objects are silhouetted. If a lamp is put out, a dark patch is thereby substituted for a bright one and objects in this locality tend not to be seen.

Most new trunk roads are of the dual carriageway type, and much investigatory work has been directed to their effective lighting, and particularly to their junctions and roundabouts. An interesting and promising trial has been made of lighting lengths of dual carriageway by "one way" lanterns throwing light only towards oncoming traffic. By this means the contrast of objects is enhanced so that they appear always as dark silhouettes and the driver sees only the lamps belonging to his own carriageway.

A British Standard Specification embodying the requirements of the Ministry of Transport report is being worked upon but has not yet been issued. At the time of writing this review of progress, street lighting throughout the country has been entirely eliminated through war conditions. The break came at a moment at which it might be said that a movement was maturing for rendering much more mechanical and simple the design of lanterns for street standards. When the art is taken up again it is to be expected that this movement will be strongly established.

On the theoretical side there has been active experimental study of "revealing power" under street-lighting conditions. This is probably one of the most important contributions which has been made towards the determination of visibility in an artificially lighted road. By an analysis of the extent to which an installation will reveal objects present in it the effectiveness of the lighting may be gauged, and the calculations can be applied to any new system which may be devised.

Study has continued of the extent to which the glare from street lamps, under different conditions, can affect good seeing—and the result shows itself in what is termed the control of cut-off, i.e. the rate at which light intensity falls off over the small critical angle within which light is sent to distances beyond about 150 ft. from the lucidary in question.

AVIATION LIGHTING

Civilian night-flying calls for lighting of various kinds in which uniformity of practice is desirable both nationally and internationally. Largely through the efforts of the International Commission on Illumination, it has been possible to obtain international agreement on the colour and arrangement of the lights used for marking boundaries, obstructions and the like, before independent techniques had been firmly established. At the Tenth Session of the Commission held in Holland in June, 1939, aviation lighting was again one of the subjects studied. Probably the most important new item on which agreement is desired is the arrangement and colour of lights used on and near the aerodrome for the facilitation of landings in fog. Agreement has been reached on the result desired, and the lines of further experiments have been laid down.

The most notable technical progress in this field has been in the application of line-source high-pressure mercury-vapour lamps to aerodrome floodlighting. Air-cooled and water-cooled lamps have been used in the trough reflectors previously employed with linear tungsten filament lamps, with considerable gains in beam candle-power.

PHOTOMETRY

The problems of photometry have been mainly two. The first is to apply the photocell to portable photometers. It cannot be said that this has increased the accuracy of portable photometers, but it has added greatly to their convenience and robustness. This is due to the use of the selenium rectifier cell, whose ability to supply sufficient current at low voltage to deflect a sensitive ammeter made it peculiarly suitable for the purpose. Unfortunately, the ease and reliability with which it appears to work frequently deceive the uncritical user regarding the errors of its indications, which may arise from any one or a combination of the following causes. Differences between the spectral sensitivity of the photocell from that of the "average eye" may introduce errors of appreciable magnitude when comparing light from tungsten-filament lamps, with which the photo-electric photometer has probably been calibrated, with that from lighting installations employing light sources of different spectral energy distributions, e.g. in some electric discharge-lamp installations. Other causes of errors may be: (*a*) departure from true cosine response, (*b*) non-linearity between the incident light and the photo-electric current, possibly not entirely corrected for in the scaling of the indicating microammeter, (*c*) fatigue effects giving rise to temporary reduction in sensitivity of the photocell after extended exposure to light, and (*d*) differences in the indications due to the temperature coefficient of the photocell.

The second and greater problem has been the correct measurement or appraisement of the light from discharge lamps. It is now appreciated that two lamps whose light shows virtual colour identity may behave very differently in their revealing power when illuminating coloured objects. The extent of the difference depends upon the spectral distribution of light from the sources, because two lamps having from appearance the same hue may have very different spectral distributions. To help this problem the lamp manufacturers have defined 8 spectral bands, and the relative luminous intensity in each band of the light under examination is stated. For discharge lamps using luminescent powders an effort is to be made to definite "colour," expressing it in terms of plus and minus limits of luminous intensity in each of these spectral bands. For expressing the apparent colour of the light, the trichromatic system is employed, but the user will not be troubled by these technical points and practical lamps will be known by their colour, expressed in everyday terms.

Measurement of the very low brightnesses associated with "blackout" conditions of vision is reviving the problem of visual photometry in a particularly exacting form. Nevertheless, it is important to establish a technique of measurement for such conditions. Before the war came the world was at last promised (for January, 1940) a liquidation of some of the international candle-power differences which have occasioned trouble for the past 30 years. A "New Candle" was to be accepted by the National Laboratories of Great Britain, America,

France and Germany, based on the light from a square centimetre of platinum at its freezing point and determined in a carefully defined manner with accurately specified apparatus. The unit of candle-power given by this was found to be very closely the same as that hitherto accepted in the first three of these countries. All nations supporting the Comité International des Poids et Mésures agreed to accept this standard with its resulting unit. It remains to be seen now whether this will in fact happen. If it does, the arrangement should also define the method to be used for stepping up in colour.

In the past, because of the difference in the colour of the light from the carbon standard lamps in which the old unit was maintained, and that from practical every-day illuminants, discrepancies tended to arise in the values assigned to these latter sources of light because the method of making the photometric comparison, or colour step, was not sufficiently accurately defined. For example, when calibrating tungsten-filament electric lamps of the gas-filled type, operating at colour temperatures some 1 000 degrees higher than those of the carbon standard lamps against which they must be calibrated, either directly or indirectly, differences of several per cent can arise in the result obtained, depending on the details of the method and apparatus used, although all would be considered good photometric practice. The magnitude of such discrepancies increases with the difference in colour and spectral energy distribution between the light sources being compared.

As the colour temperature of the light from the " New Candle " is not very different from that of the carbon standard lamps, similar difficulties would arise, but this situation will be avoided by defining the method to be used for " stepping up " in colour. The methods defined should be those which yield results in accordance with those of a hypothetical observer defined by the C.I.E. luminosity curve for the " average eye."

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ELECTRICAL PLANT AND MACHINERY*

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TURBO-ALTERNATORS

Since 1936 (the date of the last review),† no major developments can be recorded in the construction of turbo-alternators in this country.

Stators

The accumulation of data from sudden short-circuit tests has made it possible to predict with greater accuracy the reactance under transient conditions and to clarify the conception of the various terms for reactance which is of importance not only for the machines but also for network conditions.

Machines of all sizes and voltages (including 33 000 volts) are short-circuit-proof at full-voltage short-circuits and are, as a rule, tested at the manufacturers' works under this condition. Attention has been focused on high-voltage insulation, particularly with regard to moisture resistance, which has been greatly improved by the impregnation of the coils with bitumen gum under vacuum. Investigations have shown that coils treated in this way may be submerged in water and still withstand the test voltages required.

In order to reduce core losses to a minimum, spacers in ventilating ducts have been changed from magnetic to non-magnetic material.

Rotors

Developments with regard to rotors have been mainly in the direction of metallurgy in order to obtain high-tensile steels for the higher peripheral speeds necessary for greater capacities. A greater percentage discard of the ingots is made in order to ensure sound forgings.

Improvements in the alloys employed have produced steel which shows freedom from certain troubles experienced in the past. Comparisons with American practice show that the progress made in this country is quite satisfactory as regards soundness of large steel forgings.

Improvements have been made in connection with rotor windings in that hard-drawn instead of annealed copper is used, the advantage being that greater resistance to longitudinal deformation due to thermal expansion is obtained. Improvements in supporting the coils against deformations have also been introduced, and it is of interest that this has been found necessary both in the U.S.A. and on the Continent for the same reasons.

Hydrogen Cooling

The use of hydrogen as a cooling medium for turbo-alternators has been extensively developed in America, and over 33 machines are already in operation. In addition to these there is about 1 250 000 kW of this type of plant on order.

The adoption of hydrogen-cooling has made it possible to construct units of greater capacity at 3 600 r.p.m., owing to the reduction in windage losses obtainable because of the low density of the hydrogen, these being only one-tenth of those with air-cooling. The improvement in efficiency obtained at full load by hydrogen-cooling is about 0·9 % for 50 000-kW machines at 3 600 r.p.m., and about 0·6 % for 3 000 r.p.m. The reduction in effective material by the use of hydrogen should be about 20 %, but this cannot always be obtained on account of practical considerations such as voltage, etc.

Operating experience in America has been satisfactory, and with a few exceptions plants now on order are of the hydrogen-cooled type. The danger of an explosive mixture forming due to ingress of air into the hydrogen system is taken care of by protective gear, which gives an alarm when the purity of the gas falls to a certain value which is far removed from the danger point. In addition to this, the stator casing is designed to withstand the maximum explosive pressure that could be generated. The cost of a hydrogen-cooled machine is somewhat higher than that of an air-cooled one, but the gain in coal consumption more than balances this extra. For high load factors the saving in operating cost is considerable, and the system is therefore regarded as sound commercially.

The conditions in this country will, of course, not be so favourable from an economic point of view, owing to the smaller improvement in efficiency.

Machines of this type, each of 75 000 kVA capacity at 3 000 r.p.m., are under construction in this country, so that actual operating experience will be obtained in the near future. It is doubtful whether units below 37 500 kVA capacity will show any economic advantage.

Ventilation

The practice of closed air-circuit ventilation is standard for practically all machines. The exception is small machines where the cost of the coolers becomes a large percentage of the total cost, but even in such cases the trend is towards the closed air circuit, especially for textile mills where ventilation of open machines is often impaired by the ventilation ducts in the machines becoming choked by fluff.

In connection with large turbo-alternators the practice of arranging the exciters with closed air-circuit ventilation is increasing. The general practice in such cases is to use the air coolers of the alternators also for the exciters.

Practice in regard to ventilation schemes varies considerably in reference to fan arrangements used and may be classified as follows:—

(*a*) Fans on the rotor only.
(*b*) Separately-driven external fans.

* A review of progress.

† *Journal I.E.E.*, 1936, **78**, p. 159.

(*c*) A combination of these systems in which a portion of the air is supplied by fans on the rotor, the rest of the air being provided by separately-driven fans.

In this scheme the fans on the rotor usually provide sufficient air volume for 75 % to 80 % of maximum load, the separate fans being started when this load is exceeded.

Where separately-driven fans are used more than one fan is frequently provided so that some may be shut down at light load in the same way as for the composite fan arrangement.

Slip-rings

A certain amount of difficulty has occurred in connection with very high-speed slip-rings, due to an air film forming under the brushes, causing selective collection and consequently overheating of certain brushes.

This has led to a spiral groove being cut in the rings; this breaks up the air film as the spiral groove passes over the surface of the brushes. This type of ring has been very satisfactory in preventing selective collection.

Exciters

The trend as regards exciters is for main exciter and pilot exciters to be provided in order to ensure the maximum amount of stability. Super-excitation has not been employed to any marked extent, although some installations have been made in which the exciter had a maximum voltage of twice normal and were built to give very rapid response.

Exciters are almost exclusively direct-coupled to the alternator shaft with motor-driven units as standby.

Excitation voltages vary considerably, but the general tendency is towards high voltages in order to reduce the current to be carried by the slip-rings.

Water-wheel Alternators

Progress in this class of machine has been considerable so far as size of units is concerned. As would be expected, most of this class of machine is for export.

For large vertical low-speed machines the type known as the " umbrella " type has found favour. In this the thrust bearing is situated below the rotor and no top guide-bearing is provided, the lower guide-bearing being placed as near as possible to the horizontal plane through the centre of gravity of the rotor. Four 37 500-kVA units of this type, running at 100 r.p.m. and 11 000 volts, are under construction in this country, the load on the thrust bearing when the generator is giving maximum output amounting to 1 650 000 lb., including water thrust.

An interesting development in connection with water-wheel alternators is a permanent-magnet type of auxiliary alternator for driving the governors of the turbines. These machines have the same frequency and speed as the main machines, being mounted on the main shaft. Tests show that there is no appreciable diminution of the strength of the permanent magnets after sudden short-circuit tests. The reason for using permanent magnets is to obviate the danger of losing the field; this might occur with an ordinary machine owing to accidents, which would have the result that governing would be lost.

The absence of additional slip-rings is an added advantage. This development has been made possible by the great improvements made in permanent-magnet steel during the last few years.

SYNCHRONOUS CONDENSERS

The demand for synchronous condensers has been considerable during recent years. As regards the machines, there have been no major developments, but experience has been gained showing that satisfactory starting can be obtained by the use of solid poles without any special damping winding. Machines of 20 000 kVA at 1 000 r.p.m. have been constructed, arranged for tap starting.

The development of high-pressure lubrication for the bearings, which actually floats the rotor on oil films, has reduced to very small values the starting current taken from the line. Experience shows that it is possible to start machines with only 2 % of their kVA from the line. In order to accelerate in a reasonable time it is, of course, necessary to draw a larger kVA and 20 % has been found to be a satisfactory value.

Smaller machines have been built with roller bearings, which reduce the starting effort required.

Insulation

Mica in combination with paper and varnished cloth still remains the main material for insulation of rotating machinery and is used in sheet form as well as in the form of tape, which is built with either silk or paper as backing material. Considerable developments have been made in varnishes in recent years and a bitumen base is extensively used for coil-baking varnishes and for cloth impregnation.

The introduction of bitumen varnishes has also led to its adoption as a sticker for mica tape and wraps. Impregnation of completed coils in bitumen gum of high melting point has been adopted by several makers. This treatment renders the insulation moisture-proof and is of special advantage in connection with plants destined for hot and moist climates.

A development of considerable importance is the production of glass fibre which may be woven into tapes and cloths of various thicknesses which, when suitably treated with varnish, can be used for insulation purposes. In the U.S.A., glass fibres are used for covering copper strap up to ½ in. width in the same manner as cotton covering is applied, and a number of machines have been built with the conductors insulated in this way. In this country glass has been used in the form of tape and cloth. Suitable varnishes capable of withstanding high temperatures have been developed for use with glass fabrics, and it should be noted that the fabric is only used as a carrier of the varnish film which is the insulator.

The glass cloth is used in building mica sheets for wrapping of coils, and is of course of great advantage in case of high temperatures.

A considerable number of machines have been built in this country as well as in America and on the Continent. In the U.S.A., results with motors insulated with impregnated glass fabrics for service in coal mines have been very satisfactory, and the operating results on machines built in this country will be of interest.

TRANSFORMERS

Improvements in the design and manufacturing technique have continued steadily during the past few years. Three-phase units are used almost exclusively in Great Britain for practically all transformers other than small lighting transformers.

Materials

The use of paper wrapping for conductors is now almost universal. For major insulation the use of pressboard has extended. This material has very high dielectric qualities, a specific inductive capacity nearer to that of oil than that of bakelite products, and much lower cost. Electrical sheet steel for core laminations has generally improved in quality.

Many transformer manufacturers have given up varnish treatment of coils wound with paper-wrapped conductors; the windings and insulation are pre-shrunk and seasoned before mounting, to the avoidance of further shrinkage in service.

Non-inflammable fluid filling has been used to some extent for transformers where fire risk was a serious factor, but the use of this material has not become more general owing to its high cost.

Insulation Design

Further experimental data on impulse-voltage conditions in transformer windings have been accumulated, enabling progress to be made in the design of the insulation system of high-voltage transformers. In this respect the recurrent-surge recorder now in use is of assistance in determining, in the factory, the impulse-voltage distribution in the windings without subjecting the transformer to an impulse test. With the knowledge so gained there are two ways of designing the transformer with regard to surge conditions:—

(1) To proportion the inter-turn and inter-coil insulation adequately to resist, at all points, the surge voltages occurring.

(2) To provide the winding with some form of static shields, in order to distribute the capacitances so that the voltage distribution in surge conditions is more nearly uniform throughout the winding.

Both methods are employed, the former being the more predominant in this country.

Large Mobile Transformers

The outstanding development in large transformers has been the large mobile transformers designed as "transportable" spare units.

The function of these transformers is to act as spare units which can quickly be installed and put into service, thus avoiding the necessary lapse of time involved in erecting a large transformer of the normal design, which may run into several weeks. The transformers would be utilized in cases of normal main substation transformers being out of commission, or where an abnormal increase in load might occur.

The transformers are so designed and constructed that each forms a complete and self-contained mobile unit. All fittings and accessories are attached, and the transformer is filled with oil ready for installation and immediate connection to the line.

They are suitable for transport complete, either by rail or road, from the storage depot to any site where they are required.

The principal problem in the design was to produce a transformer of the highest possible kVA capacity that could be transported within the limits of the British railway loading gauge, which are rather restricted.

The maximum size which has been designed and built for these conditions is 30 000 kVA, 132/33 kV, 3-phase, and a number of units in this and smaller ratings have been constructed.

The type of truck employed is the side-girder suspension type which permits of the transformer being slung between the side girders, thus utilizing the whole of the height available between the rail face and the top of the loading gauge.

A number of road trucks suitable for transporting these transformers are also available.

The cooling of the large transformers is type OFB—forced oil circulation with air blast. This system is adopted as being the most efficient as regards space occupied by the coolers, which are mounted at the end of the transformer tank, the high-voltage terminal bushings being mounted at the other end at a steep angle so as to come just within the railway loading gauge.

On-load Tap-Changing Gear

The practice continues of providing on-load tap-changing gear on all medium and large transformers other than main step-up transformers at power stations. Developments in this class of equipment have been in the direction of making it self-contained and external to the transformer, and generally so arranged that the transformer can be transported complete with the tap-changer gear attached.

There has been a trend towards raising the limit of kVA for which preventive resistors, instead of reactors, are used. The resistor effects considerable saving in space and cost, but it must be short-time rated whereas the reactor can be continuously rated. The range of on-load tap gear has been extended so that such equipment is available for normal currents up to 1 500 amperes, and in voltage classes up to 88 kV operating voltage with fully insulated transformer windings. The application of on-load tap changers to furnace transformers and mercury-arc rectifier transformers is also noticeable.

Improvements in the design and manufacture of mercury switches have extended their use in connection with tap-changing gear, particularly for on-load tap changing.

Off-circuit Tap Switches

Off-circuit tap switches are practically universally provided on small distribution and medium-sized substation transformers. These are usually gang-operated with one external operating handle. The practice of interlocking the off-circuit tap switch with the circuit-breaker, to ensure that the transformer is dead before operating the tap switch, is tending to spread.

The range of off-circuit tap switches has been extended

to a normal current of 2 000 amperes, the class being designed for low operating torque.

Another interesting development in off-circuit tapping switches is a motor-operated combined tap switch and delta-star changer for 11 000 volts for use with furnace transformers. The switches are also arranged to connect in a reactor on certain tappings for the purpose of increasing the reactance on load tappings.

Arc-Suppression Coils

The demand for arc-suppression coils for operation with an earth fault on one common phase of a transmission line has continued to spread. Such coils have been manufactured and installed in Great Britain for systems up to 132 kV.

Protective Reactors

The linking-up and increasing capacities of power stations has increased the demand for protective power-limiting reactors of both magnetically shielded and non-shielded types. There has been a tendency for such reactors to be made as 3-phase, instead of the hitherto common groups of three single-phase units, whether separate or in one tank. The 3-phase unit is comparable with the 3-phase transformer in the saving in floor space, connections and cost. Three-phase reactors with natural oil cooling have been constructed for capacities up to 9 % choke on 20 000-kVA, 3-phase, 6-kV feeders. Such ironclad single-phase reactors have also been constructed for system voltages up to 66 kV.

Distribution Transformers

Distribution transformers, fitted with surge diverters or surge absorbers, have been developed for localities where lightning is prevalent. The surge diverters are incorporated in the h.v. terminal bushings for voltages up to 6 600. Surge absorbers are fitted as separate units.

Mining-type Transformers

A new British Standard (No. 355) was issued in 1939. The provisions of this specification lead to an improved type of mining transformer better suited to meet the requirements of the colliery industry.

The specification has effected the standardization of a number of electrical and mechanical features, and now also includes the provisions for flameproof transformers up to 5 kVA, with an outline drawing of a small flameproof transformer tank.

Mining transformers to the new specification are already being produced.

Welding Transformers

The demand for a.c. welding equipment has widely spread. A.C. welders which previously comprised a transformer with a tapped reactor in series, now frequently take the form of a double-wound transformer with a movable core, forming a magnetic shunt between the primary and secondary windings. Smooth regulation is attained by the turning of a handle without the use of any tappings or moving contacts.

DIRECT-CURRENT MACHINES

There have been no major developments within recent years in the design of direct-current machinery, but steady progress has been made in improvements of commutation; this has been accomplished by close study of the factors affecting this phenomenon.

Improvements in the mechanical design of commutators as well as methods of manufacture have made it possible to produce commutators with a great degree of accuracy and to reduce to very fine limits distortions due to speed and temperature, thereby eliminating one of the greatest difficulties in obtaining satisfactory commutation. Investigation by means of the oscillograph of the voltage curve during commutation has made it possible to determine more accurately the particular factor responsible for commutation troubles on individual machines. Methods of testing and adjustments of commutating poles have been improved, thus ensuring that correct conditions have been obtained before shipment. As an example may be cited the fact that large rolling-mill and winder equipments are subjected to prolonged testing under peak-load conditions in order to ensure that the commutation does not deteriorate with time, for it has not infrequently been found that machines would run satisfactorily for a short time, after which the commutation would gradually deteriorate. The result has been a gradual increase in speeds for a given output of machine, with consequent reduction in cost. This is particularly noticeable in connection with motor-generator sets feeding rolling-mill or winder motors, where choice of speed is not fixed by a prime mover. An increase in speed in such cases affects not only the size of the generators but also the driving motor and the flywheel, where such are provided.

As an example it may be stated that generators are now built for this class of service having a normal output of 1 800 kW and with peak loads up to three times normal at a speed of 750 r.p.m.

Rolling-mill motors having an output of 5 000 h.p. at speeds variable from 125 to 275 r.p.m. are in service in America.

They could equally well be built in this country should the demand arise.

An interesting feature is the noticeable increase in the demand for direct-current motors in the last few years. The increase is occurring throughout most of the industries using electrical drive. The explanation for this lies probably in the complex nature of modern industrial processes, which require variable speeds and the economic advantages obtained therefrom. Ward–Leonard equipments are being applied in many cases down to ½ h.p. for special machine tools.

Another field in which the direct-current motor has found use is for boiler-house auxiliary drives.

Co-ordinated control by means of Ward–Leonard sets has given excellent results in power stations. Similar drives are used for live roller drives of rolling-mills where experience has shown the saving in power consumption and reduction of maintenance to warrant their use. The live-roller tables are split into several groups, with a Ward–Leonard set for each group.

An interesting development in connection with direct-current work is the machine fitted with short-circuiting

brushes on the commutator in one axis. By the application of appropriate field windings it is possible to obtain a constant-torque characteristic to the point of stalling with more than one motor fed from the same generator. The generator is, of course, of the same type as the motors. Such equipments have been built for cranes for large forging presses where the torque is required to balance the suspended forging, thus making it possible for the press to push the forging down on the anvil without increasing the load on the lifting chains, but at the same time preventing them from slackening off.

A.C. MOTORS

Synchronous Motors

The application of synchronous motors is steadily increasing for industrial drives of various descriptions and for motor-generator sets, the advantage being that power-factor correction is obtainable at a low cost owing to the fact that considerable wattless output is obtainable with comparatively small increase in size of the motor.

Both the salient-pole type with a pole-face winding for starting purposes, and the cylindrical type rotor, known as the synchronous induction motor, are employed. Where heavy starting torque is required the latter type is favoured, on account of the lower starting current required. In cases where heavy overloads, in combination with high starting torque, are required, the trend is towards the synchronous induction motor, as overloads can be obtained by a system of compounding consisting of a current transformer, the current of which is rectified and fed to a winding on the exciter of the synchronous induction motor, thereby keeping the field strength of the motor at the required value for any load. Power-factor correction is, of course, obtainable by either type of machine. Synchronous motors of both types have been applied for driving flywheel motor-generator sets. Speed reduction is obtained by the introduction of a fluid clutch which is made to slip by changing the amount of fluid in the clutch. Load-governing on the synchronous motor within 10 % can be obtained on a winder equipment requiring a peak of twice normal load on the winder motor. The largest equipment built of this type has an output of 1 800 h.p. at 750 r.p.m.

Salient-pole synchronous motors of 4 500 h.p. capacity running at a speed of 1 500 r.p.m., and 6 000 h.p. at 1 000 r.p.m., have been built for compressor drives, and it is of interest to note that sufficient starting and synchronizing torque is obtainable for this class of drive by the use of solid poles and without damper windings being provided.

Induction Motors

Large Motors.

In this class of machine no major developments can be recorded. The capacities of individual motors show a gradual increase, especially for rolling-mill motor-generator sets and compressor motors.

Motors of 5 500 h.p. running at 750 r.p.m., 6 000 h.p. at 1 000 r.p.m., and 3 000 h.p. at 3 000 r.p.m., have been built of the slip-ring type for compressor drive.

Considerable development has been necessary in connection with the slip-ring construction, on account of the high peripheral speeds and the large currents to be carried.

The application of direct line starting for squirrel-cage motors is extending, and has proved successful in operation for motors of 700 h.p. at 3 000 r.p.m., and larger sizes at lower speeds.

Medium-size Motors.

A noticeable development in connection with medium-size motors is the production of special motors such as the flameproof type for mining, steel-mill type motors, etc. This has crystallized into a definite policy of providing complete ranges, fully jigged and tooled with stock parts for each of the major industries.

An interesting development has been carried out in the U.S.A. in connection with the flameproof motors for use in oil refineries. In order to prevent the possibility of explosions due to gas entering the motors, they were filled with an inert gas which was circulated through outside coolers in a closed circuit. These motors had an output of 1 000 h.p. at 3 600 r.p.m.

In connection with steel-mill work live-roller motors coupled direct to the rollers have been developed to fill the requirement of low inertia which is necessary for reasons of economy in power consumption and reduced maintenance. In view of the low speeds required it has been necessary to adopt lower frequencies than standard, depending on the type of mill, and motor-alternators are therefore provided.

In certain mills the speeds of the live rollers have to be matched with the main rolls and in such cases the alternators are driven by variable-speed direct-current motors, the controls of which are coupled with the speed controls of the main roll motor.

The live-roller motors are as a rule totally enclosed. The stators are ribbed on the outside to provide adequate cooling and are bolted to the roller tables.

Another development in connection with rolling-mill work has been made in order to meet the demands for operation of catcher tables for three-high sheet mills. For this class of service the number of reversals required may be as high as 23 to 25 per minute, and a very low inertia is therefore extremely important.

A motor of the cascade type has been developed for this purpose. It consists of two stators built into one housing, and two squirrel-cage rotors on the same shaft. The windings of the two rotors are suitably connected together. The supply is taken to the stator of the first motor, the second rotor being the primary and the second stator the secondary of the second motor. With this arrangement a large percentage of the heat unavoidably generated with frequent reversals can be dissipated, without the use of slip-rings, in a resistance outside the machine; this enables the best use to be made of the material in the motor itself.

The result is also a minimum temperature rise of the motor. This type of motor is, of course, also applicable for live-roller drives.

Totally Enclosed Motors.

For small and medium motors, the totally enclosed or totally enclosed fan-cooled motor is the most economical construction to meet operating conditions in a

polluted atmosphere. Constructions of motors vary, depending on the size. For motors of 1 h.p. or below, the normal totally-enclosed type is used. From 1 h.p. to approximately 30 h.p. a machine with external cooling ribs on the stator frame is found economical. From 30 h.p. to 60 or 70 h.p. a type of motor provided with double air circuit is used, sometimes having a cooler arranged with alternate ducts, some carrying the internal air of the motors and the other ducts carrying the external cooling air.

Above these sizes it is usually found desirable to adopt a more complex construction, frequently with the outside air passing through the stator punchings. In this construction passages in the stator core are provided, in some of which the internal air of the motor is circulated, the outside air passing through other ducts in the stator core. Motors of this type, having a capacity of 750 h.p. at 3 000 r.p.m. are in successful operation.

Fractional-horsepower Motors.

The increased demand for domestic appliances owing to the wider distribution of electric power has brought with it considerable developments in the various types of fractional-horsepower motors. The capacitor motor has almost entirely superseded the repulsion motor. The size of the condenser fitted to the motor has been reduced considerably for the size of the motor. Flameproof motors of this type have been developed for use with petrol pumps and other purposes where the atmosphere contains inflammable vapours.

The condensers are enclosed in flameproof extensions of the end shields at one end of the motor. Thermal overload devices are now being adopted widely on fractional-horsepower motors, especially for refrigerator duty. The flameproof type of motor has also been developed for direct current.

Small universal commutator motors are, on a.c. supplies, being superseded considerably by induction motors of the split-phase, capacitor, or shaded-pole types.

Combined motor and gearing units are available for fractional-horsepower motors of $\frac{1}{4}$ to 1 h.p.

A development which has found considerable application is the "thrustor," which is a combined fractional-horsepower motor and pump. This unit is now available in sizes having a thrust of from 40 lb. and 2 in. stroke, up to 800 lb. with 8 in. stroke. Thrustors embodying flameproof motors have also been developed.

Variable-speed A.C. Motors.

Progress has been made in the development of polyphase commutator motors of the various types.

In addition to the motor where speed variation is obtained by brush shifting, there is now available a combination of an a.c. commutator motor and a double induction regulator.

Special arrangements of commutating windings have been developed, consisting generally of additional windings, of a particular distribution, placed in the armature slots and connected in parallel with the a.c. commutator winding. The application of these additional windings has resulted in increased output per pole and consequently in increased capacity of the motor. The following may be regarded as good examples of output from motors where speed variation is obtained by brush shifting on the motor:—

420/250 h.p., 810–680 r.p.m., 8 poles, 50 c./s.
175/147 h.p., 1 500–1 350 r.p.m., 4 poles, 40 c./s.

The demand for variable-speed a.c. motors shows a tendency to increase considerably more in the smaller than in the larger sizes, and motors of 2 h.p. at 1 500 r.p.m. with a speed range from 0 to 2 500 r.p.m. are available.

Speed control is either by direct operation on the brush gear or through pilot motor operating the brush gear.

For some applications a Selsyn motor is coupled to the pilot motor so as to control two or more motors at the same speed.

The Scherbius type of control for speed variation of induction motors continues to be employed. This type of variable-speed motor has found use for bore-hole pumps, the largest output being 850–400 h.p. at 720–550 r.p.m. and 11 000 volts.

Other applications include motors for circulating pumps and boiler feed-pumps in power stations, as well as motors for fans and air compressors where variable air quantities are essential.

CONTROL GEAR

In the field of heavy engineering the most notable development in the way of control gear is the fully automatic electric winder. This type of equipment has been developed for both a.c. and d.c. winders.

The principle of operation of an automatic a.c. winder is the comparison of two voltages, one proportional to the predetermined cyclic speed, and the other proportional to the actual speed of the winder. The difference of these two voltages operates a small d.c. torque motor so that if there is any discrepancy between the two speeds a proportional torque is exerted by the torque motor which, through the medium of an oil servo mechanism, changes an external resistance in the rotor circuit of the main motor until the correct speed conditions are obtained.

The correct cyclic voltage is obtained by changing the excitation of a constant-speed generator through a cam-operated rheostat, and the voltage proportional to the actual speed is obtained from a separately excited generator driven by the winder shaft.

The winder is started by means of push buttons at the top and bottom of the shaft, after which the whole winding cycle is automatic.

Direct-current winder equipments are also controlled through cam gear and oil servo gear.

An a.c. winder with a 370-h.p. motor, and a d.c. winder of 1 800 h.p., fully automatic, are in operation.

Considerable developments have been made in the control of large reversing mill motors. Equipments of this kind are subject to very heavy overloads during reversals, and it is therefore necessary that the motors should be able to give the maximum overload. The control gear, however, must limit the load to the maximum required.

Special compounding exciters have therefore been developed which are inactive up to the maximum load required from the motor, but when the current exceeds this value the exciter voltage builds up very rapidly and strengthens the motor field so that the current is limited

to a predetermined amount. The exciter also limits the current under field-weakening conditions, in that the motor field is strengthened and thus the torque is increased and at the same time the speed is reduced. By this means the mill operates at the maximum speed appropriate for the torque required, so that maximum output is obtained from the mill. The operation of the whole equipment is carried out by a small hand-operated cam controller which operates the generator and motor exciter fields. That high values of acceleration and retardation are obtainable is shown by the fact that the time required to reverse a 5 000-h.p. motor from 150 r.p.m. in one direction to 150 r.p.m. in the other direction is 4·5 seconds.

The general trend in regard to control of reversing rolling-mill motors is to simplify the gear as much as possible in order to increase the reliability and reduce maintenance.

Developments have been made in control gear for continuous strip mill, especially in connection with tensioning devices for the reeling plant.

Improvements have been made in contactors for various classes of reversing service where very frequent reversals are required.

The use of grid-controlled rectifiers for speed control of motors is increasing.

RECTIFIERS

Mercury-Arc Rectifiers

The outstanding development of the last few years in connection with mercury-arc rectifiers is the permanently sealed steel-tank air-cooled type. The absence of pumping plant and water-cooling gear makes this type very attractive, and a considerable number are now in service in trolleybus substations at 600 volts, as well as for industrial installations. The operating records are excellent both at home and abroad, a notable application being for service underground in South African mines.

The ability of these rectifiers to withstand long-distance transport, and their simplicity in erection and operation, have been important factors in their rapid development.

Outputs up to 1 000 amperes or 500–600 kW are now obtainable, although up to the present the maximum output from a single tank in operation is 375 kW.

Pumpless rectifiers are being installed for railway service, and substations up to 2 000 kW capacity at 600 volts, made up of 375-kW units, are in manufacture. At 1 500–3 000 volts, 1 000-kW units are available although none are in operation.

For larger ratings the water-cooled type of rectifier fitted with vacuum pumps is still in favour.

An intermediate type of rectifier, which is air-cooled but provided with vacuum pumps, has been developed on the Continent but has not so far been employed in this country.

The ignitron type, in which single-anode tanks are employed, is specially advantageous for lower voltages on account of the lower drop in the arc and consequent higher efficiency. So far as this country is concerned this type has been confined to industrial processes, such as seam-welding control. In the U.S.A. however, there are a number of this type in satisfactory operation on the New York Subway. These rectifiers have a capacity of 3 000 kW at 600 volts, each consisting of a bank of 12 single-anode tanks.

Although originally confined to smaller sizes, the glass-bulb rectifier has also given excellent results in the larger sizes where a number of bulbs are connected in parallel as one unit.

While on railway electrification schemes the steel-tank rectifier is mostly used, several railways are using glass-bulb equipments for 600 and 1 500 volts. Glass-bulb rectifiers are also in wide use in trolleybus substations and in substations for industrial service. As regards the maximum rating of bulbs, while some bulbs of large ratings have been constructed British makers still prefer to limit the rating to 400–500 amperes per bulb.

There seems as yet no clear line of demarkation between the three types of rectifiers manufactured in this country, but probably the next few years may clarify the position in this respect. At present water-cooled rectifiers are uncommon below 1 000 kW, except for high voltages.

For small sizes the glass bulb is preferred and, although it is as yet early to predict how far the pumpless type will supersede the other types, it seems certain that its application will be extensive.

In the U.S.A. the general trend seems to be towards the ignitron type, and it is of interest that neither the pumpless steel-tank rectifier nor the glass-bulb rectifier has been developed.

As regards the standard of performance now obtainable, the efforts made to overcome the backfire problem have been quite successful. In one type of water-cooled rectifier, of which a large number are in operation, statistics kept over many years show an average of one backfire per 7 years.

Important experience has been gained in the last 2 years on inverted running of rectifiers for regenerative conditions for railway service. Experience indicates that there are no insuperable difficulties with this type of equipment, and the standard of performance of the inverted rectifiers is not inferior to that of the straight rectifiers.

With regard to grid control, this has not been widely employed except for special requirement for railway service. In the industrial field the principal application of grid control seems to be for maintaining constant current in electrochemical work and for the supply of special rolling-mills. The somewhat optimistic suggestions some years ago as to the possibilities of grid control do not appear to have materialized.

In Germany some elaborate experimental grid-control equipments are in use, as frequency changers for converting from 50 c./s., 3-phase, to $16\frac{2}{3}$ c./s. single-phase supply. Applications of this kind are very special and do not appear to make any appreciable difference to the relative merits of a.c. and d.c. systems of railway electrifications. The use of grids for arc suppression is, however, increasing, particularly for higher voltages. The present position is that it is almost standard at 3 000 volts, fairly common at 1 500 volts, and rarely applied for 600-volt equipments.

The application of rectifiers for electrochemical purposes is extensive abroad, and for this class of work it is usual to group the sets in such a way as to give the effect of working with 24, 36, or even more phases. Experience

in the U.S.A. has shown this precaution to be necessary for these very large installations.

During the last few years much preliminary work has been done on the preparation of national and international standard specifications for mercury-arc rectifier equipment. It is expected that the British Standard Specification on this subject will be ready for issue in the near future.

Telephone Interference by Rectifiers.

This problem has been extensively investigated during the last few years, notably on railway systems in the Dominions where this trouble was prevalent owing to unusual circumstances.

The conclusions which can be drawn from these investigations are that on the d.c. side normal methods of smoothing by resonant shunts are sufficient to give freedom from interference by railway equipments in practically all cases. As regards the a.c. side, interference by 6-phase rectifiers of normal size hardly ever occurs, provided the a.c. supply lines are adequately transposed throughout their length.

It is becoming standard practice for traction rectifiers to be provided with resonant shunt smoothing equipment. The tendency towards the use of 12-phase rectifiers for large railway installations is increasing, as it affords an additional safeguard against disturbances on the a.c. side.

Metal-oxide Rectifiers

Copper-oxide Type.

This type of rectifier has been established for many years and continues to be used extensively.

Selenium Rectifier

This type has been in continuous use on the Continent for the last 10 years to an increasing extent. It has been manufactured in this country for some years and has proved successful.

The selenium rectifier element consists essentially of a nickel-plated sheet-steel disc, on one side of which is a coating of selenium that is subjected to a series of heat treatments to bring the selenium into the correct crystalline condition, after which the working surface is sprayed with a special alloy which forms the counter electrode.

The safe working temperature of this rectifier is 75° C. The rectifier may be oil-immersed, which is of advantage for extra-high-voltage work or as a protection against atmosphere laden with chemical vapours.

The rectifier has been applied over a wide range, from cable testers giving an output of 80 kV at a few milliamperes to rectifiers giving thousands of amperes at low voltage for electro-plating.

In the latter field oil-immersing is of considerable advantage as it permits installation close to the plating vat, thereby saving long and costly busbar connection.

Rectifiers of this type having an output of 50 000 amperes are in operation.

Apart from battery charging, which is one of the principal uses for both the copper-oxide and the selenium type, they are successfully being used for the supply of direct current for telephone and repeater exchanges, broadcast transmitters, excitation of alternator fields, operation of solenoids and lifting magnets, and many other purposes.

They are also used in large numbers as blocking rectifiers in d.c. circuits for telephone exchanges, railway signalling, etc.

ELECTRIC FURNACES

The production of special steels has progressed considerably, and with it a demand for electric furnaces of various types. Stainless steels are produced in arc furnaces and induction furnaces, and also in oil-fired furnaces in combination with induction furnaces.

Arc Furnaces

The development in connection with this type of furnace has been directed towards increased power, in order to reduce the melting time. Constructional developments in the furnaces have also been made for the same reason. The general trend is to build furnaces with removable roofs, in order to facilitate charging. Revolving hearths are frequently incorporated in large furnaces.

There are now many furnaces absorbing 10 000 kVA in operation, and also some larger units in the U.S.A.

High-frequency Induction Furnaces

This type of furnace has advantages for the steel makers in that it is very suitable for melting scrap which is not easily handled in the arc furnace.

Developments consist mainly in details of design such as efficient shielding of the supporting structure, provision for earthing and operating mechanism.

The power factor of the furnace is about 0·2, and in order to reduce the capacity of the generator it is operated at or near unity power factor, the wattless current being supplied by static condensers.

Condensers in series with the generator are also provided for voltage regulation.

Owing to the fact that the power factor varies during the melting time, the condenser capacitance has to be varied, and this is accomplished by the use of contactors.

The number of furnaces installed and under construction in this country is approximately 45, with a rated capacity of 13 250 kW. The individual capacities vary from 100 kW to 1 500 kW, and frequencies from 2 200 c./s. for the smaller units to 1 100 c./s. for the larger ones.

A development of considerable interest is the use of high-frequency induction heating in connection with crankshaft hardening. Specially-constructed heating coils are placed round the portion of the crankshaft to be hardened. These coils are provided with water passages for admission of water to the crank shaft when the correct temperature has been reached. The penetration of the high-frequency current is, of course, extremely small, so that a very thin layer of the steel is heated and hardened.

In order to prevent penetration of heat due to conduction the heating period is extremely short, so that the power required is comparatively large.

A number of applications for high-frequency induction heating have been made; as an example may be cited annealing in situ of welds for high-pressure steam piping.

Resistance Furnaces

Steady progress has been made in resistance furnaces. Although no outstanding development can be recorded there has been improvement in construction of heating elements. Some very large resistance furnaces have been installed in the U.S.A. for annealing welded structures.

AIR-CONDITIONING

There appears to be a more definite appreciation of the importance of clean air for ventilation of electrical machinery. This is indicated by the more general demand for filtering arrangements. To meet this demand machines have been developed with filters built into the stator structure; this avoids the provision of special air ducts to and from the machine. Vertical motors lend themselves particularly well to this arrangement.

In most modern rolling-mill and other industrial installations filtered air is provided for ventilation of the electrical plant. In some cases a system of re-circulation of the air is adopted, the cooling air being taken from the machine room, which is completely closed from the outside air. After passing through the machines, the air is discharged into closed spaces in the basement, from whence it is passed through coolers and returned to the machine room.

In addition to cleanliness, this system has an advantage in that it keeps the machine room at a satisfactory temperature.

Air Filters

The type of air filter used is almost exclusively that in which the cleaning of the air is accomplished by the air being forced into contact with surfaces coated with an oil film. For smaller sizes it is usual to install the normal stationary type of filter, which, of course, requires cleaning at regular intervals. Where large air quantities have to be handled self-cleaning filters are economical, as the higher cost of this type of filter is off-set by the saving in labour required for cleaning. Several large-capacity self-cleaning filters have been installed in recent years, particularly in steel works, where air conditions are such that almost constant cleaning would be essential with the normal stationary filter.

NOISE ABATEMENT

The problem of noise from electrical machines and transformers has received much attention and considerable progress has been made. Research has been carried out on transformer noise, in which the electromagnetic, thermal and economic, as well as the acoustic aspects of various methods of abatement have been investigated.

Progress in noise abatement generally is greatly dependent on there being available accurate and convenient portable noise meters indicating on an accepted basis. Such a basis is provided by the unit of equivalent loudness, the phon, formulated by the British Standards Institution and, since 1937, adopted internationally. The sustained noise associated with electrical plant covers an exceedingly wide range of levels and compositions and the devising of a meter capable of measuring any noise of this type in phons is a very complex problem. However, the development of a new form of meter has recently been completed and a description of it may be expected in the near future.

The most usual way of reducing noise in transformers is to reduce the flux density in the core, but to reach by these means a limit comparable with the usual background noise is not economical.

Cases of complaint are comparatively rare. In localities where the noise is of real importance, the effective means are either to enclose the transformer in a chamber properly designed to restrict the egress of sound, or, if the transformer is installed out of doors, to surround it with a sound-absorbent screen, either inside or outside the tank. In the latter arrangement the oil is cooled by separate radiators, connected through flexible pipe connections.

As regards rotating machinery, the problem is exceedingly complicated because noise is produced magnetically as well as by the air passing through the machines, especially in high-speed machines. Investigations have shown that considerable reduction in noise can be obtained in induction motors by suitable choice of the number of slots in rotor and stator. This often imposes limitations on the output, and a certain volume of noise necessarily remains.

In direct-current machines the problem is still more difficult owing to the presence of commutator risers which, at high speeds, have a powerful fan action. The tendency is, therefore, towards totally enclosing such machines where noise is likely to be objectionable.

High-frequency generators are specially troublesome in respect of noise, but successful results have been obtained with this type of machine by complete enclosure and by providing sufficiently long air ducts to and from the machines, at the same time covering the surfaces of these air ducts with sound-absorbing material.

ELECTRIC WELDING

Considerable developments have taken place in the last few years in electric welding. These have taken place in two directions: (1) in the design of welded structures, and (2) improvements in methods and equipments used. Improved methods have made possible the extension, with safety, of welding to rotating elements. As examples may be mentioned rotor spiders for large direct-current reversing rolling-mill and winder motors as well as for large water-wheel alternators, also supporting bearing brackets for large vertical alternators carrying loads up to 1 650 000 lb. In the U.S.A., brackets of this type have been produced as welded structures for supporting a total load of 3 000 000 lb.

Complete rotors, including shafts, have been built as welded structures. It is regarded as good practice to stress relieve rotating parts when completed by normalizing at 550° C., but this is not considered necessary for static structures.

As an instance of the extent to which electric welding is applied may be cited the fact that, with the exception of the shaft, pedestal bearings, commutator brush-arms and holders, all parts for direct-current machines which formerly were castings have been replaced by welded structures. Complete casings for large marine turbines and gears have also been welded, resulting in a considerable reduction in weight.

By preheating it is possible to weld together mild steel and alloyed steels such as stainless steel. By this means it is possible to employ cheap material for the main body of the structure and to use the more expensive materials only where they are necessary.

One example of this is the insertion of non-magnetic steel in switchgear parts carrying heavy current through bushings.

The most important process for welded work is metallic arc welding by either direct or alternating current. Electrodes have been steadily improved from the bare wire stage through the braided types and fly spun types, to the modern extruded electrodes which have recently been introduced.

Improvements in the quality and the ease of operation of electrodes have consistently improved the type of welds and mechanical properties of the desposit.

As electrodes have been improved it has been found necessary to improve the characteristics of welding equipment, and a close study of this has resulted in the production of satisfactory types of equipment for either a.c. or d.c. work. In addition, the technique in assembly has been developed, and to facilitate this work welding manipulators have been developed and are being used to an increasingly large extent. These make assembly and welding simpler and ease the deposition of the weld metal, giving greater consistency and better appearance.

Atomic arc welding is used to some extent, especially on thinner sections.

Resistance welding is increasing in importance. For instance, large flash butt-welding machines are used for welding reasonably regular sections together. Either steel or non-ferrous metals are welded in this class of machine, and the welds obtained in copper by butt-welding are superior to those obtained from other processes.

The use of spot welding for heavy sections is becoming of importance, mainly owing to developments in the production of heavy spot-welding machines and a more perfect type of timing control. This control makes use of an ignitron type of rectifier arranged to give a succession of short welding periods. This gives great consistency in the weld and makes it possible to weld heavier plates together than could be done with the older methods.

Considerable attention is being paid to the development of projection welding, which at present plays a small part but will undoubtedly play an increasing part in the near future. A development which has come into use is percussion welding, which is a butt-welding process carried out in a very short time. This process makes it possible to weld together dissimilar metals such as copper and aluminium. It is employed in connection with turbo-alternators built with aluminium rotor windings. Pieces of copper are welded to the aluminium conductor at the end of each coil, by which means silver solder joints, in copper, are made between the coils.

STATIC CONDENSERS

In this field there has been increasing activity in the application of condensers for power-factor improvement of industrial loads. In a large measure this has been due to the realization that economic advantages may be obtained under maximum-demand conditions by keeping the power factor high. Many instances may be given of power-factor improvement up to 0·95 lag or even higher, increasing the useful capacity of substations sufficiently to eliminate the necessity for increased capital expenditure on additional transformers, switchgear and distribution equipment, with a corresponding improvement in overall efficiency.

Condensers for power-factor improvement are now made with inherent losses of 1·3 watts per kVA, and the reliability is as high as that of transformers. An entirely reliable system of centralized automatic control has been developed in the case of machine-shop loads consisting of motor-driven machine tools. This system of automatic control makes it possible to install the condensers at suitable distribution points adjacent to the centres of gravity of various sources of low power factor, which arrangement gives the maximum reduction in I^2R losses. At the same time the total kVA of condensers in circuit is automatically regulated to maintain the overall power factor at a value as near to unity as is desired. In connection with high-frequency induction furnaces, condensers have maintained their position as the only practicable method of power-factor improvement. Development of high-voltage X-ray apparatus for medical purposes has involved the construction of d.c. generators which are almost invariably built on the well-known voltage-multiplication principle, which requires high-voltage d.c. condensers operating individually at voltages up to 250 kV. Such generators have been constructed to deliver 1 million volts.

A number of high-voltage condensers for use as carrier-current cable condensers for feeding high-frequency currents in 132-kV grid lines have been constructed. It seems likely that condensers will be increasingly used for this duty in view of recent advances with design and improvements in the system of carrier-current tripping and lock-in protection. Another application for coupling condensers is for feeding carrier currents through medium-voltage supply systems where carrier currents at various frequencies are used to operate resonant relays controlling air-raid warnings, lighting and water-heating loads.

Increasing attention has been directed to testing of insulating materials and complete electrical equipment under impulse voltages. Numerous high-voltage impulse generators, some of them of very high discharge energy have been built on the Marx principle, and its modifications—this involves the employment of a large number of high-voltage d.c. condensers. Similar impulse generators for the production of exceedingly large currents up to 100 000 amperes at a moderately high voltage have also been constructed.

Pressboard insulated with a suitable mineral oil has been introduced and largely used as a substitute for paper, especially in the construction of high-voltage condensers, where the plate form of assembly is employed to the best advantage.

Some developments have taken place in the nature of the impregnating medium. Petroleum jelly and transformer oil are still largely used, but chemical research has resulted in the production of new liquid impregnants possessing the advantage of a high specific inductive capacity (about double that of transformer oil), combined with non-inflammability.

TRENDS IN THE APPLICATION OF ELECTRICITY IN BRITISH SHIPS*

By G. O. WATSON, Associate Member.

GENERAL

As one might expect in a branch of the industry noted for its conservatism and already well-established on a satisfactory basis, innovations in the fundamental character of the installation have been lacking and, since the last review on this subject by Col. A. P. Pyne,† progress has been mainly in the direction of improvement in quality of materials and methods of installation. If one were dealing with world shipping, however, an exception to this statement would have to be made in deference to the important developments in Germany in the use of alternating current for Diesel-electric propulsion plant. This development is one of outstanding importance and, from the information available, has been successful and extensively applied.

The most notable event, in the opinion of those closely concerned with marine installations, has been the issue of the Third Edition of the Institution's "Regulations for the Electrical Equipment of Ships" in 1939; this will be dealt with more fully in a later paragraph.

The uses to which electricity is put continue to extend, and any development in the direction of new appliances finds a ready demand in ships, particularly those catering for passengers. It is observed, in general, that provisions made for the benefit of passengers involving the use of electricity, whether for illumination or for the innumerable appliances and services evolved for utility, personal comfort and convenience, are usually considerably in advance of any to be found on land, either in hotels or in private homes. It is quite common to find suites of cabins equipped with telephone, fan, electric heating, illuminated shaving mirrors, call bells, sockets for electric irons, curling tongs, etc., and electric clock, berth lights and so on. The principal innovation during the past 3 years has been the introduction of fluorescent tube lighting in public rooms, cocktail bars and the like.

RULES AND REGULATIONS

The Committee appointed by the Council of The Institution for the purpose has been engaged for some time past in the revision of the "Regulations for the Electrical Equipment of Ships" and the Third Edition was published in September, 1939.

It represents a very considerable advance on the Second Edition dated June, 1926, taking cognisance, as it does, of the improvements in technique, increased kilowatt capacity in passenger ships and the greater utilization of electricity in all directions, which had taken place in the intervening period. This is not the place for, and space does not permit, a comprehensive review of the important amendments which have been incorporated in the Third Edition, but a few points are worthy of mention.

Growth in the size and capacity of installations has brought with it problems of distribution involving the use of sub-distribution boards, which in many cases are larger than the main switchboard which was customary when the Second Edition was issued. Busbars to carry 10 000 amperes are quite common nowadays, and the requirements for the sub-division and protection of outgoing circuits have been overhauled and clarified.

The section dealing with tankers has been considerably amplified to specify more precisely the type of fitting and enclosure and the method of running cables appropriate to various parts of the vessel, having regard to the risks involved.

Cables have received special consideration in view of the advances made by cable makers, who can now offer cables which, in construction and materials, are superior to those available when the previous issue was drafted. This has enabled types which have proved unsatisfactory to be eliminated, such as those having pure rubber next to the conductor, armouring without the use of lead sheathing, flat lead-covered twin and 3-core cables, and tough-rubber sheathing for permanent wiring. Improved types which are now required are, for instance, lead-alloy sheath as opposed to the use of pure lead, "HR" sheathed cables, rubber wormings for multicore cables, and the use of fire-proofed braid overall.

The standards established by the British Standards Institution are utilized wherever appropriate, and the Regulations generally have been brought into line with the Eleventh Edition of the I.E.E. Regulations for the Electrical Equipment of Buildings.

An important addition to the Regulations is an entirely new section on electric propulsion plant, based very largely on similar Rules issued in 1934 by Lloyd's Register of Shipping.

Good progress has been made in an effort at international standardization, through the medium of the International Electrotechnical Commission. Several meetings have been held and a considerable measure of agreement has already been reached.

In case it might be thought that too much attention is paid to Rules and Regulations, it is as well to remember that indirectly they become mandatory in shipbuilding. The British Classification Societies and the Board of Trade use the I.E.E. Regulations as the basis for their Rules, and their surveyors inspect the installations during construction to see that in all vessels classed with them or built under their supervision the requirements are observed.

SUPPLY SYSTEMS

In British ships direct current continues to be the only system used, except where it is necessary to convert to alternating current for special purposes, such as discharge-

* A review of progress. † *Journal I.E.E.*, 1936, 78, p. 178.

tube lighting, gyro-compasses and so on. For several years no British ship has been installed with alternating-current distribution or with a higher direct-current voltage than 220.

CABLES

As cables and their installation constitute on the whole the largest single item in the cost of equipment and maintenance, it is not unnatural that they continue to receive considerable attention, and it is satisfactory to note a steady improvement in quality both as to the cables themselves and in the method of installation.

The quality of rubber has undoubtedly improved as a result of better methods of sampling and testing, improved technique in mixing and composition and better control during manufacture and vulcanization, resulting in a more uniform and reliable product. The use of pure rubber next to the conductor has at last gone by the board. Research has also shown the advantage of lead alloys in resisting cracking due to vibration, and the composition of these alloys is now standardized.

A sheathing known in the trade, for want of better nomenclature, as "HR type" gains ground as a result of roughly 10 years' experience. It is a rubber composition which behaves well in the presence of diesel oil and, while preserving the dielectric from influences of this nature, is not subject to cracking.

A new fireproof cable with a highly compressed refractory mineral insulating material, copper-sheathed, has made its appearance and has been tried out on a relatively small scale, and during the short period it has been in service it has proved successful.

Sealing the ends of all types of cable, including those insulated with rubber, has proved beneficial and will undoubtedly be adopted more extensively in the future. Experience has shown that the majority of cable failures occur within 6 ft. of the ends owing to moisture creeping up the strands and also from another cause, namely, too high a temperature at the terminals. The latter is being taken care of by stricter attention to the temperature of switch and, more particularly, fuse terminals. No radical change has taken place in the method of installing cables, but there has been a distinct improvement in the method of clipping and fixing, and more care and supervision is given to the pre-selection of suitable locations free from heat and oil.

The fixing of cable clips to steel bulkheads, requiring some thousands of studs, has led to the development of an interesting welding equipment to replace drilling and tapping. The studs are of naval brass, $\frac{1}{4}$ in. and $\frac{5}{16}$ in. diameter and they are welded to the steel plate. The duration of the arc is about $0 \cdot 25$ sec. with $\frac{1}{4}$ in. studs, and slightly longer with $\frac{5}{16}$ in., and the current is approximately 230 amperes (d.c.). The dynamo must provide this current without appreciable reduction of voltage, and is usually either a 65-volt or 110-volt machine used in conjunction with a limiting resistance.

INTERNAL COMMUNICATIONS

More extensive use is being made of telephones and all large passenger ships now have an extensive telephone system, frequently having a telephone in each first-class or de luxe cabin. As an example the Shaw, Savill and Albion Co.'s new liner "Dominion Monarch" has about 400 lines. The passengers make direct verbal requests for service, and the switchboard operator is the clearing house for such calls, which may be dealt with directly or passed on to a service clerk or to deck pantries or to stewards' telephone extensions. There are telephones in small alcoves in the corridors, and associated with each is a lamp which is an indication that a steward is required on the telephone, the lamp signals being repeated on other lamps in pantries, etc. Passengers may communicate with other passengers and, immediately the ship docks, a connection is made to shore so that passengers can telephone through.

A development of considerable importance, since it is independent of any source of electrical supply in an emergency, is that of sound-powered telephones. The necessary electrical energy required for the transmission of speech from one station to another is actually generated by the voice of the operator. Calling is provided by means of a generator of special design driven by ratchet and lever, to make a call it being only necessary to pull a lever through 90°, producing a loud ring for approximately $1\frac{1}{2}$ sec. at the distant station. A special feature due to the fact that there is no background noise is that quality of speech is of a very high degree of intelligibility and immune from interruption caused by local noises. This dinproof property is of great importance in connection with instruments housed in noisy situations such as engine rooms. Tests have shown that it is possible to transmit and receive messages distinctly with a noise level of 120 phons, which is generally considered to be about the limit of human endurance.

Call systems other than the telephone are still in use in cabins, and in these the luminous-call system in conjunction with call buzzers is rapidly ousting the old type of bell installation with shutter or flag indications.

WIRELESS COMMUNICATION

Growth in the number of radio services has brought with it many ancillary problems, particularly that of the suppression of man-made interference, which is accentuated by the low field strength of signals. These services include direction-finding, telegraphy, radio-telephony and broadcast reception. Also allied to this problem is that of diffusion of broadcast programmes throughout the ship on loud-speakers.

A better appreciation of these services and the problems involved will be obtained by considering briefly the equipment of the latest large passenger vessel to be completed, namely the new "Mauretania" commissioned in 1939.

Long-wave, medium-wave and short-wave transmitters are all crystal-controlled, and a unique system of remote frequency adjustment is used to select the requisite transmitting frequency. This system of frequency selection enables any one of 18 short-wave frequencies, 8 medium-wave frequencies and 8 long-wave frequencies to be selected at will with a delay not exceeding 10–15 sec., and it has the great advantage that it can be applied to most normal designs of transmitter.

Highly selective receivers permit the simultaneous use of three telegraphic channels, news and weather reception, as well as broadcast bulletins, without any mutual

interference between these five services, while at the same time direction-finding and auto-alarm services are completely independent of the normal communication channels.

The ship-to-shore, or inter-ship, telephone service is carried out on a special telephone transmitter in which the low-frequency response is suitable for broadcast purposes. For normal telephone requirements this band width may be reduced to $\pm$ 3 kc./s. and privacy equipment is standardized.

A limiter and noise reducer also form part of the standard telephone equipment, while the telephone receiver is equipped with automatic first-oscillator frequency control as well as automatic volume control.

An interesting feature is the use of three separate high-frequency amplifiers associated with a common intermediate-frequency amplifier in the telephone receiver. This permits three stations to be tuned in on the high-frequency side of the receiver and reduces considerably the time of " lining-up " for a telephone circuit.

The requirements of safety of life at sea have been met by emergency and lifeboat installations, and a public address system installed in the vessel enables items of special interest to be disseminated throughout the ship.

At the opposite extreme of size, marked advances have been made in the low-power telephone transmitters for use in small vessels which are not compulsorily equipped with wireless apparatus—such as trawlers. These instruments are usually arranged so that they can transmit only on certain predetermined frequencies, which are stabilized by crystals. It is common practice to provide five telephone channels in this service, one being the international telephone distress wave, two the British ship-to-shore waves, and the other two the British inter-ship waves.

The replacement of spark transmitters by valve transmitters, the introduction of crystal and stabilized loose-coupled circuits, and the rigorous application of international regulations, have during the last few years led to an immensely increased efficiency in the use of marine frequency allocations.

From the 1st January, 1940, no spark transmitter of 300 watts alternator output or over will be permitted, so that there has been in progress a change-over in the type of transmitter in use in a very large number of ships. Valve transmitters giving I.C.W. or C.W./I.C.W. are being substituted.

The use of short-wave transmitters enabling ships to maintain daily touch with their home country has extended very much in recent years. From being almost confined to large passenger ships, their use has spread to cargo ships and particularly to tankers.

The use of radio-telephone sets, operated by members of the crew unskilled in morse, has extended very much in British coastal shipping. This has been due to the facilities given by the G.P.O. at their coast stations, Humber and Seaforth radio now being available through Portpatrick. The latter gives facilities superior to the others at present, in that it provides facilities for duplex communication instead of simplex conversations, such as are available through the other stations mentioned.

Stabilized frequency of emission has promoted the use of highly selective receivers having stabilized frequencies of reception and made possible automatic services, which previously had demanded skilled operation.

The introduction of the " link telephone service " by the G.P.O. has increased enormously the value of the 2 Mc./s. telephone band in marine mobile services, and the use of true duplex services between ships in this band is steadily expanding.

Investigation of aerials and aerial constants is proceeding and it is becoming normal to use a single aerial for transmission or reception on all wavelengths. On the " Mauretania " the number of aerials has been decided not by the frequencies required but only by the number of simultaneous services demanded.

As on shore, so on sea but to a much greater extent, the interaction of electrical apparatus on communication services is extremely intense, and it is a matter of considerable congratulation to the I.E.E., the B.S.I. and other official bodies of opinion, that during the last few years both officially and unofficially the various representatives of electrical and communication engineering have begun to understand one another's problems and have set out to reduce to the utmost extent the interference between these two widely different sides of the electrical industry.

The latest developments in wireless apparatus are generally directed towards improved frequency stability, improved efficiency, and reduced costs.

In these directions the introduction and use of crystals having low-temperature coefficients has largely removed the need for thermostatically-controlled heat chambers, while at the same time the development of stabilized loose-coupled circuits in both scientific and commercial establishments will in the future enable the advantages of high stability together with accurate frequency selection to be applied to marine communications.

The efficiency of the average marine transmitter is of a high order, but the eternal questions of size, weight and power consumption are invariably under review by the engineers responsible for these matters. The more modern low-temperature low-voltage valves are being adopted, and all new developments in materials and processes, both electrical and mechanical, have become part of the development engineer's stock-in-trade.

The research problems of to-day are largely concerned with the minute investigations of innumerable problems which, though in themselves not of a major order, may ultimately produce a matter of first-rate importance.

The most serious of these is still the question of a stabilized frequency control which permits of the most minute and accurate adjustment. Another is the difficulty of obtaining any precise frequency adjustment from a loose-coupled oscillator immediately upon switching on. Further problems are concerned with the commercial application of scientific discoveries, e.g. visual direction-finding, remote frequency control, penetration of fog by the reflection of electromagnetic rays, applications of television, and variable-frequency modulation.

The field of research in this subject is wider to-day than ever before, not only because the subject is still in its infancy but also because every day a new tool or instrument is being handed to us for inspection and use.

NAVIGATION

Electricity plays an ever increasing part in aiding the navigator, reducing and eliminating risks. It makes possible the use of gyro compass, depth-sounding, indication of propeller speeds and revolution counting, direction-finding, time signals, weather reports and gale warnings, reception of navigation warnings such as the extinction of signal lights and movement of buoys, etc., with a degree of accuracy and reliability that leaves little to be desired.

Progress in depth-sounding has steadily advanced; the number of depth-sounding equipments rises steadily and the performance of apparatus continuously improves.

The method of indication of depth is in some cases visual; in others a permanent or semi-permanent record is obtained. There are two chief methods of obtaining soundings by echo from the sea bottom—the piezo-electric and the magneto-striction—and these have both been described at length in the *Journal.*

The piezo-electric installations are usually fitted in trawlers, accompanied by visual indicators. As a rule, ocean-going ships are equipped with recorders, almost always electrolytic. In many cases a visual indicator is fitted as well as a recorder.

The magneto-striction equipments, being much more easily adapted to the use of high power, are usually installed so as to operate through the skin of the ship, thus avoiding piercing the hull and making it possible to fit the equipment while the ship is afloat. For this reason they are very popular in big ships.

Development and research have been in the main directed to overcoming the two chief difficulties which beset to some extent all types of echo-sounding devices.

In the first place, there is always a difficulty in measuring very shallow depths of water under a ship, though the meaning to be attached to the expression "very shallow" varies with the conditions.

Apparatus for use in launches equipped for surveying duties will operate satisfactorily at depths below the projector of about 2 ft., and for use in ocean-going ships, where it is required for navigation rather than for surveying, will operate satisfactorily at depths of about 1 fathom below the keel.

The second difficulty is due to the comparative opacity of disturbed water to supersonic pressure waves, making it difficult to obtain satisfactory operation in ships which are very light in the water, especially in fast ships trimmed very light forward. There are, however, certain positions under the bottom of a ship which are comparatively clear of trouble of this nature, and projectors can now be so placed as to ensure good working under almost any conditions of draught, speed and trim.

Owners of fishing vessels find a dual advantage in that depth sounders will indicate the presence of shoals of fish.

The use of the swinging frame in connection with direction-finding equipment is on the wane and is giving way to the fixed frame aerial and internal goniometer coils. A very ingenious form of the latter is arranged to have a light signal on a compass card giving the bearing of the radio beacon direct and obviating the necessity of using earphones to determine the minimum signal. This apparatus is claimed to be correct within 1° when the vessel is out at sea, and uses stations within 20 miles.

It is well known that in certain geographical positions the magnetic compass is unreliable. For instance, it is now essential for ships trading to Hudson Bay to be equipped with a gyro compass. One reason is the basis rates to vessels so fitted—the normal rate of 15s. % on the insured total loss value being increased by an additional 15s. % for ships not having gyro compasses—and secondly the proximity of the magnetic North Pole affects the standard compass so seriously that no reliance can be placed upon it.

A sea-temperature recorder has been developed for the purpose of forecasting fog conditions. A difference of 1 or 2 deg. in the surface temperature of the sea may indicate the probability of imminent fog, and a high degree of accuracy in temperature recording is therefore necessary. This particular equipment depends on the known relation between the resistance of pure platinum and its temperature. The thermometer consists of a glass tube upon which is wound a fine platinum wire having a resistance of 2 200 ohms at 32° F. This is enclosed in a streamline casing in such a way that, when the whole is towed in the sea, water flows through the inside of the glass tube and ensures that the platinum winding quickly attains the true temperature, which is recorded by means of a Wheatstone bridge. The recorder must be installed in a position free from excessive variations of temperature, as, owing to the high sensitivity required, it has been impossible to eliminate errors due to changes of air temperature at the recording position.

All large naval vessels carrying aeroplanes have their own meteorological offices on board for the purpose of supplying the aircraft personnel with weather forecasts; and the behaviour of wind is an important factor, a great deal being learned from the velocity and duration of gusts of wind.

An ingenious electrical method is now available for determining and recording the true direction and true velocity of the wind, making correction for the course and speed of the ship. Briefly stated, the method depends on a wind-speed generator in conjunction with compass-bearing and ship-speed measurements. A vector analyser, utilizing alternating-current supplies, resolves the resultant "wind as felt" into its two component vectors, i.e. that due to the course and speed of the ship and that due to true direction and true speed of the wind, these latter being recorded on a chart.

BOILER PLANT

Following land practice, though not to the same extent, there is a trend towards more extensive use of gauges and instruments as an aid to the achievement of greater economy and improved performance. With the increased use of water-tube boilers there is a demand for salinometers, and in this connection an unusual application is that adopted in the new French liner "Pasteur." The salinometer is used to control the flow of water from an auxiliary tank into a main tank, and to cut off the flow when the concentration of salt in the auxiliary tank reaches 10 mg. of sodium chloride per litre. An alarm bell gives warning when the valves have operated.

GENERATORS

There is a tendency to employ diesel-driven generators in conjunction with turbo-generators on steam-propelled vessels, so that during an overhaul, or while the vessel is lying up in port, the boilers can be completely shut down. The latest example of this is on the "Andes," in which vessel there are three turbo-driven generators each of 1 000 kW, and two diesel-driven generators each of 500 kW, in addition to the 100 kW diesel-driven emergency set.

Table 1 is an interesting analysis of the equipment of new vessels for which electrical plans were dealt with by Lloyd's Register of Shipping during the years 1936–37–38.

Table 2 gives an idea of the generating plant in large vessels, built or building, during the period under review.

A development more peculiar to the Continent is the application of dynamos driven from the propeller shaft and used in conjunction with automatic voltage regulators.

Table 1

Total generator capacity per ship	Number of vessels			Percentage of total number			Aggregate kW		
	1936	1937	1938	1936	1937	1938	1936	1937	1938
15 kW and under	126	136	103	38	37	32·7	732	907	695
16 kW to 100 kW	121	116	131	36·2	31·7	41·6	4 404	4 745	4 407
100 kW to 200 kW	29	44	26	8·6	12	8·2	3 927	6 758	4 182
200 kW to 500 kW	45	46	29	13·4	12·5	9·2	14 654	16 010	9 134
Over 500 kW	13	24	26	3·8	6·6	8·2	15 157	40 258	36 538
Totals	334	366	315				38 874	68 678	54 956

DECK AUXILIARIES

A number of diesel-engined trawlers have been equipped with electric trawl winches, varying in size up to 7½ tons pull. They are in use by British, French, Italian, Polish and other owners, and operating experience has been very satisfactory. The trawler skippers have expressed themselves as very pleased with the flexibility, convenience and safety of the winches, and there is probably no class of service which is more severe on electrical plant, since the ships operate in all weathers, often iced up in winter and with no skilled attention.

There is a slowly growing demand for more flexible forms of speed control for such equipment as winches, capstans, etc., leading to the use of booster and reducer systems which are fitted with a modified form of Ward–Leonard control. Shipowners are realizing more and more that higher speeds of lifting and lowering, combined with creeping speeds for breaking out cargo and for landing, are worth a little extra capital expenditure, since these features speed up cargo-handling and minimize the risk of damage.

A development which has made considerable progress on the Continent is the use of electric deck cranes in place of winches. A recent installation of this nature is to be found on the motor vessel "Oranje."

With regard to windlasses, there is a tendency in larger vessels to go in for split drives, e.g. where a windlass would normally have a 100-h.p. motor for a single motor drive there will be two 100-h.p. motors, only one of which is in use at a time. Where the windlass is combined with capstans, both can be used for the latter, one for the starboard capstan and one for the port capstan. Also they can clutch in the windlass to one or other of the motors, thus giving them the benefit of the windlass warp ends as well as both the capstans. In certain cases it is possible to arrange that both motors can be clutched into the windlass so that extra heavy pulls can be obtained when necessary.

REFRIGERATION

The chief progress has been in the direction of auxiliary devices such as recording apparatus, ozone generators, etc.

Ozone-generating devices are of considerable value with certain classes of cargo on account of the powerful oxidizing properties of ozone, which instantly combines with and destroys any deleterious organic matter present in the air or in the foods or fabric, walls, etc., of the storage chamber. It is also useful as a de-odorizer for removing unpleasant smells, and in this respect it is invaluable for removing the odour of one cargo before loading another of a different character which might be spoiled by contamination.

For correct control of refrigeration, accurate temperature recording at all parts of the chamber is necessary; electric distance thermometers meet this requirement.

A comparatively new development of electrical instruments is in connection with gas analysis in refrigerated chambers. Frozen meat and chilled meat are two very different things, and recent developments have greatly altered the technique of storage. Up to quite recent years nearly all chilled meat, i.e. meat which has been subjected to only comparatively slight refrigeration between 28° and 30° F., came from the Argentine, the journey being sufficiently short to make this possible.

The much longer transit from Australia and New Zealand made it necessary, in preventing deterioration, to freeze meat at temperatures between 10° and 15° F., resulting in a much inferior-quality product. As a result of intensive research it has been found that chilled meat stored in an atmosphere containing up to 10 % CO_2 will

Table 2

GENERATING PLANT FOR PRINCIPAL BRITISH-BUILT VESSELS OF 10 000 GROSS TONS AND UPWARDS, COMPLETED OR UNDER CONSTRUCTION IN 1936, 1937, 1938 AND 1939 (UP TO NOVEMBER)

Year of completion	Name of vessel	Description	Gross tons	Auxiliary generators for lighting and power: No. and capacity (kW) of sets	Auxiliary generators for lighting and power: Total kW*
1936	"Queen Mary"	Steamship	80 774	7—1 300	9 100
	"Athlone Castle"	Motorship	25 564	5— 700	3 500
	"Stirling Castle"	Motorship	25 550	5— 700	3 500
	"Dunedin Star"	Motorship	11 168	3— 340	1 020
	"Sydney Star"	Motorship	11 095	3— 330	990
	"Dilwara"	Motorship	11 080	4— 220	880
	"Kanimbla"	Motorship	10 985	4— 300	1 200
	"Clan McArthur"	Steamship	10 528	2— 100 1— 60 1— 30	290
	"Dunottar Castle"	Motorship	15 007	4— 350	1 400
	"Dunvegan Castle"	Motorship	15 007	4— 350	1 400
	"Awatea"	Steamship	13 482	3— 450	1 350
	"City of Benares"	Steamship	11 081	3— 100	300
	"Melbourne Star"	Motorship	11 076	3— 350	1 050
	"Essex"	Motorship	11 063	4— 300 1— 150	1 350
	"Perthshire"	Steamship	10 496	2— 100 1— 60 1— 30	290
	"Clan Macauley"	Steamship	10 492	2— 100 1— 60 1— 30	290
1937	"Brisbane Star"	Motorship	11 076	3— 350	1 050
	"Sussex"	Motorship	11 063	4— 300 1— 150	1 350
	"Stratheden"	Steamship	23 722	3— 500	1 500
	"Orcades"	Steamship	23 456	3— 550	1 650
1938	"Capetown Castle"	Motorship	27 000	5— 700	3 500
	"Strathallan"	Steamship	23 722	3— 500	1 500
	"Dunera"	Motorship	11 162	4— 220	880
	"Beacon Grange"	Motorship	10 199	4— 260	1 040
	"Regent Tiger"	Motortanker	10 176	1— 5 2— 25	55
	"Waimarama"	Motorship	11 092	4— 300	1 200
	"Canton"	Steamship	15 784	3— 450	1 350
	"Ettrick"	Motorship	11 279	4— 200	800
1939	"Durham Castle"	Motorship	17 388	4— 450	1 800
	"Dominion Monarch"	Motorship	27 155	2— 100 5— 600	3 200
	"Queen Elizabeth"	Steamship	85 000	4—2 200	8 800
	"Suffolk"	Motorship	13 890	4— 300 1— 250	1 450
	"Auckland Star"	Motorship	11 400	3— 330	990
	"Wellington Star"	Motorship	12 382	3— 330	990
	"Waiotira"	Motorship	11 090	4— 300	1 200
	"Thiara"	Motortanker	10 364	2— 20	40
	"Andes"	Steamship	25 689	3—1 000 2— 500	4 000
	"Pretoria Castle"	Motorship	17 392	4— 450	1 800

* Excluding emergency sets.

keep sweet and free from moulds or bacterial growth for much longer periods. This has opened up a vast avenue of trade with the Antipodes, with the result that many new ships have been built in recent years solely for this trade.

In practice, CO_2 is admitted to the holds from steel cylinders or bottles, and the gases in the hold are then sampled and analysed. It is important that the concentration does not get excessive, since the appearance of the meat would be affected and result in complaints and lower prices.

With fruit certain varieties of apples, for instance, cannot be held in ordinary cold storage for the necessary period of time unless a temperature lower than 37° F. is maintained, and this unfortunately is so low as to cause damage to the tissues of the more sensitive varieties so that the fruit collapses soon after removal from the store. Again the solution is found in the use of CO_2, and the proportion has to be carefully regulated to avoid "brown heart." The most recent developments go even further by the independent adjustment of the constituents of the atmosphere of the store, that is, CO_2, oxygen and nitrogen.

Recording and indicating instruments are available to indicate the percentage of both CO_2 and oxygen, and the principle depends on the thermal conductivity of the gases. These are led through a metal tube about 4 in. long and about $\frac{1}{4}$ in. diameter. A fine platinum wire is stretched along the central axis of the tube and is warmed by an electric current so that its temperature, and therefore its resistance, will depend upon the magnitude of the heating current and the thermal conductivity of the gas.

Electric distance thermometers are also used for checking the temperatures in the chilled holds of fish trawlers, and it is of interest to note that special thermometers are obtainable which are so small that they can be inserted in the actual fish to ensure accuracy of supervision.

CONTROL GEAR

Although there have been isolated examples of the use of the multiple-motor starter, it has not made appreciable headway mainly on account of cost, in which is included installation cost and wiring. The systems comprise a master unit by means of which each of the motors in the engine room can be started, one at a time, by push-button control. After each motor is started and running it is connected to the "running" busbars and the starting unit is then free for the next motor.

In other cases, as in the "Queen Mary" and "Mauretania," certain large motors have individual automatic starters grouped in a central control room in order to keep the engine room as free as possible.

BATTERIES

Secondary batteries are being used to an increasing extent for emergency equipments for vessels of 10 000 gross tons and over. In the case of motorships it is common to provide an emergency supply for both steering and lighting, which, under normal conditions, are taken direct from the ship's supply and on failure of this the emergency circuits are transferred to the battery by means of an automatic switch. The new "Mauretania," for instance, has a very large battery consisting of 190 cells with a capacity of 240 ampere-hours to maintain a lighting load of 200 amperes for $\frac{1}{2}$ hour within very close voltage limits.

An installation of special interest was introduced in the new Cowes Ferry. In brief it is a combination of a battery and a diesel generating-set for propelling the ferry, the battery helping the diesel-electric equipment with the heavy load required for starting. It afterwards receives a charge to compensate for the discharge when the ferry is under way. Though the rating of the generator is only 15 kW the motor is rated at 40 h.p. and can give 2·6 times full-load torque when starting with the aid of the battery. The ferry makes approximately 200 crossings per day and high starting torque is necessary should the ferry accidentally run ashore at the terminal points.

For acid batteries, moulded containers have now superseded the older types of construction.

HEATING

This subject has received special consideration during recent years for two reasons, first on account of fire risks and, secondly, as a result of legislation.

It has been proved that many fires were started by clothing, bedding, etc., being dried or aired before electric radiators and coming into contact with the heating elements. Consequently, Lloyd's Register of Shipping amended their Rules to require heaters in cabins to be of the convector type, and this gave an impetus to the improvement and development of this form of heater. A later amendment to their requirements permits the radiant type, provided the design is satisfactory from this point of view.

Government legislation now requires heating facilities in crews' quarters and lays down the temperature to be maintained under specified conditions. This in turn has led to a closer study of the problem, which has been very fully dealt with in Mr. H. C. MacEwan's paper on "Electric Heating for Merchant Ships."*

FUSES

Large quantities of fuses are used in ships and it is not unnatural that cognizance has been taken of the progress and research which has taken place in recent years. Large passenger vessels such as the "Queen Elizabeth" and the "Mauretania" have a total generator capacity of 40 000 and 14 500 amperes, respectively, at full-load rating 220 volts d.c. with distribution fuses on the main switchboard. Consequently the fuses must be of the highest grade.

Then again, cable failures, as already mentioned, usually occur near the ends of cables, and this can be traced in some cases to heat conducted from fuse terminals. An effort is therefore being made to eliminate all fuses which do not comply with the requirements of B.S. No. 88.

SHIPBUILDING

The use of arc welding is continually expanding, a small number of all-welded hulls having been built, but even in

* *Journal I.E.E.*, 1938, **84**, p. 421.

riveted hulls welded portions and structures are growing in number and the technique of shipbuilding and naval architecture is being continually modified to allow the builder to avail himself of advantages in weight and cost.

In portable tools the use of portable electric drills, grinders and reamers, is extending, a favourite tool being that operated by a high-frequency a.c. supply. To keep weight within reasonable limits it is necessary to have a higher rotor speed than can be obtained with a frequency of 50 c./s., and these tools are operated at 180 to 200 c./s. which gives a maximum rotor speed of about 12 000 r.p.m. Maintenance costs are low owing to the absence of commutators and the simplicity of the squirrel-cage rotor. A 75 % reduction in power costs compared with those of compressed-air drills is claimed, largely on the score of efficiency but partly as a result of better performance, i.e. number of holes drilled or reamered per day.

ELECTRIC PROPULSION

Unfortunately, there has been no large equipment ordered or installed in the United Kingdom in recent years, orders, few in number, being confined to tugs, ferries and similar small craft. The most interesting of these is the Trinity House pilot cutter "Vigia," which employs for this application an entirely new system of control. There are three 66-kW 250-volt 1 650-r.p.m. constant-voltage generators coupled to diesel engines and connected in parallel, and these operate a 240 s.h.p. shunt-wound propulsion motor. Reversal is effected by reversing the propeller motor field, a booster operating on the field and driven by the motor effectively limiting the peak loads to safe values during propeller acceleration and reversal. A feature of this equipment is the space occupied, which is considerably less in this particular instance than that required for any other form of drive.

It is noteworthy that during the last 3 years there has been considerable activity in Germany in the application of electric propulsion. The technical directors of their leading shipping concerns state that they have examined the economics of the question and are satisfied that, for their purposes, electric propulsion is cheaper to run than other types. An interesting sociological sidelight is that German crews are required by law to have definite hours of leave while their ships are in port, and it is claimed that this is rendered possible for engine-room staffs by the adoption of multi-unit electric drive. With multi-engine diesel-electric drive it is possible to shut down one engine without appreciable loss of speed and to proceed with adjustments and maintenance while the vessel is at sea.

As already indicated, Germany has developed diesel-electric drive with alternating-current transmission, coupled with a.c. auxiliaries, development elsewhere having been confined to direct current, though it is understood that British firms are prepared to supply a.c. equipments. Table 3 indicates the extent to which alternating current has so far been used.

In the U.S.A. the application of electric drives has slowed up, mainly owing to reductions in shipbuilding activity, but notable additions in recent times are those of the Atlantic Refining Co.'s new 18 100-ton welded tankers "J. W. Van Dyke," "E. J. Henry" and "Robert H. Colley." They have turbo-electric drive, 625 lb. gauge steam pressure, and temperature 825° F., or in the "E. J. Henry" 910° F. The turbo-generator is rated at 4 500 kW, 2 300 volts, 3 600 r.p.m., and it can also supply auxiliaries, which, in this instance, are on the a.c. system. In marine drives rapid and frequent variations in load from no load to full load take place while manoeuvring, and the period at no load may be a few seconds or several minutes, necessitating very close boiler control.

In land installations automatic combustion control is commonly practised, but not so at sea. These installations are noteworthy in that they are provided with complete automatic combustion control, including superheat control.

Another foreign development of special interest is the application of electric slip couplings, originally developed in Sweden but since made also in Holland and the United States. The use of slip couplings arises from the application of standardized medium-speed engines in conjunction with reduction gears. The usual combination is two or four Diesel engines coupled through slip couplings to a common gear wheel, the coupling providing for slight errors of alignment and damping out any torsional vibrations which might be transmitted to the gear teeth and shafting. The coupling is of simple mechanical construction having a fairly wide air gap of the order of 10 mm., one half comprising a multi-pole magnet system energized by direct current and the other half having a squirrel-cage winding. The slip is of low order, about 1 to 1·5 %; consequently the efficiency of transmission is very high, being of the order of 97 % when excitation is taken into account.

Table 3

LIST OF VESSELS EQUIPPED WITH DIESEL-ELECTRIC ALTERNATING-CURRENT PROPELLING SYSTEMS

Name	Gross tons	Number of screws	Number of engines	Type of engine	Total s.h.p.
"Wuppertal"	6 137	1	3	2SCSA	9 800
"Patria" ..	15 000	2	6	2SCSA	15 000
"Steiermark"	9 400	2	4	4SCSA	16 500
"Robert Ley"	27 288	2	6	2SCSA	8 800
"Osorno" ..	6 951	1	3	2SCSA	7 000
"Huascaran"	6 951	1	3	2SCSA	7 000

MISCELLANEOUS

There is a tendency with some owners to adopt pipe ventilation wherever possible for motors operating in oily atmospheres, the cooling air being obtained from a convenient ventilating trunk. Oily vapour causes trouble with commutation and insulation.

There is scope for improvement in the insulation of motors for marine use, since, broadly speaking, present methods rely on the thoroughness of the impregnation to exclude oil and salt-laden atmospheres, and there is no

practical test (other than length of service) which can be applied to impregnation. Substitutes for ordinary cotton-covered wire have been produced which are practically non-hygroscopic, and progress in this direction has been made, but there is need for further experience on this subject.

Extended use has been made of electrolysers in recent years, particularly in liners, troop and hospital ships. The apparatus consists of a series of plates in a vat of sea water, the passing of a current through which produces sodium hypochlorite by freeing some of the chlorine content of the sea water. Sodium hypochlorite is a powerful disinfecting, deodorant and bleaching agent, which has been widely adopted on account of its low cost combined with its own non-poisonous and generally harmless nature. Five minutes' electrolysis will yield 6–7 gallons of sanitary fluid containing 2–2½ parts per 1 000 of available chlorine.

With regard to lighting, there is little to report except a tendency towards simplicity instead of the rather ornate fittings in use a few years ago. More attention is being given to details of ventilation of large fittings, and in this connection natural ventilation has proved more suitable than forced ventilation by extractor fans, which have been found to cause an excessive accumulation of dust inside the fitting. Machinery spaces are more efficiently illuminated by lamps of high candle-power in suitable fittings, taking the place of small guarded pendants.

Air conditioning in passenger and cargo vessels has made rapid headway, and there has also been improvement in ventilation systems in general. Systems supplying hot and cold air, which may be regulated to suit individual requirements, are the general rule. Considerable attention has been given to the silencing of ventilating systems so as to reduce the annoyance and irritation caused by noise from ventilating motors and fans which is conducted along the ducts to cabins and public rooms.

A very comprehensive air-conditioning plant, capable of maintaining a temperature of 70° F. and a relative humidity of 45 % with an outside temperature of 10° F., is installed in the new "Mauretania." In summer, reductions up to 15 deg. F. in temperature and 35 % in relative humidity will be possible. The air-conditioning units are distributed throughout the ship and are automatically controlled by thermostats.

In switchboard construction there is a notable tendency to dispense with slate and marble and to use synthetic materials, which are more constant and reliable in quality. Systems of preferential circuits with selective tripping have been perfected and are more extensively used. They ensure that essential motors are retained in service and less-important circuits cut out in the event of the generators becoming overloaded.

The provision of means for fire protection and extinction has become more general, being almost universal in passenger and large cargo ships. Means for extinction include sprinkler systems, CO_2 and foam generators, allied with detectors giving audible warning of the presence of smoke or excessive heat. A further safeguard provided in many instances is so to arrange all feeders supplying ventilating fans and heaters that they can be switched off from the bridge, thus ensuring that in the event of a fire these systems in any particular section of the ship can be shut down immediately.

Electric lifts are extensively used, a large passenger ship frequently having seven or eight lifts for passengers, baggage, stores, engineers and galleys.

Several large electrically-driven scavenging blowers for diesel engines have been supplied. Those for the "Athlone Castle," "Stirling Castle" and "Capetown Castle" are rated at 590 b.h.p., 220 volts, and 2 800 r.p.m., and are capable of supplying 31 100 cubic ft. of free air per minute at 3·4 lb./sq. in. pressure. For the three Sulzer 12-cylinder 2-stroke engines, each of 12 500 s.h.p. at 145 r.p.m. to drive the passenger ship "Oranje," the scavenging blowers are each driven by two 500-kW d.c. motors. These, together with similar ones for the "Stockholm" and "Saturnia," are among the biggest yet built for drive by d.c. motors.

ELECTRO-PHYSICS*

By NORMAN CAMPBELL, Sc.D.

(1) NUCLEAR PHYSICS

(1.1) Research into transformations of the nucleus continues feverishly. Cyclotrons, which are now heavy engineering structures and require the service of a small power station, are being built in all countries to provide ever more intense streams of ever faster bombarding particles; 10 μA of α-particles of 40 MV (MV = million electron-volts) is about the present record. Their construction is encouraged by the increasing medical and biological use of artificial radioactives, radio-sodium as a substitute for radium in treatment and radio-phosphorus for physiological investigations. High-voltage installations, occupying vast halls, are also popular; though they are limited to some 2 MV, the velocity of the particles they yield is more uniform and therefore better adapted to quantitative investigations. γ-rays of 17 MV are available from the transformation of ^{7}Li when bombarded with protons.

There are less spectacular, but not less useful, advances in the technique of identifying the products of transformation. Geiger counters are becoming ever more simple and efficient; modifications of the Wilson cloud chamber are increasing greatly the period over which it is sensitive, and therefore the chance of observing any given event. Chemical technique is growing in delicacy; one ingenious device may be mentioned specifically. When an atom of one of the stable isotopes of bromine, ^{79}Br, captures a fast neutron and thereby becomes the radioactive isotope, ^{80}Br, the momentum it receives breaks it off from the rest of the molecule. The radioactive isotope is free and can be precipitated with silver; the stable isotope, being combined and unable to combine with silver, is left in solution. A separation of chemically identical isotopes by means of a chemical reaction is therefore possible.

The more general separation of isotopes—necessary in order that it should be known which isotope is reacting —has been greatly advanced by an ingenious combination of thermal diffusion and convection. A thin layer of gas is maintained between a heated inner cylinder and a cooled outer cylinder. In virtue of thermal diffusion, the lighter isotope is concentrated near the hot cylinder and is carried upwards by convection; the heavier is concentrated near the cold cylinder and is carried downwards by convection. The conditions need careful control, but when that is achieved the results are remarkable. Both isotopes of chlorine have been prepared of better than 90 % purity; a few years ago an enrichment of 10 % was a considerable achievement.

(1.2) The one really novel discovery that has been made did not require any new technique, but merely greater care in accepting " obvious " conclusions. It has been known for some years that the bombardment of uranium and thorium by both fast and slow neutrons produced several β-active elements. It was supposed, partly on chemical evidence, partly on general probability, that these were transuranic elements with atomic weight $A > 238$ and atomic number $Z > 92$, or the products of the disintegration of these transuranic elements. Two of the products were similar to radium and actinium, and were supposed to be isotopes of these elements. Very beautiful work by Hahn and his associates proved conclusively that the supposed radium was actually barium, whose atomic weight is not much more than half that of uranium; Joliot similarly proved that the supposed actinium was a rare earth. It appeared, therefore, that the uranium nucleus, after taking up a neutron, must split into two nearly equal halves; this suggestion was immediately confirmed by workers all over the world, who detected by several different methods the recoil of the two halves from each other with approximately equal velocity. Further chemical work has shown that a large number of elements are produced other than those originally detected, e.g. strontium, caesium, xenon, lanthanum, tellurium, iodine. Each consists of several isotopes, and some of them are arranged in mother-daughter chains. There is no longer any evidence for any transuranic element with $Z > 92$, though there must be at least one isotope of uranium with $A > 238$.

There is nothing really surprising in such a splitting of a nucleus with large value of Z. The mass defect† of uranium is much less than twice that of the known stable nuclei of about half its mass; energy, arising from the repulsion of the nuclear charges, should therefore be liberated by a split. Indeed, at one stage in radioactive theory, the puzzle was rather why atoms did not split into halves instead of merely losing fragments; but recent theories had overlooked this possibility, although it is seen now that Bohr's " liquid drop " theory (see below) might have contemplated it.

However, the greatest interest arose from another aspect of the matter. $^{238}_{92}$Ur contains 92 protons and 146 neutrons. Two stable elements of about half its mass contain about 100 protons and 138 neutrons; accordingly there will be a deficiency of protons and an excess of neutrons in the halves into which the nucleus splits. This may be, and certainly is to some extent, rectified by β-activity converting neutrons into protons. But it is also possible that free neutrons may be emitted; actually the emission of about 2 neutrons per splitting nucleus has been detected experimentally. Why then, it was asked by alarmists, does not all the uranium in the world explode, the neutrons liberated by the splitting of one atom causing the splitting of others?

The answer is apparently that a sufficiently large and

* A review of progress, 1936-1939.

† The mass defect of a nucleus is the difference between its mass and the sum of the Z protons and the $(A-Z)$ neutrons of which it is composed. According to the theory that underlies all nuclear research, this difference is proportional to the energy that would be liberated in the formation of the nucleus from these protons and neutrons.

pure mass of the isotope involved (it may be the rare isotope ^{235}Ur) might explode. But unless the mass were very large and very pure, the neutrons would escape or be absorbed mainly in nuclei other than the effective isotope, so that neutrons would not be produced as fast as they disappeared. There is thus no prospect of an unintended catastrophe; on the other hand, the controlled liberation of large quantities of atomic energy, so beloved of sensational novelists, seems a little less impossible than it did before the splitting of heavy nuclei was discovered.

(1.3) Thorium splits under fast (but not slow) neutron bombardment in the same way as uranium; so probably do other nuclei of very large Z. Otherwise all known transformations result in the emission of either a neutron n, a proton p, a deuteron d, a helion α, or a photon γ (or sometimes of more than one of these); that is to say, a particle which, if it is part of the nucleus and the nucleus is not very small, is only a small part of it. These emitted particles are also those used as projectiles for inducing the transformation. The reasons why heavier particles have been so little used as projectiles is partly experimental and partly based on the belief that the effect of a heavier projectile would be simply the sum of the effects of the lighter projectiles of which it is composed. It is noteworthy that, in spite of the Coulomb repulsion, elements with $Z \simeq 50$ have been transformed by protons with energy of only a few MV. Transformation under β-particles have been reported; but it is difficult to prove that it is not due to the accompanying γ-rays.

Accordingly, a typical nuclear transformation may be written

$$^{A}_{Z}\mathrm{X}\ (u, v)\ ^{A'}_{Z'}\mathrm{X}'_1$$

where X and X′ are the chemical symbols for nuclei of atomic number Z and Z', A and A' are the mass numbers (nearest integers to the atomic weight), and u is the projectile and v the emitted particle, each of u, v, being either n, p, d, α or γ. (Since X is determined by Z, one of the two is superfluous; Z is often omitted, but it would be clearer to omit X.) The sum of the nuclear charges and the sum of the energies (made up of the kinetic energies of u and v and the mass defects of X and X′) must be the same for the initial and final states. In practice, X must be a stable or long-lived nucleus, in order that it shall be available in sufficient quantity; X′ must be radioactive, in order that it shall be detected. But there is no reason to believe that this limitation holds in principle.

The immediate task of nuclear chemistry, like that of atomic chemistry in the nineteenth century, is to discover all possible reactions of this type. The search is so far advanced that nearly all possible nuclei, stable and radioactive, have been discovered for all the common chemical elements. The same final nucleus X′ can be produced by different transformations from different original nuclei X; accordingly, though doubtless the number of transformations would be increased if the heavy elements could be bombarded with heavy projectiles, the number of nuclei would probably not be much increased. All combinations (u, v) have been observed, except (α, d), (α, γ), (d, γ), (n, d); it is not known whether these exceptions are significant.

Attention is now being paid to the probability rather than the possibility of transformations. The probability is usually expressed as a "cross-section" of the nucleus X for the particular transformation, i.e. the cross-sectional area deduced on the following assumption. N, the number of atoms contained in unit area of the cross-section of the layer through which the projectiles pass, is known. If the atoms have each a cross-section of area S and if NS is small compared with unity, then, the chance of a projectile of zero cross-section hitting an atom in passing through the layer is NS; if each projectile causes a transformation when it hits an atom, the chance that a projectile will cause a transformation is also NS. The actual chance p that a projectile will cause a transformation can be measured experimentally. Accordingly, the cross-section is defined as a quantity given by $p = NS$, or $S = p/N$, where p and N are known. The cross-section varies greatly with the energy of the projectile; for this reason means for controlling the velocity accurately are of more importance than the production of ever-greater velocities.

(1.4) The interest of the results lies in the light they throw on the structure of the nucleus. This is a difficult matter to discuss. Our journals are full of immensely learned papers, unintelligible except to a small group of experts, in which the most elaborate weapons of wave-mechanics are directed to the problem. It might appear, therefore, to an outsider that all the fundamental ideas are firmly established, and that the sole remaining problem is to determine, from the experimental data, constants whose nature is no longer in doubt. Actually the position is very different: fundamental questions are still unanswered; the immediate value of all this intricate work is rather to provide mathematical apparatus for dealing with any fundamental assumptions that may later appear to have some physical basis. Accordingly, in this review the author will merely try to set forth the physical conceptions that appear likely to be embodied in any future nuclear theory and the difficulties that attend them.

A nucleus $^{A}_{Z}\mathrm{X}$ consists of A particles, of which Z are protons and $(A-Z)$ neutrons. These particles are held together by short-range interactions between pairs of particles, the interaction being very nearly (perhaps quite) the same whether the pair is (p, p), (n, n) or (p, n). Some knowledge of the nature of these interactions (of which more is said below) can be derived from the binding energy, or mass defect, of nuclei containing only a few particles; still more is derived from the scattering of protons and neutrons by the protons of hydrogen. The "short range" is about 10^{-13} cm., and is not very different from the radius of an electron; this is one of many reasons for holding that electrons are not constituents of the nucleus.

Superimposed on these short-range forces is the long-range Coulomb repulsion between each proton and all other protons; but this repulsion is of minor importance except when Z, and therefore A, is large. In general, any stable nucleus A can combine with either a neutron or a proton to form a nucleus $(A + 1)$; the binding energy liberated in such a combination is always nearly the same; the main exceptions are (i) when $A < 10$ and the short-range interactions are incomplete, (ii) when $Z > 80$ and the Coulomb repulsion is important.

Quantum principles are applicable to the nucleus in the

same way as they are to the Bohr atom. Each particle is in a steady state or level of definite energy; there are also unoccupied levels to which a particle in the nucleus can pass if it receives (or loses) an appropriate amount of energy, and into which a particle from outside can enter. In the ground state all particles are in the lowest possible levels; but excited states are also possible. Subject to a later consideration, the probability that an incident particle will enter an unoccupied level increases generally with its energy; for increase of energy makes more levels available to it. But, superimposed on this general increase, are maxima due to "resonance," that is to say an exceptionally large probability that a particle will enter a level when it has the energy appropriate to that level; the width of the level, in other words the degree to which the incident energy may vary and yet resonance occur, varies from level to level, as it does in the atom. The detection of resonance is thus the best method of finding out what the levels are.

But there are two great differences between the Bohr atom and the nucleus which, in Bohr's phrase, make the nucleus more analogous to a liquid drop than to an atom. First, in the nucleus there is nothing corresponding to the nucleus in the Bohr atom, a single permanent particle exerting forces on the other particles which are only slightly modified by the forces between these other particles; all the forces are of the same nature and may change greatly with a rearrangement of the particles into different levels. The result (as general considerations indicate) is that there are far more unoccupied levels per particle than in the Bohr atom, and that these levels are far more closely spaced, especially in the neighbourhood of the "surface" levels. Second, the period occupied by a nuclear rearrangement (or, more accurately, the period within which a rearrangement, if it can occur, has a high chance of occurring) is much less in the nucleus; it is of the order of 10^{-24} sec. and is less than the period that the projectiles which cause nuclear changes spend within the nucleus. In the Bohr atom the analogous period is about 10^{-15} sec. and is greater than the time that an ionizing particle spends within the atom. The efficiency of slow neutrons in causing transformations is due to this cause; indeed, there is usually a range of small velocity within which the cross-section increases as the velocity of the neutron decreases, because the slower neutron spends the longer time within the atom.

The effects of an incident particle are correspondingly different. In a Bohr atom the usual result is that a single particle is transferred to a higher level; the chance of many particles being all transferred to higher levels is small, partly because the effect has no time to spread through the atom, partly because there are so few levels available. In the nucleus the most probable effect is that many particles (including the projectile) all enter new levels, with the result that all the levels change; an intermediate or compound nucleus is formed, usually in a highly excited state. In the return to the ground-level, the excess energy may appear as the kinetic energy of one particle, or of a small group of particles (α or d), which is ejected; or, as in the case of uranium, it may appear as the mutual kinetic energy of two parts of nearly equal mass, so that the nucleus splits.

It might be thought that this second stage was to be identified with ordinary radioactivity; but the time for which the compound nucleus would be expected to endure is of the order of 10^{-24} sec., and far too small to be detected experimentally. Since radioactive elements are known having lives as long as 10^{12} years and as short as 10^{-7} sec., a distinction between ordinary radioactivity and the readjustment of an intermediate nucleus would be artificial if based on this ground alone. But there are good reasons for believing that the distinction is one of kind. Thus radioactivity with the emission of a particle other than an electron is always α-activity; no n- or p- or d-activity is known. (Delayed neutron emission has been reported in the splitting of the Ur nucleus; the facts are uncertain, but if it is substantiated it will still be entirely exceptional.) Further, with the curious exception of $^{140}_{62}$Sm, no α-active nucleus is known for which $Z < 80$. Heavy-particle radioactivity is still completely consistent with Gamow's theory, which represents it as a "leak" of the α-particle (which need not be present as such in the nucleus) through the potential barrier due to the positive charge on the nucleus. That is to say, it is a property of a system in a state of final equilibrium in respect of short-range forces and due entirely to the long-range forces which are of minor importance in most nuclei.

β-radioactivity, on the other hand, can occur at all values of A and Z. Indeed, it is not, like α-activity, a property of the nucleus superimposed on others; it is inherent in all discussions of nuclear properties. For the criterion of a stable nucleus is that energy should not be liberated by its transformation into another nucleus of the same A value by loss of a negative electron (Z increasing by unity) or of a positive electron (Z decreasing by unity). (An alternative to the loss of a positive electron is the absorption of a negative electron from the K-level.) Stability therefore depends on the mass defect of the nucleus relative to those of the same A and neighbouring Z. But, since electrons are not constituents of the nucleus, what change in nuclear structure can the gain or loss of an electron produce? This question has become more, rather than less, mysterious in recent years.

Fermi's theory is that the emission of an electron (ϵ) together with a neutrino (ν) is a by-product of the change of a neutron into a proton, just as (on Bohr's theory) the emission of aetherial radiation is a by-product of the transition of an electron between two levels. This theory was devised primarily to explain (i) the fact that not all β-rays have the maximum energy characteristic of the transformation, and (ii) the "Sargent curves," relating this maximum energy to the life of the emitting nucleus. By means of plausible assumptions about the wave-functions of the system (p, n, ϵ, ν), (ii) can be explained if the mass of the neutrino is taken to be zero; but the distribution of the β-ray energies is not rightly predicted. Konopinski and Uhlenbeck, by less plausible assumptions, explain the distribution of energies better, especially if the mass of the neutrino is not zero; but the latest measurements do not seem consistent even with their modified theory. On the other hand, any form of the Fermi theory can explain the large magnetic moments of p and n (namely $+ 2 \cdot 9$ and $- 2 \cdot 0$, where $+ 1 \cdot 0$ and 0 would be expected from analogy with the electron). For since, on this theory, a proton (or neutron) spends

part of its time, really or virtually, in the form of a neutron (or proton) plus an electron, it partakes of the large magnetic moment of an electron.

But there is more. If a proton can become a neutron or vice versa by the exchange of an electron, there should be an " exchange force " between two such particles similar to that which binds two hydrogen atoms into a molecule. (In terms of quantum mechanics, the possibility of exchange makes the number of available levels greater when the atoms are near than when they are far apart, and therefore increases the chance of an electron being in a level of *low* energy. The system tends to assume the state of lowest energy; therefore the atoms tend to approach.) The potential energy between a p–n pair should vary with r, the distance between them as $e^{-\lambda/r}/r$, where $\lambda = h/(2\pi mc)$ and m is the mass of the electron. The form of this expression accords with what is known experimentally of the p–n force, but λ, which can be estimated roughly, turns out to 100–200 times $h/(2\pi mc)$.

On these grounds Yukawa suggested that the particle exchanged is not an ordinary electron but a " heavy electron " (or " meson " or " mesotron " or " barytron ") with 100–200 times its mass. It must have a spin 0 or 1, and when free must have a finite life of about 2×10^{-6} sec., after which it disintegrates into an ordinary electron and a neutrino (each of which have spin $\frac{1}{2}$), the excess mass appearing as kinetic energy or γ-rays. (The life is determined from the *frequency* of the exchange, which determines the absolute *magnitude* of the force.) Remarkable evidence has been obtained from other sources [see Section (2)] that such particles indeed exist; Yukawa's theory (which predicted them before discovery) has therefore been studied very carefully. But, after the first enthusiasm, it is now being asked whether the accuracy of the prediction was not, in part at least, fortuitous, and indeed whether the theory does not raise more difficulties than it removes. Thus, to take one example only, if the n–n or p–p force is the same as the p–n force and is due to exchange, there must be neutral mesons (neutrettos) as well as charged ones; this is contrary to Fermi's fundamental idea and apparently inconsistent with all that that idea explained. Various modifications of that idea have been proposed. Thus, according to the Gamow–Tiller theory, an (electron + positron), instead of a mere electron, is interchanged; according to others, the β-activity is a secondary by-product of a process in which the first stage is the interchange of a meson or neutretto. But all such ideas are clearly strained and artificial. The whole question whether there is a relation between β-activity and short-range forces has been reopened; if there is no relation, then neither of them is in any way explained and we are still very far from a complete theory of the nucleus.

(2) COSMIC RAYS

(2.1) Here there has been great progress; two of the most important problems have been solved.

The first is: How does an electron with an energy of say) 200 MV produce a " shower " of many high-energy (> 10 MV) electrons, all apparently originating at the same point in a lead block? The answer is this: when the speed of an electron is very near that of light, the chance is high that, at an encounter with a nucleus, a large fraction of its energy will be converted into a single photon. This photon, having an energy much greater than that corresponding to the mass of an electron-pair, positive and negative, will have a large chance of being converted into such a pair in the field of a nucleus, each of the electrons having nearly half the energy of the photon. A high-energy electron by n repetitions of this double process is rapidly converted into a shower of 2^n electrons, each having about $1/2^n$ of its energy. The shower appears to originate in a point only because the probability of each step in the process is so high that many double processes can occur in a fraction of a millimetre of matter.

The number, energy and angular distribution of the showers that should be produced in this way in a given thickness of matter can be predicted by the accepted principles of quantum mechanics. The predictions are confirmed by experiment; suitable experiments with a cloud chamber make the successive stages visible. Showers do not, as was thought at one time, prove that quantum mechanics breaks down for particles where energy is much greater than mc^2; on the contrary they establish its validity for electrons of the highest energy known.

(2.2) The second question concerns the nature of the " penetrating component." At sea-level about one-quarter of the cosmic rays are " soft " and absorbed in some 5 mm. of lead; they are doubtless electrons having energies up to some 200 MV; about three-quarters are hard and able to penetrate at least 1 m. of lead. At the top of the atmosphere, the soft rays greatly exceed the hard. If the shower theory just expounded is true, no electron can be as penetrating as the hard rays; for as its energy and speed increase, so does the probability of its losing energy by the aforesaid double process; the hard rays cannot be very energetic electrons. They ionize and are deflected in a magnetic field, and must therefore be charged. Can they be protons? The answer is " No," but the reason is rather complicated.

Let us term " relative energy " the ratio of the kinetic energy of a particle to the energy corresponding to its rest mass. Then it is known, theoretically and experimentally, that (except at very low energies) the ionizing power (i.e. ions formed per cm. of path in a standard gas) of a particle of given relative energy depends only on its charge and not on its mass; it falls rapidly with increasing energy, reaches a minimum when the ratio is about unity, and does not increase much thereafter. The ionizing power of the hard rays is near the minimum for particles with electronic charge; therefore (i) they have electronic charge and (ii) their kinetic energy cannot be less than their rest mass. If we assume that their rest mass is that of a proton, a lower limit is set to their kinetic energy, to their momentum and to the radius of curvature of their path in a given magnetic field. But the observed radius is definitely less than this minimum. The largest rest mass that the hard particles can have, in view of these considerations, is some 100–200 times that of an electron.

All this has been realized for some years. But until recently the existence of particles intermediate in mass between the electron and the protons appeared less

plausible than a breakdown at high energies of the general theories of the reaction between charged particles, on which the foregoing argument is based. But the success of the shower theory shows that such breakdown does not occur; and Yukawa's mesons [see Section (1.4)] have precisely the properties required for the intermediate particles. It is therefore now universally accepted that the "penetrating component" of the cosmic rays are mesons, with mass between 150 and 200 times that of the electron, and with kinetic energies from 200 MV upwards to some unknown limit, which probably does not much exceed 1 000 MV.

There is neat confirmatory evidence. It has long been known that the absorption of the atmosphere for the penetrating component is greater than that of a layer of lead, equivalent in mass but some 5 000 times thinner. This difference can be explained by supposing that the particles have a natural life of the order of 10^{-6} sec., so that more of them disappear, otherwise than by reaction with matter, during the longer passage through the air. But this is just the life attributed to the Yukawa mesons. This is a remarkable coincidence; but while the validity of the Yukawa theory is so doubtful, it may be no more.

The mesons must be produced by the reaction of the primary high-energy electrons with nuclei in the upper atmosphere; how they are produced is not known. No real progress has been made with theories of the ultimate origin of the cosmic rays; various astronomical theories have been advanced attributing them to processes occurring in the interior of stars or to a cyclotron action of their magnetic field; but all are purely speculation and unsupported by direct evidence.

(3) ASTROPHYSICS

It has long been recognized that the main part of the energy radiated by the stars must arise from nuclear reactions. When the only nuclear reactions known were the natural radioactivities of the heavy elements, these had to be held responsible; but that view presented great difficulties. When "artificial radioactivity," that is to say the transformation discussed in Section (1), was discovered, other possibilities appeared; but it is only recently that it has been possible to say that, under conditions known to prevail in stars, reactions observed in the laboratory should occur that are sufficient to provide the requisite energy.

The temperature of the core of a star (according to Eddington's theory and its developments) is determined by the mass of the star and the mean molecular weight of its material. For the sun this temperature is about 18×10^6 °C., if (as is probable) hydrogen and helium are major constituents. There are therefore present in it protons having thermal kinetic energies of the order of 25 000 eV; such protons are known to produce transformations in nuclei of relatively low atomic weight, leading ultimately to the production of helium nuclei of great stability with the liberation of energy. The question is therefore whether any of these reactions have a probability such that energy will be liberated at the right rate, neither too small nor too large.

Bethe, by examining all the known possibilities, has concluded that there is one and only one chain of reactions that gives the right result. It is right, moreover, not only for the sun, but for all the other stars of the "main astronomical sequence" for which the data are available. For each of them it gives the right relation between temperature (which determines the rate at which the reaction proceeds) and luminosity (which is a measure of the energy liberated). This chain is as follows:—

$$
\begin{aligned}
&{}^{12}_{6}\mathrm{C}\,(p, \gamma)\,{}^{13}_{7}\mathrm{N} \\
&{}^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + \epsilon^{+} \text{ (positron radioactivity)} \\
&{}^{13}_{6}\mathrm{C}\,(p, \gamma)\,{}^{14}_{7}\mathrm{N} \\
&{}^{14}_{7}\mathrm{N}\,(p, \gamma)\,{}^{15}_{8}\mathrm{O} \\
&{}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + \epsilon^{+} \text{ (positron radioactivity)} \\
&{}^{15}_{7}\mathrm{N}\,(p, \alpha)\,{}^{12}_{6}\mathrm{C}
\end{aligned}
$$

Carbon therefore acts as a catalyst, enabling four protons to combine into one α-particle, with the liberation of two positive electrons. It is curious to note that carbon is as essential to this "burning" of hydrogen to form helium in the stars as it is to ordinary combustion on the earth.

There seems to be little doubt of the truth of this theory. But it does not explain everything. It does not explain the existence of exceptional stars off the main sequence; but some of these may possibly be explained by other reactions of the same kind. Nor does it explain at all the presence in the sun and other stars of large quantities of elements much heavier than those concerned in the reaction; no process by which they can have been formed in stellar conditions can be suggested.

(4) QUANTUM MECHANICS

In spite of many difficulties with the nucleus, the preceding Sections provide strong support for the fundamental conceptions of quantum mechanics, which underlie all modern physical theory. Additional and very striking support is provided by Rabi's beautiful measurements of atomic magnetic moments, which depend on the fact that even the "Lorentz precession" is quantized. Nevertheless, all is not well. The long-outstanding difficulties arising from the infinite energy of the charged particle and of the "zero" (i.e. non-thermal) radiation in a vacuum remain. The solution of the former mentioned in the 1936 review* has not proved satisfactory; but other attempts to solve it (and also nuclear difficulties) by introducing a minimum length of some 10^{-13} cm., limiting the validity of quantum mechanics as h limits that of classical mechanics, still find some favour.

Others feel that the difficulties will not vanish until quantum mechanics makes itself entirely independent of classical mechanics. Dirac's exposition goes furthest in this direction; but it still relies on the Hamiltonian function (an essentially classical conception) as a starting point in solving particular problems. His latest attempt to avoid the difficulties is therefore of exceptional interest. Like his theory of negative and positive electrons, it consists, not in modifying fundamental equations, but in altering the "dictionary" by means of which symbols in those equations are related to experi-

* *Journal I.E.E.*, 1936, **78**, p. 186.

mental magnitudes. It is therefore impossible to formulate his suggestions otherwise than in the mathematical symbolism appropriate to them; but he himself summarizes them by saying that the interior of an electron is a region where space-time is modified and signals are transmitted with a velocity greater than that of light. That is at least proof against experimental contradiction!

It is now quite certain that the apparent discrepancy in the values of the " universal constants " is real; and that, in particular, the measured values of e, m, h, c, inserted in the Bohr formula for the Rydberg constant, do not yield the measured value of that constant. The purely experimental evidence undoubtedly indicates that the Bohr formula is wrong by about 1 %, all the other assumptions underlying the measurements of the constants being right. But nobody has been able to suggest how that can be; every modification of the formula seems to destroy the most fundamental assumptions on which it is based, and therefore every reason for attaching any validity to the discrepant measurements.

(5) MOLECULAR PHYSICS

(5.1) In molecular physics the conceptions of order and disorder, introduced originally to explain the properties of alloys, are of ever increasing importance. They are applicable to all " co-operative " phenomena, i.e. to those in which the forces exerted on any molecule by its neighbours vary with the state of these neighbours in respect of the condition that the forces tend to produce. Thus ferromagnetism is typically co-operative, because the forces orientating any molecule depend upon how far its neighbours are orientated. Whenever that happens the energy V required to transfer a molecule from an orderly position to a disorderly one depends on Z, the extent of the existing order. There will be a range of temperature between the upper and the lower critical temperature, T_c and T_c', within which there are in general two possible states of the system, one having more order than the other; at one temperature T_m in this range the two states are identical. If V is simply proportional to Z (as Bragg and Williams assumed in their early work), T_c' is zero; but if the dependence is more complicated T_c' will be finite.

These very general ideas have recently been applied to liquids; they seem capable of explaining the familiar, but so far mysterious, fact that crystals melt at a definite temperature and do not, like glasses, pass gradually from the liquid to the solid state. For even the most perfect crystal has some disorder and, as X-ray analysis shows, liquids have some order; in both states there is, as in alloys, a mixture of order and disorder. If the reasonable assumption is made that the strength of the ordered lattice decreases rapidly as the extent of order approaches zero, it can be explained why T_m, the melting point, is very near T_c (so that a crystal cannot be appreciably overheated without melting), but is far from T_c' (so that a liquid can be greatly supercooled before it freezes). The merit of these ideas for amateurs is that they lead to qualitative, but fully correct, explanations by the use of little more than common sense.

(5.2) But the most remarkable discovery in the molecular field concerns liquid helium. Liquid helium at a temperature above 2·1° K. (liquid helium I) resembles other liquefied gases; it has a very small thermal conductivity and a viscosity which, though small, is much greater than that of gases. When it is cooled below 2·1° K. at atmospheric pressure, the liquid passes, with evolution of latent heat, into a new phase (liquid helium II) with most surprising properties. It is a liquid in the sense of having a definite volume but no resistance to shear; but its thermal conductivity, measured in the ordinary way, is greater than that of copper at the same temperature, and its viscosity, measured in the ordinary way, less than that of gaseous hydrogen. No appreciable temperature difference can therefore exist in it; the simple observation that first drew attention to its peculiarities is that when the liquid is cooled past the the transition point, the bubbling characteristic of all ordinary liquefied gases in ordinary containers ceases; the surface suddenly becomes perfectly still.

Further experiment showed that the " conduction " of heat was accompanied by a motion of the liquid as a whole from a colder to a warmer region. If a tube partially immersed in the liquid is closed at its lower end by a porous plug, and if the liquid in the tube is gently heated, a jet of liquid several centimetres high is projected from the open end (" fountain effect "); the liquid must be flowing in through the plug towards the heated zone. Later work has shown that this, and many other, strange phenomena arise from the power of liquid helium II to form on solid bodies a film, about $3 \cdot 5 \times 10^{-6}$ cm. thick, travelling towards any region from which the film may be removed with a speed that depends on the width of the exposed surface but is almost independent of the pressure aiding or opposing the flow. Thus, if a cup of the liquid is suspended in an enclosure at constant temperature, the liquid will climb over the rim against gravity and drip off the bottom of the cup. If the bottom of the cup is immersed in liquid and the liquid in the cup heated, liquid will climb into the cup from outside, so that the liquid inside is higher than that outside.

Such properties, though not fully describable in terms of ordinary physical conceptions, do not (as might appear at first sight) violate the principles of energy and thermodynamics; for they are associated with equally strange " inverse " effects. But there is a thermodynamical difficulty. The equilibrium diagram of helium indicates that liquid helium II persists down to absolute zero, where its entropy must be zero. But, since disorder increases entropy, a body with zero entropy must have perfect order; but how can a perfectly ordered body be mobile and fail to resist shear?

No really adequate answer is yet forthcoming; but the mobility must be associated somehow with the " zero " energy that persists even at 0° K. It is worth while to point out that there is a certain analogy between the mechanical properties of liquid helium II, on the one hand, and supraconductivity on the other. (Liquid helium II is not a supraconductor; it is an insulator.) For in a supraconductor the electrons have a high degree of order that enables them to move through a body of finite size without reacting with the ions. In liquid helium II the molecules move as a whole without interference. A solution of one mystery is likely to solve the other.

(6) HIGH-FREQUENCY OSCILLATORS

(6.1) One development in the instrumental field is so interesting (although it involves nothing but the principles of classical dynamics) and is likely to have such important practical consequences that it deserves specific mention.

In any apparatus arranged for exciting a tuned oscillator by a beam of electrons, the condition must be fulfilled that the time taken by the electrons to pass through the oscillating field is not much greater than the period of the oscillator. Since 3 000 volt electrons have a velocity of 3×10^9 cm. per sec., and since the electrodes may be less than 1 mm. apart, there should be no difficulty in fulfilling this condition at frequencies as high as 1 000 Mc./s. But the arrangement of the usual valve oscillator is peculiarly unfavourable. The efficiency is low unless the transit time is much less (and not merely not greater) than the period; all the unutilized energy of the electrons has to be dissipated at the oscillating electrodes; and the connection of the grid to the tuned circuit introduces much damping. Accordingly, only a few watts of oscillating power can be generated at 1 000 Mc./s. A study of first principles has enabled several independent workers in America to devise instruments (sometimes called "klystrons"), using velocity modulation, by which many kilowatts of 1 000 Mc./s. power can be generated and controlled.

Consider an electron entering the enclosure A (Fig. 1) through the hole P, leaving through the hole P′, and passing on its way through the grid G, which is oscillating in potential relative to the enclosure. The potential difference between the enclosure A and the source of electrons is constant, so that the electrons always enter A with the same velocity. The potential of A is taken as zero, so that G is said to be positive or negative according as its potential is above or below that of A. If the electron enters when G is negative, passes G when it is changing sign, and leaves when G is positive, it will be retarded throughout its path. The energy it loses will be given by electrostatic induction to the oscillating system of which G is part; the electrons will maintain the oscillations. If the electron enters in the opposite phase it will take energy from the oscillations. In order that the oscillations may be maintained, the electrons must enter in groups when G is negative, and these groups must be separated by gaps, so that none enter when G is positive.

How are these groups to be produced? If a continuous stream of electrons, all of the same velocity, enters the enclosure, those that enter when G is negative will be retarded and those that enter later when G is positive will be accelerated. After leaving the enclosure for the field-free space on the right, the latter will begin to overtake the former; at some point in that space they will actually overtake them. Accordingly, at some points in the space the electrons will be bunched into groups, and these points will be separated by others at which there are relatively few electrons. A very small change in the velocity of the electrons will produce this bunching, so long as the space on the right is sufficiently long. Therefore, by spacing two enclosures similar to that shown at a suitable distance along the path of an originally uniform stream of electrons, and by causing the grid of the first to oscillate with the natural frequency of the grid of the second, the grid of the second can be caused to take from the stream energy greatly in excess of that given to the grid of the first; the arrangement will act as an amplifier or, if the grids are interconnected, as an oscillator.

Of course, certain complications are overlooked in this simple account. Thus, if the groups of electrons entering the second enclosure have been produced by overtaking, they will not all have the same velocity. But a more

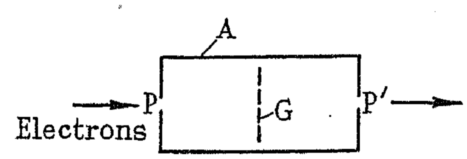

Fig. 1

detailed examination shows that the efficiency of the arrangement can be surprisingly high. Ideally, 58 % of the kinetic energy of the stream may be converted into oscillations. None of the unused energy need be dissipated at the electrodes; it can be dissipated at the end of the field-free space.

Simple grids coupled to ordinary tuned circuits would, however, involve too much damping. In the most characteristic instruments, use is made of a fact long known, but not hitherto applied. Just as the air in a rigid enclosure can oscillate with natural frequencies determined by the size and shape of the enclosure, producing standing waves within it, so the electromagnetic field inside a conducting enclosure can oscillate; in the fundamental mode the field takes alternately the form of an electric field extending in one direction across the enclosure and unassociated with currents, a magnetic field in a perpendicular direction associated with currents in the conductor, an electric field in the opposite direction, and so on. Suppose, then, that the grid in Fig. 1 is removed; if the field in the enclosure A begins to oscillate with such a frequency and phase that, when a group of electrons is passing through it, the electric field retards the electrons and, when the field is in the opposite direction, no electrons are passing; then, in virtue of the principles already set forth, the field will take energy

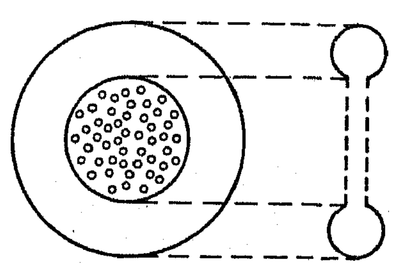

Fig. 2

from the electrons and the oscillations will be maintained. (It is worth mentioning that the conception of potential is inapplicable, just as it is to the field excited in a closed ring by varying magnetic induction linked with it.)

But the condition must be fulfilled that the time occupied by the electrons in passing through the enclosure is not more than half the period of the oscillations of the field. This period T will be proportional to the linear dimensions of an enclosure of given shape. Hence, if l

is the length of the enclosure along the path of the electrons, v their velocity, and k a factor depending on the shape of the enclosure, we must have $l/v \leqslant \frac{1}{2}kl$. k obviously cannot differ *very* greatly from $1/c$, where c is the velocity of light; v cannot exceed c and is conveniently as low as $c/10$. The condition can be fulfilled, therefore, only if there are enclosures of such a shape that $kc \geqslant 20$. Fortunately, but rather unexpectedly in view of elementary considerations, there are such enclosures. Spheres do not satisfy the condition; but the shape shown in Fig. 2, among many others, does. Oscillators of this and other suitable kinds are sometimes called "rhumbatrons." If the metal were continuous, the damping would be determined wholly by the "skin" conductivity of the metal; but it is necessarily increased somewhat by the holes through which the electrons enter and leave. Nevertheless, by suitable construction the damping can be made much less than that characteristic of a tuned oscillator of the ordinary kind having the same frequency.

PROCEEDINGS OF THE INSTITUTION

949TH ORDINARY MEETING, 26TH OCTOBER, 1939

Mr. Johnstone Wright, President, took the chair at 12.30 p.m.

The minutes of the Annual General Meeting of the 11th May, 1939, and of the Ordinary Meeting held on the same date, were taken as read and were confirmed and signed.

A list of candidates for election and transfer, approved by the Council for ballot, was taken as read and was ordered to be suspended in the Hall.

The meeting then terminated.

950TH ORDINARY MEETING, 16TH NOVEMBER, 1939

Mr. Johnstone Wright, President, took the chair at 12.30 p.m.

The minutes of the Ordinary Meeting held on the 26th October, 1939, were taken as read and were confirmed and signed.

Messrs. J. I. Bernard and R. A. McMahon were appointed scrutineers of the ballot for the election and transfer of members and, at the end of the meeting, the President reported that the members whose names appeared on the lists (see page 107) had been duly elected and transferred.

The President announced that during the months of May to October 816 donations and subscriptions to the Benevolent Fund had been received, amounting to £887. A vote of thanks was accorded to the donors.

The meeting then terminated.

951ST ORDINARY MEETING, 11TH JANUARY, 1940

Mr. Johnstone Wright, President, took the chair at 12.30 p.m.

The minutes of the Ordinary Meeting held on the 16th November, 1939, were taken as read and were confirmed and signed.

The President announced that during the months of November and December 113 donations and subscriptions to the Benevolent Fund had been received, amounting to £93. A vote of thanks was accorded to the donors.

A list of candidates for election and transfer, approved by the Council for ballot, was taken as read and was ordered to be suspended in the Hall.

The meeting then terminated.

952ND ORDINARY MEETING, 25TH JANUARY, 1940

Mr. Johnstone Wright, President, took the chair at 6 p.m.

The minutes of the Ordinary Meeting held on the 11th January, 1940, were taken as read and were confirmed and signed.

Messrs. C. Reid and A. W. Berry were appointed scrutineers of the ballot for the election and transfer of members and, at the end of the meeting, the President reported that the members whose names appeared on the lists (see page 220) had been duly elected and transferred.

The President announced that the Council had elected Mr. Roger T. Smith, B.Sc., (Past-President) to be an Honorary Member of The Institution.

He also announced that the Council had made the Eighteenth Award of the Faraday Medal to Dr. Alexander Russell, M.A., D.Sc., LL.D., F.R.S. (Past-President).

A discussion took place on "Fire-Fighting Equipment for Electrical Installations," based on the E.R.A. Report on this subject in the December, 1939, issue of the *Journal* (see vol. 85, page 719). The discussion was introduced by Mr. H. W. Swann, Member, and Mr. R. A. McMahon, Associate Member, the latter of whom gave a running commentary on a cinematograph film in colour illustrating the fire-extinguishing tests made during the research in connection with the Report. Mr. J. Hacking, Associate Member, replied briefly to the discussion.

On the motion of the President, a vote of thanks was accorded to Messrs. Swann, McMahon and Hacking, and to all those associated with the preparation of the Report and the conduct of the tests.

The meeting then terminated.

INSTITUTION NOTES

HONORARY MEMBER

At the Ordinary Meeting of The Institution held on the 25th January, 1940, the President announced that the Council had elected Mr. Roger T. Smith, B.Sc., Past-President, to be an Honorary Member.

FARADAY MEDAL

At the same meeting the President also announced that the Council had made the Eighteenth Award of the Faraday Medal to Dr. Alexander Russell, M.A., D.Sc., LL.D., F.R.S., Past-President.

NATIONAL CERTIFICATES AND DIPLOMAS IN ELECTRICAL ENGINEERING

The following are the results of the examinations in connection with the above for the year 1939:—

England and Wales

	Pass	*Fail*
Ordinary Certificates	1 133	925
Ordinary Certificates endorsed	2	1
Higher Certificates	421	254
Higher Certificates endorsed ..	79	15
Ordinary Diplomas	48	25
Higher Diplomas	8	4
	1 691	1 224

Scotland

	Pass	*Fail*
Ordinary Certificates	49	19
Higher Certificates	14	1
Higher Certificates endorsed ..	3	3
Higher Diplomas	8	2
	74	25

ASSOCIATE MEMBERSHIP EXAMINATION: NOVEMBER, 1939 (HOME CENTRES)

List of Successful Candidates

[Success in this Examination does not of itself constitute the candidate an Associate Member.]

Parts I and II*

Allan, James.
Baird, Trevor Gordon.
Chapter, Colin Falconer.
Coombs, Frederick Leslie.
Donovan, Timothy Denis.
Ferguson, Oswald.
Harris, Frederick Llewellyn.
Hart, Francis.
Hewson, John Elam.
Long, George Richard.
McBain, John.
Martin, Kenneth Spencer.
Meiklejohn, William Kenneth.
Palmer, William Clifford.
Parker, Walter Brian.
Parry, Kenneth.
Patel, Chinubhai Manibhai.
Pikett, Cecil Charles.
Read, William Macartney.
Reid, James.
Street, Walter George.
Thomas, Alfred Morris.
Williams, Denis.

* This list also includes candidates who are exempt from, or who have previously passed, a part of the Examination and have now passed in the remaining subjects.

Part I only

Arthur, Enoch.
Bindoff, Leonard Lewis.
Chandler, Edward Anthony.
Eckford, Alexander Thomas.
Quance, Gordon Courtenay.
Sanders, John.

Part II only

Byrne, Joseph.
Lang, George Andrew.
Swannell, Arthur Ernest.

ELECTIONS AND TRANSFERS

At the Ordinary Meeting of The Institution held on the 25th January, 1940, the following elections and transfers were effected:—

Elections

Associate Members

Aschman, Geoffrey Donald.
Astbury, Herbert.
Atkinson, Edward Kenneth.
Bell, Alexander Malcolm.
Binnie, John Alexander, Commander, R.N. (Retd.).
Boyse, Cyril Oliver, B.Sc. (Eng.)
Bradley, James Kenneth.
Bruce, William Ramsay.
Burr, Patrick Dickie, B.Sc.(Eng.).
Bushell, Eric Chesterton, B.Sc.(Eng.).
Bylewski, Jerzy.
Campbell, Patrick Joseph.
Catford, Maurice Braund, B.Sc.Tech.
Coade, Loftus George.
Dalton, James.
Davies, Harry Beaumont.
Drummond, Hugh.
Einhorn, Heinz Dieter, Dr.Ing.
Fergusson, Leonard.
Fielder, Frederick Henry.
Folkard, George Frederick, B.A., B.Sc.(Eng.).
Garratt, Gerald Reginald M., M.A.
Giddings, William Frank, B.Sc.(Eng.).
Gilchrist, Thomas Alfred.
Haswell, Arthur James.
Henderson, William George N.
Hibbs, Norman Lewis, B.Sc.(Eng.).
Hodgetts, Eric.
Horsfall, Frank.
Jarratt, Reginald Cecil, Lieut., R.E.
Jones, George Henry, B.Sc.
Jones, Gilbert Louis R.
Kennedy, Oliver Henry.
King, Leonard Seth, B.Sc.(Eng.).
Linck, Henry Charles A.
Lough, Jack Reginald.
McCarthy, Terence, B.E.
McKellar, George Ledingham, B.Sc.
Macnamara, Terence Cameron.
Mangnall, Wilfrid Earlam.
Marshall, David Stuart.
Meldrum, John Patrick A., B.Sc.
Odell, Trevor George.
Savory, Alan Balantyne.
Seavers, Robert.
Skipsey, William, B.Sc. (Eng.).
Sumner, John.
Stritzl, Peter Friedrich, D.Sc.
Sung, Zau Yoen, B.Sc. (Eng.).
Watts, George Ernest.
Welch, Leslie Hugh, B.Sc.(Eng.).
Wood, Thomas Stafford.

Companion

Spurling, Sir S. Stanley, C.M.G.

Associates

Acton, William.
Barrack, Allan Oscar.
Bates, Fred.
Clark, Robert George.
Cooke, Arasaratnam Albert.
Croft, Norman Benjamin.

Associates—continued.

Fennemore, Thomas William.
Fox, John.
Hastings, Donald Jack.
Kalra, Ram Das.
Kingston, Edgar Oxley.
Letchford, Herbert Reginald.
Martin, Anthony Wyard.
Patrick, Reginald Clifford.
Penny, Sherwin.
Plumpton, Mark William.
Rimmington, Henry George.
Smith, Bertie.
Southorn, Lawrence Frederick.
Thornton, Thomas James E.
Timberlake, Herbert Thomas.
Walker, Garrett Wellesley.
Weaver, Albert Dudley.
Wilson, Cecil Edward.

Graduates

Ali, Abid, B.Sc.(Eng.).
Ashley, Herbert Neville, B.A.
Atkinson, George Henry.
Avery, Robert Benjamin.
Bagalkot, Dadahayat Qasim.
Bayley, Robert Mitchell.
Beecher, Arthur Edward S.
Black, David Carswell.
Brennand, James Joseph.
Brown, James, B.Sc.
Brown, Robert William.
Carter, Cecil George, B.Sc. (Eng.).
Chiang, Ts-En, B.Sc.
Clothier, William Keith, B.Sc., B.E.
Conybeare, Anthony Macaulay, B.A.
de Boor, Willi Max, Dipl. Ing.
de Souza, Jose Caetano, B.Sc.(Eng.).
Dhami, Bhagat Singh, B.A.Sc.
Downham, Cyril Vincent.
Drake, Sidney Richard, B.Sc.
Drury, Frederick Leslie, B.Sc.
Egenes, Olaf Kristian, B.Sc.
Farr, Percy Robert A.
Ghose, Manindra Kumar.
Goldsmith, Frank Henry.
Goldsworthy, Lionel John, B.A.
Gosling, Leonard Owen, B.A.
Gosling, Robert Starr.
Gould, John Harold.
Hale, Frank Maurice, B.Sc.
Hansford, Richard Norman, B.A.
Head, Richard Beaumont, B.Sc.(Eng.).
Heins, John Philip, B.Eng.
Hesketh, John Frederick.
Hilton, John Sumner, B.Sc.
Hilton, Roy, M.Eng.
Hoare, Leslie Cecil.
Horner, Frederick, B.Sc.
Innes, John Mitchell.
Khanna, Atta Chand.
Klein, Nicholas, Dipl.Ing.
Kojranski, Jan Wieslaw L., B.Sc.Tech.
Korn, Andrew McLeod G.
Laurie, John Bruce, B.Sc. (Eng.).
Lee, Robert Sparrow.
Lele, Vyankatesh Govind.
Lobo, David Caridade.
Lyddiard, Jack Alfred, B.Sc.
Lyon, George, B.Sc.(Eng.).
McMillan, Robert.
Marathe, Daltatray Hari.
Martinez, Giorgio, B.A.
Merchant, Ravindra Bhugvandas, B.Sc.(Eng.).
Merry, Robert George, B.Sc.
Mitchell, John Harwood G.
Mohtadi, Mohammad, B.Sc.
Morley, Richard Brian.
Morrison, William Henderson.
Orok, Robert John, B.A.Sc.
Palmer, Leonard Ryfealyer, B.Sc.(Eng.).
Peartree, John Allan.
Reddy, Nallapareddy Ramaraghava, B.Sc.Tech.
Richards, Anthony Powell.
Rix, Edward Alfred.
Robson, Ronald Chapman, B.Sc.Tech.
Spooner, Gerald William, B.Sc.
Stark, Gordon Alexander.
Stoyle, Harry.
Sutcliffe, John Bernard, B.Sc.

Graduates—continued.

Tilzey, Rupert.
Uglow, Peter Rawson.
van Ryn, Bernard.
Ward, Warwick John.
White, Clifford Hubert, B.Sc.(Eng.).
Wertheim, John Michael, M.A.
Willis, Frederick Robert, B.Sc.(Eng.).
Wilson, Edmund Denis.
Wood, Douglas.

Students

Abercrombie, Thomas William.
Adams, Douglas Rutterford.
Adams, Edward Francis.
Allan, George Rowland.
Allen, Frank.
Anderson, Kenneth John.
Antrich, David.
Apparao, Gumpeny.
Apps, David Sydney.
Armitage, Jasper William.
Armstrong, Alan.
Arnold, Basil Drake.
Arnold, George Frederick.
Ashby, Walter James.
Aspinall, Reuben.
Atkins, John Percival.
Atkinson, John.
Atkinson, Leonard George.
Bagnall, John.
Baker, Arthur Reginald.
Baker, Norman Henry.
Balwani, Brijlal Hakikatrai.
Barker, Ronald.
Barker, Ronald Sach.
Barnes, Richard Charles M.
Barnes, Robert Leslie E.
Barstow, Robert.
Bell, James Edward.
Bell, John Harold B., B.A.
Bell, Lindsay Gordon.
Bendall, Cyril Alexander.
Bhalla, Roshan Lall.
Bhatnagar, Maheshprasada J.
Bhatnagar, Prakash Swarup.
Bhise, Mahadeo Ramachandra.
Bickerdike, Cecil Harold.
Bishop, Richard George.
Black, Graeme.
Blackwell, Frederick William.
Blythen, Roy.
Bowden, Wilfrid Edgar.
Bowler, John Edward.
Bowra, Douglas Samuel.
Bradley, Francis John.
Brewin, Edwin.
Briggs, Ernest Godwin F.
Britten, Jack Hillyer.
Broadbent, Albert.
Brockman, Robert Norman.
Brook, Thomas Charles.
Brooke, Norman.
Brown, Alfred.
Brown, John Malcolm.
Brownlee, Thomas.
Buckley, Charles Howard.
Burgess, Derek William.
Burt, Edward Mackenzie.
Bysouth, Kenneth William.
Callander, Archibald Carlisle.
Canham, Frederick Charles.
Carr, Philip Henry.
Castle, Charles Alfred.
Castro, Raymond Ernest.
Champion, Charles Alexis.
Chatten, Norman Curtis.
Chaudhary, Mohammed Bashir.
Chhibbar, H. K. Lall.
Chidlow, Ernest.
Chipperfield, Victor James, B.Sc.(Eng.).
Chopra, Isaac Victor.
Choudhury, R. C. Roy.
Christie, Herbert Hight.
Clark, Frederick William J.
Clarke, Edward Temple.
Clarke, Richard John.
Coakes, Harold Seaton.
Cocking, William Warren.
Cole, Jack Edmund H.
Collier, Clifford Thomas.
Coker, Albert James.
Conley, Ernest Geoffrey.
Cook, John Henry.
Cook, Stanley Halliday.
Cooper, Charles Edward.
Cooper, Geoffrey Ernest F.
Cope, Philip George.
Cornish, Clifford Harry.
Costelloe, Ignatius.
Courtis, William John.
Crosby, William Hunter.
Cully, Alan William.
Curran, John Edwin.
Dalby, Ernest Kershaw.
Dalgleish, Eric Hugh.
Dalling, Courtenay John.

Students—continued.

Dalton, Lionel Frank C.
Dance, John Henney.
Davies, Henry Farnell.
Davies, Phillip.
Davis, Herbert Ronald.
Davis, Victor Thomas.
Denes, Peter.
Dharmarajan, K. S.
Dickinson, Stanley.
Dixon, John George.
Doherty, Stephen Norman.
Dolwin, John Davison.
Doughty, Edward Henry M.
Douglas, George Gilbert.
Douglas, Donald Ross.
Douthwaite, Brian Hugh.
Dowell, Allan.
Driver, George Alfred P.
Drury, Philip.
Dyke, Thomas Daniel.
Earle, Brian Christopher.
Eccleston, Dennis Norman.
Edwards, Aubrey Thomas.
Edwards, Ronald Robert.
Edwin, Denis Harold C.
Effemey, Harry George.
Eggers, Anton Friedrich W. H.
Ellis, William Anthony N.
Elzarki, Mahmud Taha, B.Sc.
Endersby, Francis George.
Ennos, John Frederick.
Eshelby, Roy Albert.
Escaño, José Fortich.
Evans, Eric Victor.
Farr, Robert Anthony L.
Farrell, Edward Thomas.
Farrell, Frank Kenneth.
Felgate, Anthony Cecil.
Fern, Robert.
Fillingham, Joseph James.
Finnamore, Alan John.
Fisher, George Thomas.
Fisher, Henry William W.
Flack, Donald.
Foster, Peter Walter.
Fox, Harold.
Foxall, William Walton.
Foyer, John.
Franklin, Donald Arthur.
Freebrey, Arthur George.
Freeman, Eric Samuel.
Frost, Reginald Ernest.
Fussell, Alfred Louis.
Gabriel, Alfred Joseph.
Gajjar, Becharlal Maganlal.
Gangla, Sambhaji Shankar.
Garrod, John Antony.
Gassin, Pierre Carl N.
Gayatonde, Vasant Shankar.
Gear, Alfred Joseph.
Gooch, Cyril William.
Goodden, Kenneth Robert.
Goodwin, Herbert Frederick.
Goss, Victor George S.
Grant, John Eric.
Gray, Charles Alexander.
Gray, Rafael.
Gregory, Darrell Welbourn.
Grierson, Alastair Ronald.
Gronhaug, Arnold Conrad.
Grose, Basil Hubert.
Guneratne, Piyatilleke Perera.
Gupta, D. C.
Hall, Leslie Cannon.
Hamilton, David.
Hancock, Arnold.
Hankin, Barclay Dundas.
Harrison, Fred.
Harvey, Philip Henry.
Hatch, Raymond Arthur.
Hawkins, Ronald Oliver W.
Haynes, Eric Ralph, B.Sc.(Eng.).
Hickin, Ernest Malcolm.
Hill, Albert Edward.
Hills, Eric George.
Hills, Norman Leslie W.
Hobson, Philip James.
Hodson, Thomas John.
Hollyer, Eric Brian.
Holmberg, Oscar Carl G.
Hopkinson, Brian Edward.
Horton, Ralton.
Howard, Dennis Robert.
Howie, William Forbes.
Howlett, Peter Vernon.
Hudson, John.
Huey, Eustace Beresford.
Hughes, James.
Humphreys, Herbert Moore, 2nd Lieut., R.A.
Hunjan, Madan Gopal.
Hussey, Arthur Percy.
Hutchings, Cyril Gilbert.
Hutchinson, Peter Hugh.
Inamdar, Dinker Kahandas.
Irgin, Charles Arthur.
Isherwood, Charles Medcalf.
Jafri, Mohd. Abis.
James, Charles Henry R.
Jarvis, Alan Maurice.
Jay, Richard Henry.
Jeffery, Douglas Albert.

Students—continued.

Jeyanayagam, Samuel John.
Johnson, Albert Ronald.
Johnson, Douglas William.
Johnson, John Kennedy (Jun.).
Jones, Donald William K.
Jones, Frank Ernest.
Jones, Leonard Charles N.
Jordan, Robert William P.
Judge, James Arthur.
Kane, Frederick William.
Kapoor, Chandra Prakash.
Kemp, Leslie Middleton O.
Kennedy, Graham.
Keskar, Purushottam.
Khan, Abdul Mannan.
Khanna, Prem Nath.
Kin, Oh Ewe.
King, Derek La Windell W.
King, Percy William.
Kirk, John.
Kitchener, Ronald David.
Knowles, Ralph.
Koram, Edmund Manteaw.
Kustner, George Max A.
Langdon, Leslie Edward E.
Lapham, Raymond George.
Lawson, Peter James.
Lee, Richard John.
Lester, Malcolm.
Levy, Bernard.
Limpenny, Hubert Robert E.
Linay, Albert Stanley.
Lincoln, Herbert Samuel.
Lithgow, John Charles.
Livingston, John William.
Llewellyn, Eric.
Lockhart, David Hugh.
Love, George Alexander H.
Luce, Francis James.
Macara, John Stuart.
McBride, James Maurice W., B.Sc.
McCarthy, Dennis Harold.
McClure, Denis Brian.
McElhinney, Arthur Frederick H.
MacKay, Frederick Gordon.
McKenna, Michael Foley.
McLeod, Donald Duncan.
McLiesh, David.
Maddams, Harold Sidney.
Major, Arthur Stephen.
Makhdumi, Sultan Ahmad.
Malhotra, Jamna Das.
Manssen, Norman Bernard.
Mardon, Edward Charles.
Marshall, James Horace.
Mathew, Attupurathu Mathew.
Matkin, Arthur Winton.
May, David.
Maynard, Roland George W.
Mehta, Champaklal Harilal.
Mehta, Rai Singh.
Mehta, Raj Kumar.
Meyler, Thomas David.
Miller, Andi Alfred.
Millett, George.
Millson, Henry William.
Mitchell, Thomas Reginald.
Modi, Noshir Homi .
Mogford, Gordon Sidney H.
Monk, Murray.
Morgan, Kenneth Albert.
Mullin, Leslie Raeburn.
Mundy, Grahame Arthur.
Murray, Stuart James.
Myatt, Austin Brock.
Naganand, Bangeri.
Narasimhan, Vedartham Ramaswamy.
Nawab, Syed Hasan.
Nayudu, G. Prabhaker Rao.
Neale, Denis Manktelow.
Neville, Peter Nigel.
Newby, Raymond Laurence.
Nichols, John Winfrith de L.
Ord, Craven Basil.
Palmer, Thomas.
Parker, Eric, B.A.
Parkins, Thomas.
Parlour, John Huxley.
Parthasarathy, T. S. Sarangapani Ayyangar.
Paul, John Michell.
Payne, Francis Inwood.
Pearson, John.
Pender, James Thompson.
Pendrill, Ernest Marcus.
Penzig, Oscar Dudley.
Philp, Ernest Douglas.
Pledger, John Russell.
Plummer, Donald.
Pout, Harry Wilfrid.
Poyser, Rex William R.
Prakash, Chandra.
Price, Philip Ratcliffe.
Priestman, Patrick Neil.
Privett, Herbert Lawrence.
Purvis, William James.
Putman, John Laban.

Students—continued.

Pykett, Douglas Ernest.
Pyper, Hugh Richard.
Quamber, Wilhelm Oswald.
Rahman, S. Habibur.
Rakha, Ram.
Ramm, Edwin Trevor.
Reekie, James.
Reynolds, Frank.
Reynolds, Peter John.
Rhodes, John Wickham.
Roberts, John.
Robinson, Henry Blenkiron.
Robinson, Wilfrid Eric.
Rogers, John Maldwyn.
Ross, George Ian.
Rowse, David Munro.
Rudd, John.
Rydings, Douglas Dunkerley.
Sach, Norman Daniel.
Said, R. M.
Salama, George, B.Sc.
Sampson, Philip.
Saunders, Harold Edward R.
Schofield, Cyril James.
Scott, Robert William.
Sedgwick, Robert Henry.
Sen, Sachindranath.
Shahaney, Bhagwan Javhermal.
Sheldon, Philip.
Shimwell, James Alan.
Simmons, Jack.
Simmons, James Douglas.
Simpson, Alan Edward.
Simpson, Harold David.
Singh, Niamet.
Singh, Pardaman.
Sinha, Lakshmishwar Prasad.
Singleton, Stanley Douglas.
Skinner, Francis Ronald.
Sleigh, Arthur Ffennell C.
Smith, Archibald Charles.
Smith, Eric Frank.
Smith, Jack Macfarlane.
Smith, John Whitby.
Smith, Neville Dunn.
Smith, Norman N. Parker.
Smith, Reuben.
Smith, Robert Antony G.
Snell, Paul Anthony.
Spence, Peter Edward.
Spencer, Percy Vernon.
Squire, Robert Frederick.
Sreeramulu, Manchukonda.
Srinivasan, Narayanswami.
Stannard, Squire Austin.
Stanworth, George.
Stevens, Ronald Charles.
Stewart, Andrew Goold.
Stewart, Gordon Scott.
Storey, Harry Edward.
Subramaniam, Tinnevelly Doraiswami.
Suntharampillai, Appiah, B.Sc.
Sutton, Ronald James.
Sutton, William Harry.
Talbot, William Allan D.
Tapsfield, Harold Arthur.
Tattersall, Frank Reuss.
Taylor, Noel Hugh A.
Telford, John Weston S.
Tew, John Maxwell.
Thomas, Alfred David, B.Sc.
Thomas, Christopher William.
Thompson, Arthur D'Arcy.
Thompson, Geoffrey Peirce.
Thorley, Kenneth John W.
Tikku, Pratap Krishna.
Tinson, Leonard Percy.
Tobin, Anthony Glennon.
Tonge, Geoffrey Arthur M.
Townsend, Malcolm William H.
Tritton, Harold Percy.
Trundle, Joseph Owen.
Tucker, Victor Edwin.
Turner, Ronald Charles.
Tweedy, Ernest Paterson.
Tye, Ronald Lawrence M.
Vale, Ivan Leith.
Varma, Ram Saran.
Venkatesan, Conjeeveram Vedachala.
Verma, Bishwambher Prasad.
Vernon, Thomas Victor.
Viswanathan, Madras Natesa S.
Vosper, John Squire.
Walker, Geoffrey Reginald.
Walker, Peter Howe.
Walters, Ronald Charles.
Ward, George Melvin.
Warmsley, William James.
Warwick, Edmund Vernon.
Watson, Cyril Edwin.
Watts, Victor John.
Westgate, Robert George.
White, Douglas Edwin.
White, Roye Fred Le G.
Williams, Leonard.

Students—continued.

Whyman, Arthur John.
Wilkinson, Frederic Leo, B.A.
Willis, Joseph Henry.
Wilson, Kenneth Bryan.
Winfield, Henry Charles.
Withers, Gerald Oliver.
Wodehouse, Patrick Armine.
Woodford, Eric Victor.
Woodgate, Frank Maxwell.
Woodrow, Albert John.
Woodward, Alan George V.
Worthington, John Malcolm.
Woolley, Donald Herbert.
Wright, John.
Zaazou, Abbas H. Abdel-Latif.

Transfers

Associate Member to Member

Barbour, Ralph Henry, M.A.
Bayley, Benjamin Croft, M.B.E.
Bell, Hugh Glover, M.Sc.Tech.
Chadwick, Albert Thomas.
Connell, Arthur Grevatt.
Dawson, Cecil.
Elliott, Norman Randall, M.A.
Gillett, James Keith, M.Sc.Tech.
Gomersall, William Charles.
Green, Ernest Hedley R., M.Sc.
Harkin, Anthony, M.E.
Harris, Harry Kingsford, M.A.
Hodgson, Alan D'Arcy.
Kirkwood, Ian Ward A.
Mann, Reginald William.
Murray, Richard.
Naismith, Robert.
Nixon, Leslie Reginald, M.Sc.(Eng.).
Orchard, Frederick Charles.
Pickworth, Cecil Henry.
Pooley, Leonard Arthur C.
Powell, Edward Blennerhassett S.
Priest, Charles Wilson A., B.Sc.(Eng.).
Schofield, John Ernest.
Somerville, Henry Benson, B.Eng.
Sundaram, Gangadhara.
Taplin, Keith William, B.E.
Townend, Robert.
Whitehead, Stanley, M.A., Ph.D.

Associate to Member

Oliver, Charles.

Associate to Associate Member

Bradley, Christopher.
Day, John Charles.
Gaunt, Gerald Lewis.
Honour, Cyril.
Negus, Edward Walter D.
Phillips, Alexander Stevenson.
Sinclair, George David.
Urquhart, Samuel Ramage.

Graduate to Associate Member

Amos, Wilfred Jack.
Anderson, Eric, B.Sc., Lieut.,R.E.
Ashe, Thomas Stuart, B.Sc.
Atkinson, William Ranson.
Barber, Bernard Davie.
Barfield, Turland John.
Barwell, Frederick Thomas, Ph.D., B.Sc.(Eng.).
Batty, Kenneth Thomas.
Bayley, John Magnier.
Bilimoria, Phiroze Dadyba, B.Sc.(Eng.).
Bristow, Kenneth Walter, B.Sc.(Eng.).
Burns, Dennis Owen, B.Sc. (Eng.).
Butler, William Edward, B.Sc.(Eng.).
Candler, John Lawrence, B.Sc.(Eng.).
Carey, Frederick Thomas.
Carr, Arthur Smith, B.Sc.
Collier, John Thomas.
Collins, John.
Combridge, John Hayden.
Cooper, Victor James, B.Sc.(Eng.).
Darby, William Joseph J.

Graduate to Associate Member—continued.

Dickens, Thomas Arthur J., B.Sc.
Dickinson, Frederick Harold, M.Eng.
Dixit, Vidya Nand, B.Eng.
d'Ombrain, George Lee, Ph.D., B.Sc.
D'Souza, Alfred.
Duncan, John Bernard.
Easton, William.
Elstob, Hedley Graham, B.Sc.(Eng.).
Fortescue, Richard Lewis, M.A.
Fraser, Sidney Robert, M.Eng.
Furneaux, Edwin George.
Garry, Francis Keith, B.E.
Greer, Percy Howard.
Gupta, Sudarshan Lal.
Gwyer, Ronald George B., M.A.
Hamnett, Leslie George.
Hebblethwaite, Arthur William.
Hill, Ernest.
Hume, John McIntosh.
Jennett, William Joseph, B.Sc.(Eng.).
Jordan, Arthur.
Khanna, Mangal Sen.
Kidd, William Leslie, B.Sc.Tech.
Marshall, William Leslie.
Mayne, Edward Adrian, B.Sc.(Eng.).
Midgley, Albert Morrell.
Mills, Cyril Ernest.
Moore, Sydney.
Morley, John Baillie, B.Sc.Tech.
Nye, Philip.
O'Kane, Bernard John, Ph.D., B.Eng.
Parr, George Herbert.
Pickup, Harry, B.Sc.Tech.
Pletts, John William, B.Sc.(Eng.).
Potts, Edward Carlton, B.Sc.
Pressey, Brion George, M.Sc.(Eng.).
Rao, Kurapaty, Venkata S., B.Sc.(Eng.).
Reveley, Paul Vernon, B.Sc.
Revell, Hedley James, B.Sc.(Eng.).
Rhys-Jones, John Emyr.
Roberts, George Albert.
Roberts, Walter George.
Robertson, David Campbell, B.Sc.(Eng.).
Roper, Robert Francis.
Roy, Philip Scott.
Savage, Arthur Noel.
Scoles, Graham John, B.Sc.(Eng.).
Scott, Dan.
Sharpe, Bernard Anselm, Ph.D., B.Eng.
Shone, Arthur Brian, B.Eng.
Smith, Edward Craig, Ph.D., B.Sc.(Eng.).
Smyth, Charles Norman, B.Sc.(Eng.).
Smyth, John Bernard S., B.Sc.(Eng.).
Statham, Cyril David J., Ph.D., B.Eng.
Swain, Edwin Charles, B.Sc.(Eng.).
Tongaonkar, Gopal Rangnath, B.Sc.(Eng.).
Turner, Wallace James.
Walker, Robert.
Wheatcroft, Eric Oscar, B.Sc.(Eng.).
White, Charles Henry.
Williams, Robert Charles G., Ph.D., B.Sc.(Eng.).
Woods, Oswald.
Wyman, Hubert Lionel D.

Student to Associate Member

Miller, Geoffrey Stuart, B.Sc.

From Student to Associate

Burnand, John William.

The following transfers were also effected by the Council at their meeting held on the 11th January, 1940:—

Student to Graduate

Agarwala, Gyan Prakash.
Bartle, Philip Walter, B.Sc.(Eng.).
Cowley, Percy Edward A., B.Sc.(Eng.).
Daver, Framroje Pirojshaw.
Davies, Alun Madoc, B.Sc. (Eng.).
Degerdon, Maurice Edward, B.Sc.(Eng.).
Dobson, Frank John B., B.Sc.(Eng.).
Eversfield, Frederick Furneaux.
Garfitt, Richard George, B.Sc.Tech.
Gower, Hedley John C.
Haywood, James Joseph H.
Hill, John Wreghitt.
Houchin, David Arthur R.
Howie, Robert Campbell, B.Sc.
Keele, Brian Rushworth.
McMillan, Charles Tennant.
Mainprise, Bertie Wilmot F., B.Sc.(Eng.).
Medlock, Edwin Roy, B.Sc.(Eng.).
Menty, Burjor Manchersha.
Moden, John Carol.
Mortimer, Kenneth, B.Sc. (Eng.).
Moss, Hugh MacLean, B.Sc.
Muire, Clement Napier.
Newman, Durnford Frederick W.
Newton, Frederick William.
Nicholson, John, B.Sc.
Payne, John James.
Roberts, Frank Woodward, B.Sc.(Eng.).
Rochester, John Charles O., B.Sc.
Sandercock, Kenneth Norman.
Scott, Bruce Glazier, B.Eng.
Taylor, John Johnson.
Yeo, Mervyn James.

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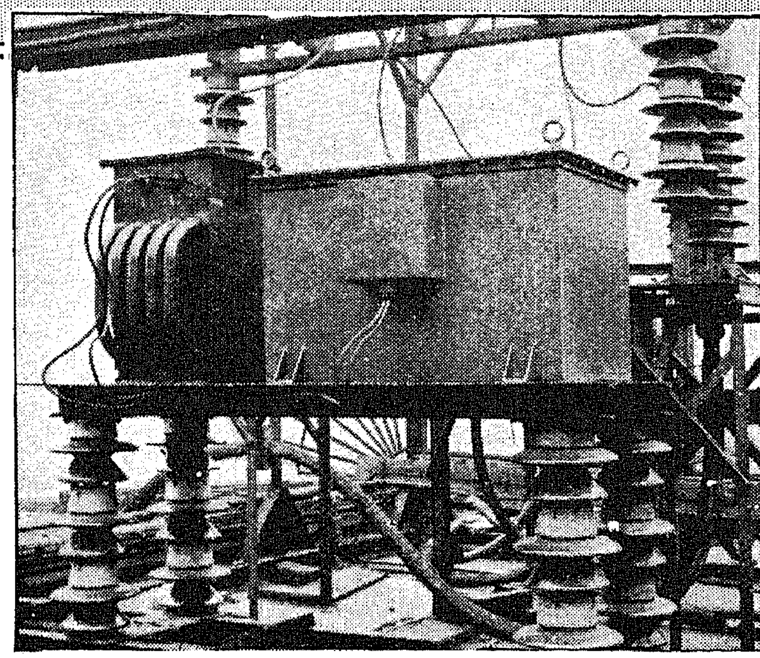

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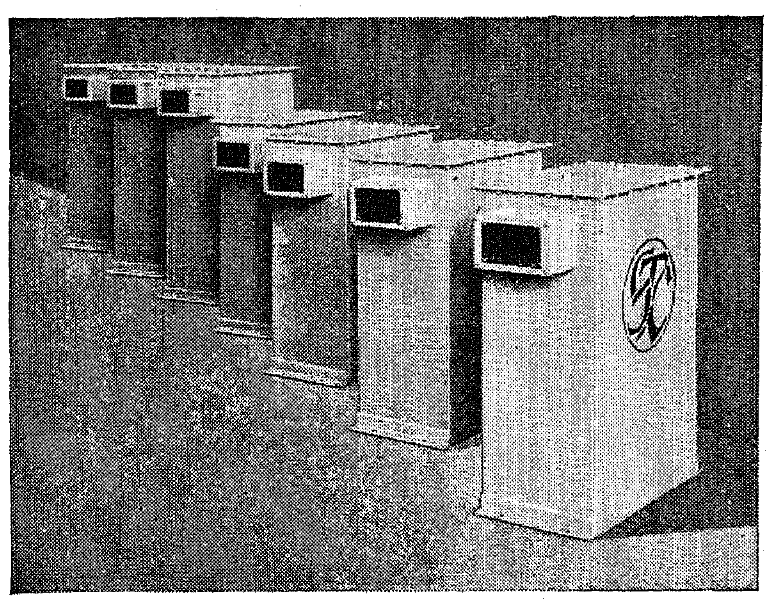

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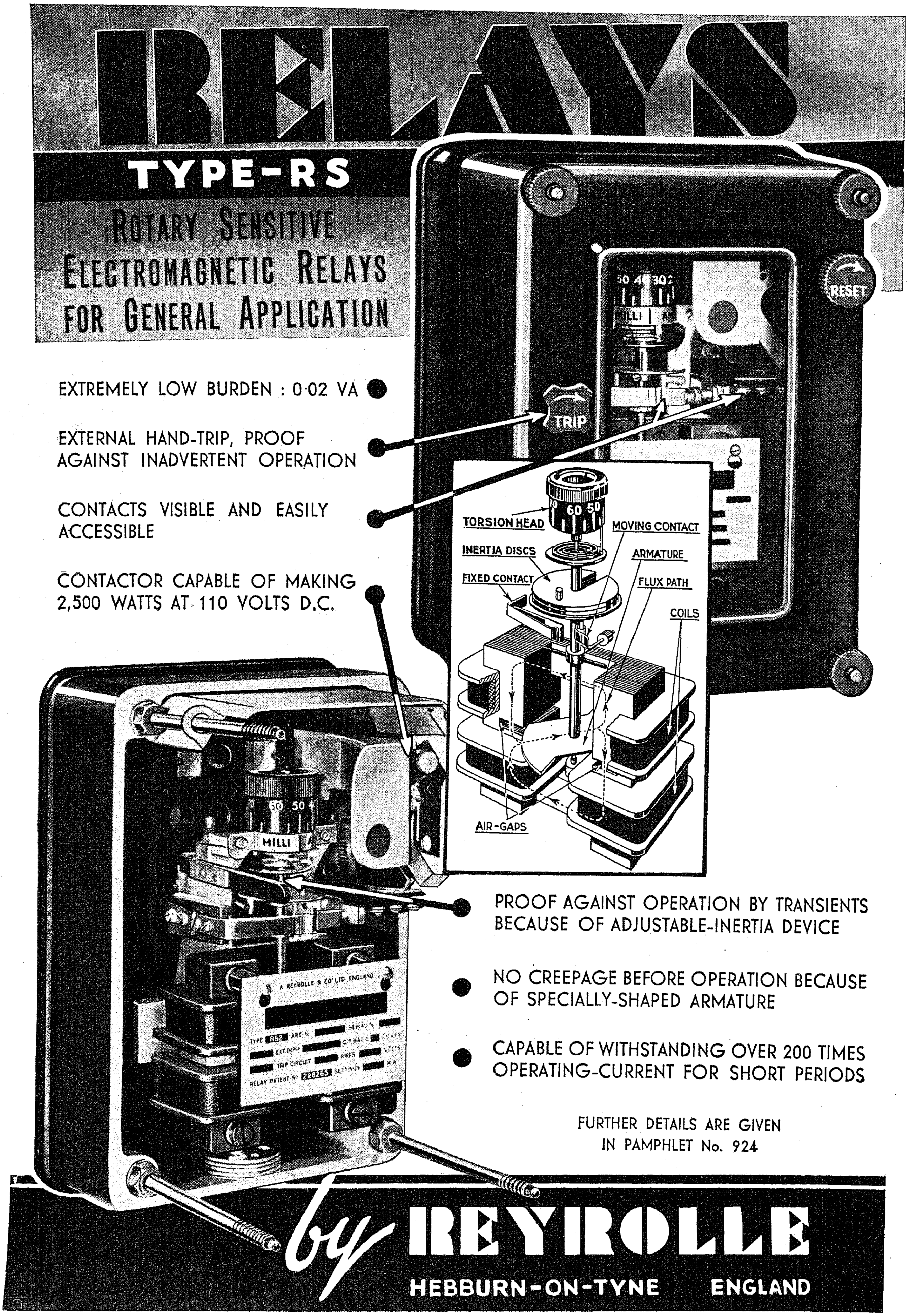
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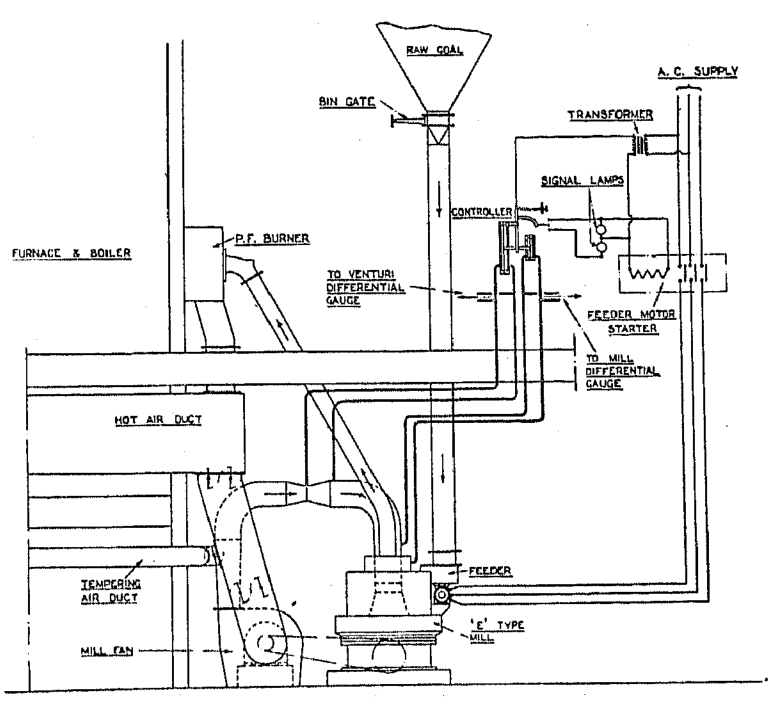

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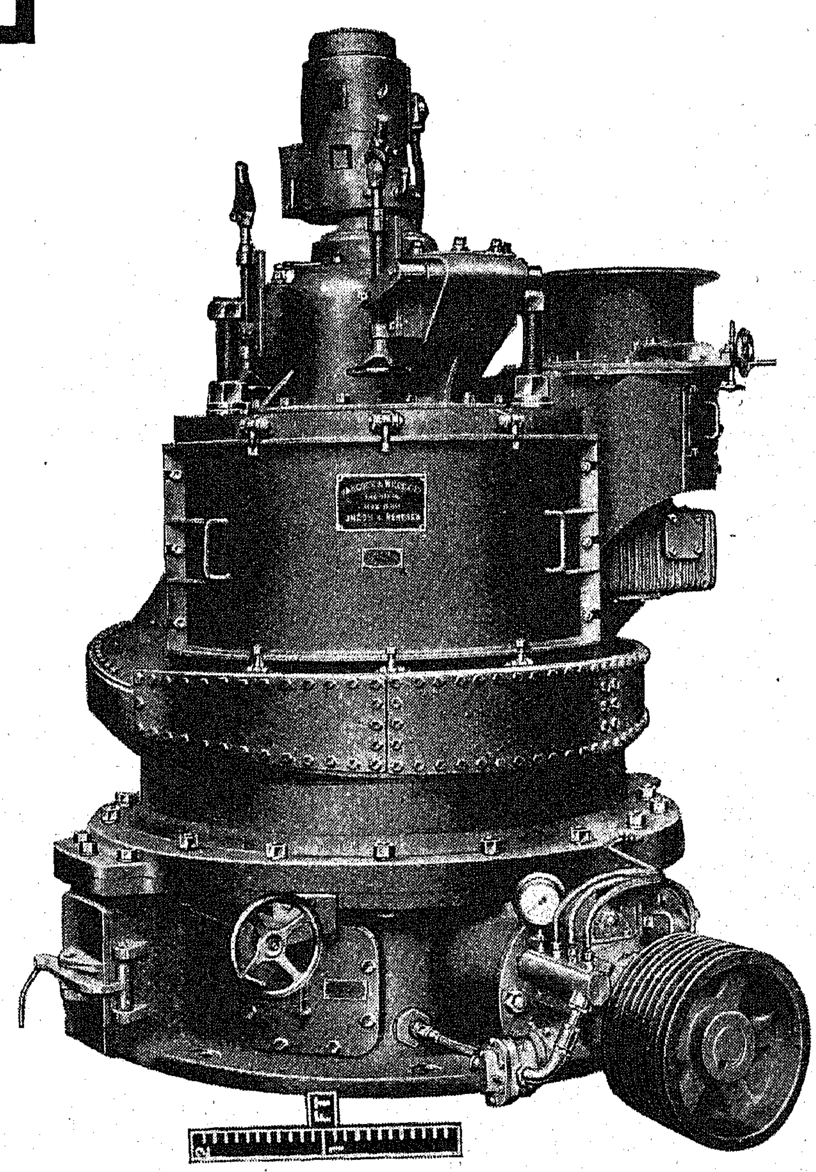

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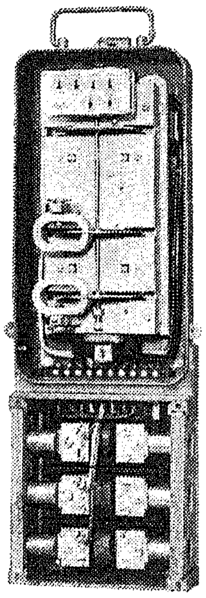

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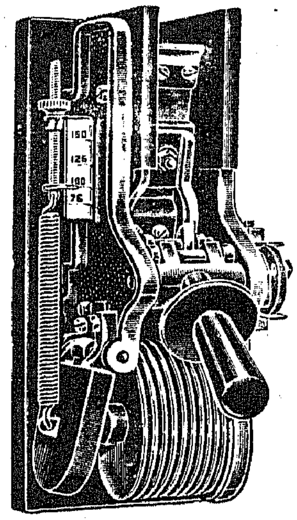

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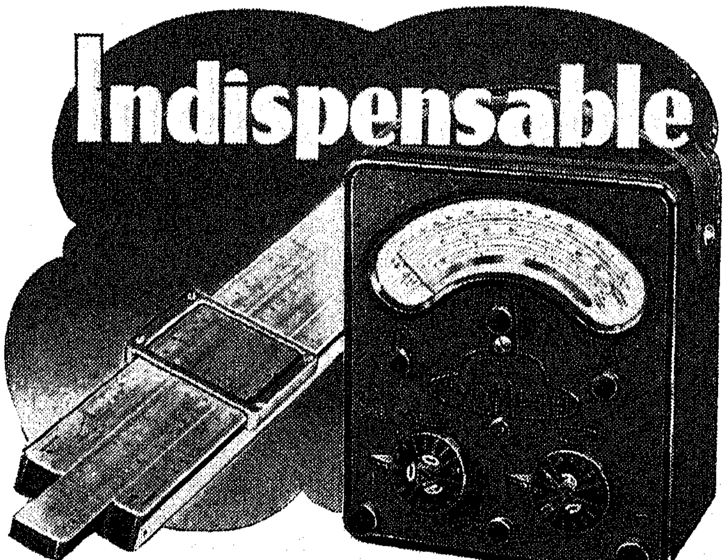

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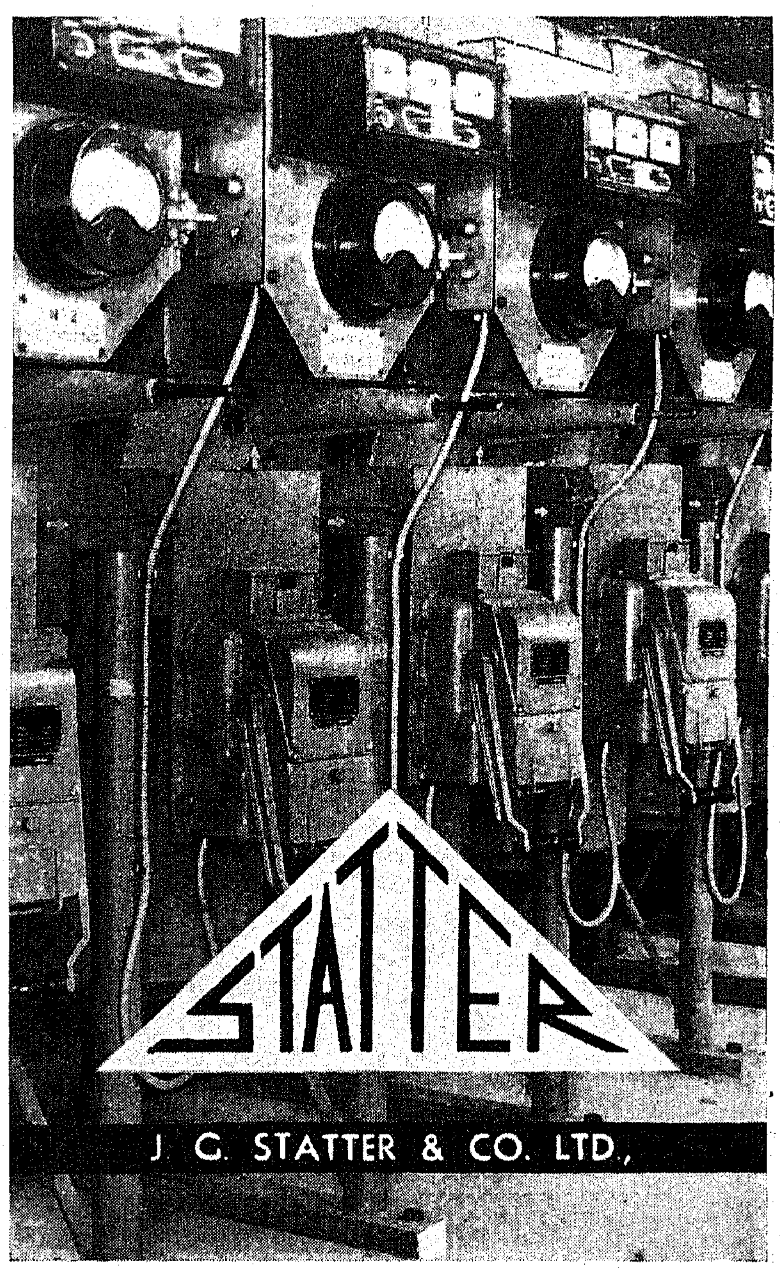

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